

COMPLETE SCHOOL ALGEBRA

HAWKES-LUBY-TOUTON

TEACHERS' EDITION

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Complete school algebra

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COMPLETE SCHOOL ALGEBRA

TEACHERS' EDITION

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BY

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PREFACE

The busy teacher of high-school algebra often finds that limitations of time forbid the actual working out of all the exercises and problems in a text. This teachers' edition is designed to meet such a condition. In its plan it is brief and schematic rather than complete and detailed.

Many exercises are so simple that the answers only are given in this teachers' edition. The solutions of numerous other exercises are complete in outline only. For this reason they should not be regarded as models of the form in which the pupil should present his work.

Circumstances may make imperative, and at times justify, the use of such a book as this; but continued reliance upon it, however, with class after class will result in teaching which is faulty and lacking both in freedom and life.

The authors and the publishers desire the coöperation of teachers in keeping this book out of the hands of pupils.

COMPLETE SCHOOL ALGEBRA

TEACHERS' EDITION

Page 1

1. $4h + 3m = 4(3600) + 3(60) = 14,580$ seconds.
2. $5y + 4f = 5(36) + 4(12) = 228$ inches.
3. $4q + 6d = 4(25) + 6(10) = 160$ cents.
4. $4t + 6h = 4(2000) + 6(100) = 8600$ pounds.
5. $3x + 5x = 8x$.
6. $4x + 5x = 9x$.
7. $2x + 3x + 6x = 11x$.
8. $2x + 2 + 3x + 4 = 5x + 6$.
9. $x + x + 2 + x + x + 2 = 4x + 4$.
10. $n + n + 1 + n + 2 = 3n + 3$.
11. $5a + 18 - 3a - 7 = 2a + 11$.
12. $8x - 3 + 18 - 5x = 3x + 15$.
13. $4w - 8 + 3w + 20 = 7w + 12$.
14. $y + 5m = 17m$.
15. $2y + 7f = 13f$.
16. $4q + 3n = 23n$.
17. $2d + 15h = 63h$.
18. $15h + 50m = 950m$.
19. Area, $5 \cdot 5 \cdot 1$ square inch = 25 square inches; perimeter, $4 \cdot 5$ inches = 20 inches.
20. Perimeter, $4s$ inches; area, $s \cdot s$ square inches.
21. Area, $12 \cdot 4 \cdot 1$ square foot = 48 square feet; perimeter, $2 \cdot 12$ feet + $2 \cdot 4$ feet = 32 feet.
22. Area, ab square feet; perimeter, $2a$ feet + $2b$ feet = $(2a + 2b)$ feet.
23. (a) $2x$ inches; (b) $2 \cdot x + 2 \cdot 2x = 6x$ inches; (c) $2x \cdot x$ square inches.
24. $3 \cdot y$ years.
25. $(30 + y)$ years.
26. Length, $(12 + w)$ feet; perimeter, $w + w + 12 + w + 12 + w = (24 + 4w)$ feet.
27. Length, $(18 + w)$ feet; perimeter, $w + w + 18 + w + 18 + w = (36 + 4w)$ feet.
28. Let w feet = the width.
Then $2w + 4$ feet = the length,
and the perimeter = $w + w + 2w + 4 + 2w + 4$,
= $(6w + 8)$ feet.

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1. Greater number + less number = 160.

If we represent the less by l , then $4l$ must represent the greater, and the above statement becomes

$$4l + l = 160.$$

Whence

$$l = 32,$$

and

$$4l = 128.$$

Therefore the greater number is 128 and the less is 32.

2. The number + five times the number = 216.

If we represent the number by n , the above statement becomes

$$n + 5n = 216,$$

or

$$6n = 216.$$

Whence

$$n = 36.$$

3. Greater number + less number = 72.

If we represent the less by l , then $7l$ must represent the greater, and the above statement becomes

$$7l + l = 72.$$

Whence

$$l = 9,$$

and

$$7l = 63.$$

4. 1st number + 2d number + 3d number = 105.

If we let t represent the 3d number, $2t$ must represent the 1st number and $4t$ the 2d. Then the above statement becomes

$$2t + 4t + t = 105.$$

Whence

$$t = 15,$$

$$2t = 30,$$

and

$$4t = 60.$$

5. 1st number + 2d number + 3d number = 117.

If we represent the 1st number by n , $2n$ must represent the 2d number and $6n$ the 3d. Then the above statement becomes

$$n + 2n + 6n = 117.$$

Whence

$$n = 13,$$

$$2n = 26,$$

and

$$6n = 78.$$

6. 1st number + 2d number + 3d number = 192.

If we represent the 2d number by n , $2n$ must represent the 1st number and $3n$ the 3d. Then the above statement becomes

$$2n + n + 3n = 192.$$

Whence

$$n = 32,$$

$$2n = 64,$$

and

$$3n = 96.$$

7. 1st number + 2d number + 3d number = 324.

If we represent the 1st number by n , $5n$ must represent the 2d number and $30n$ the 3d. Then the above statement becomes

$$n + 5n + 30n = 324.$$

Whence $n = 9$,

$$5n = 45,$$

and $30n = 270.$

8. 1st number + 2d number + 3d number = 104.

If we represent the 1st number by n , $3n$ must represent the 2d number and $4n$ the 3d. Then the above statement becomes

$$n + 3n + 4n = 104.$$

Whence $n = 13$,

$$3n = 39,$$

and $4n = 52.$

10. From arithmetic, principal + interest = amount.

$$\text{Principal} + .06 \text{ principal} = \$265.$$

If p represents the principal, this last statement becomes

$$p + .06p = 265.$$

Whence $p = 250.$

Therefore the sum is \$250.

11. Principal + .12 principal = \$700.

If p represents the principal, the above statement becomes

$$p + .12p = 700.$$

Whence $p = 625.$

Therefore the required sum is \$625.

12. $225 \times .06 \cdot (\text{number of years}) = 27.$

If we represent the number of years by n , this statement becomes

$$225 \times .06 \cdot n = 27,$$

or $13.5n = 27.$

Whence $n = 2.$

13. $520 \times .065 \cdot (\text{number of years}) = 169.$

If n represents the number of years, this statement becomes

$$520 \times .065 \cdot n = 169,$$

or $33.8n = 169.$

Whence $n = 5.$

15. Let $x =$ the rate of interest.

Then $\$825x =$ the interest for one year,

and $\$3300x =$ the interest for four years.

But $\$165 =$ the interest for four years.
 Whence $3300x = 165$,
 and $x = \frac{1}{20}$.
 Therefore the money is lent at 5%.

16. Let $x =$ the rate of interest.
 Then $\$250x =$ the interest for one year,
 and $\$1500x =$ the interest for six years.
 But $\$317.50 - \$250$, or $\$67.50 =$ the interest for six years.
 Whence $1500x = 67.50$,
 and $x = .045$.
 Therefore the money is lent at $4\frac{1}{2}\%$.

17. Let $x =$ the number of years.
 Then $\$8x =$ the interest at 4%.
 But $\$200 =$ the interest at 4%.
 Whence $8x = 200$,
 and $x = 25$.
 Therefore the time required is 25 years.

18. $\$150 \times .05 \cdot (\text{number of years}) = \$450 - \$150$.

If we represent the number of years by n , this statement becomes

$$3.50n = 300.$$

Whence $n = 40$.

Therefore the required time is 40 years.

19. Let $s =$ a side of the square in feet.
 Then $4s =$ the perimeter, or 160 feet.
 Whence $4s = 160$,
 and $s = 40$.
 Therefore the square is 40 feet on a side.

20. Let $s =$ the number of feet in the width of the rectangle.
 Then $3s =$ the number of feet in the length of the rectangle,
 and $2s + 2 \cdot 3s$, or $8s =$ the number of feet in the perimeter of the rectangle, or 256 feet.
 Whence $8s = 256$.
 Therefore $s = 32$,
 and $3s = 96$.

21. Let $s =$ the side of either square in inches.
 Then $2s =$ the length of the rectangle in inches,

and $2 \cdot s + 2 \cdot 2s$, or $6s =$ the perimeter of the rectangle, or 198 inches.

Therefore $6s = 198$.

Whence $s = 33$,

and $4s = 132$, the perimeter.

22. Let, $x =$ the side of each square in inches.

Then $4x =$ the perimeter of each square, or 120 inches.

Therefore $4x = 120$.

Whence $x = 30$,

and $6x = 180$, the perimeter.

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1. $3a + 5b$.

2. $5b - 3a$.

3. $b^2 - a^2$.

4. $3a^2 - 2a^2$.

5. $\frac{a}{b}$.

6. $4a^2b$.

7. $\frac{a+b}{a-b}$, or $\frac{a+b}{b-a}$.

8. $a(2b - c)$.

9. $a(b + c)$.

10. $7x - (a - b)$.

11. $(a + b)(a - b)$,
or $(a + b)(b - a)$.

12. $\sqrt{5a} + \sqrt[3]{7b}$.

13. $\sqrt{7x}(x - y)$.

14. $(a + b)^2$.

15. $(a - b)^2$, or $(b - a)^2$.

16. $\frac{3ab^2}{4ca^3}$.

17. $\frac{a}{3x} + \frac{4y}{c}$.

Page 10

1. $20 - 5 + 6 - 10 = 15 + 6 - 10 = 21 - 10 = 11$.

2. $16 - (8 - 2) = 16 - 6 = 10$.

3. $14 - (16 - 8) + (12 - 4) = 14 - 8 + 8 = 14$.

4. $6 \div 3 - 2 = 2 - 2 = 0$.

5. $8 \cdot 6 \div 3 - 10 = 48 \div 3 - 10 = 16 - 10 = 6$.

6. $18 \div (2 \cdot 3) = 18 \div 6 = 3$.

7. $(6 - 3)(17 - 2 \cdot 5) = 3(17 - 10) = 21$.

8. $23 - 2 \cdot 6 - 4 \div 2 + 16 = 23 - 12 - 2 + 16 = 11 + 14 = 25$.

9. $18 \div (9 - 3) = 18 \div 6 = 3$.

10. $(10 - 3)(16 - 3 \cdot 2 + 8 \div 4) = 7(16 - 6 + 2) = 7 \cdot 12 = 84$.

11. $14 - 3 \cdot (16 - 2 \cdot 5) \div 6 + 8 \cdot 2 = 14 - (16 - 10) \div 2 + 16 = 30 - 3 = 27$.

12. $(18 - 2) - 6 \div (4 + 2 \cdot 8 - 18 \div 9) = 16 - 6 \div (4 + 16 - 2) = 16 - 6 \div 18 = 15\frac{2}{3}$.

13. $(16 - 6)(18 - 8) \div 100 \cdot 5 - 5 = 10 \cdot 10 \div 100 \cdot 5 - 5 = 5 - 5 = 0$.

14. $(5 + 3)(5 - 3) \div 5 - 3 = 8 \cdot 2 \div 5 - 3 = 3\frac{1}{5} - 3 = \frac{1}{5}$.

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If $a = 3$, $b = 1$, $c = 5$, $d = 7$, and $e = 2$:

1. $4a + 3d = 4 \cdot 3 + 3 \cdot 7 = 33$.

2. $a^2 + 2c = 3^2 + 2 \cdot 5 = 19$.

3. $ab + cd = 3 \cdot 1 + 5 \cdot 7 = 38$.

4. $c^2 - 5ab = 25 - 5 \cdot 3 \cdot 1 = 10$.

5. $abcd - 4e^2 = 3 \cdot 1 \cdot 5 \cdot 7 - 4 \cdot 4 = 89.$
6. $\frac{d+c}{2} = \frac{7+5}{2} = 6.$
7. $\frac{cd\cancel{e}}{5\cancel{e}} + \frac{\cancel{d}ce}{2\cancel{d}} = \frac{5 \cdot 7}{5} + \frac{5 \cdot 2}{2} = 12.$
8. $\frac{10b}{e} - \frac{c}{b} = \frac{10 \cdot 1}{2} - \frac{5}{1} = 0.$
9. $\frac{1}{c} + \frac{1}{d} + \frac{1}{e} = \frac{1}{5} + \frac{1}{7} + \frac{1}{2} = \frac{14}{70} + \frac{10}{70} + \frac{35}{70} = \frac{59}{70}.$
10. $\frac{e^2}{ac} + \frac{4cd}{ae} = \frac{4}{3 \cdot 5} + \frac{4 \cdot 5 \cdot 7}{3 \cdot 2} = \frac{4}{15} + \frac{70}{3} = \frac{354}{15} = 23\frac{3}{5}.$
11. $\frac{cd + ae + ce}{a - b + c} = \frac{35 + 6 + 10}{3 - 1 + 5} = \frac{51}{7} = 7\frac{2}{7}.$
12. $\frac{a^2 + b^2 + c^2 + d^2}{a + b + c + d} = \frac{9 + 1 + 25 + 49}{3 + 1 + 5 + 7} = \frac{84}{16} = 5\frac{1}{4}.$
13. $\frac{a^3 - ce^2 + 3cd}{d - a + c} = \frac{27 - 5 \cdot 4 + 3 \cdot 35}{7 - 3 + 5} = \frac{112}{9} = 12\frac{4}{9}.$
14. $5b^3 + 4b^2 - 2b - 5 = 5 \cdot 1 + 4 \cdot 1 - 2 \cdot 1 - 5 = 9 - 7 = 2.$
15. $3b^5 - 14b^4 + 11b^3 + 11b^2 + 13b - 20 = 3 - 14 + 11 + 11 + 13 - 20 = 4.$
16. $a^e = 3^2 = 9.$
17. $c^a = 5^3 = 125.$
18. $d^e + e^a = 7^2 + 2^3 = 57.$
19. $b^c + c^a = 1^5 + 5^3 = 126.$
20. $d^e - c^2 + b^3 = 7^2 - 25 + 1 = 25.$
21. $e^2 \cdot c^a = 4 \cdot 5^3 = 500.$
22. $\frac{d + e^c}{2c + a} = \frac{7 + 2^5}{2 \cdot 5 + 3} = \frac{7 + 32}{13} = 3.$
23. $\frac{a^a + 3b}{c^e - d - e - b} = \frac{3^3 + 3 \cdot 1}{5^2 - 7 - 2 - 1} = \frac{30}{25 - 10} = 2.$
- If $a = 4, b = 0, c = 5, d = 7,$ and $e = 8:$
24. $\frac{4a + 3b + 2d}{c + e + 2} = \frac{4 \cdot 4 + 3 \cdot 0 + 2 \cdot 7}{5 + 8 + 2} = \frac{16 + 14}{15} = 2.$
25. $\frac{b}{a + c + d} = \frac{0}{4 + 5 + 7} = 0.$
26. $abc + acd - be = 4 \cdot 0 \cdot 5 + 4 \cdot 5 \cdot 7 - 0 \cdot 8 = 140.$
27. $\frac{ab}{c} + \frac{bd}{a} + \frac{be}{cd} = \frac{0}{5} + \frac{0}{4} + \frac{0}{35} = 0.$
28. $\frac{a^2 - b^2 + c^3}{3d - 2e + c} = \frac{16 - 0 + 125}{21 - 16 + 5} = 14\frac{1}{10}.$
29. $\sqrt{a} + \sqrt{2e} = \sqrt{4} + \sqrt{16} = 6.$
30. $2\sqrt{a} + \sqrt[3]{e} = 2\sqrt{4} + \sqrt[3]{8} = 6.$
31. $b\sqrt{a} + cd + \sqrt[3]{e} \cdot \sqrt{a} = 0 + 35 + 2 \cdot 2 = 39.$

32. $c \sqrt[3]{2ae} + d \sqrt[3]{2ae} = 5 \sqrt[3]{2 \cdot 4 \cdot 8} + 7 \sqrt[3]{2 \cdot 4 \cdot 8} = 40 + 28 = 68.$
33. $\sqrt{b^2 + c^2 + d + a} = \sqrt{0 + 25 + 7 + 4} = 6.$
34. $\sqrt[3]{3d + ec + a - 1} = \sqrt[3]{21 + 40 + 4 - 1} = 4.$
35. $de + ac \sqrt{ac^2 + 2e + c} = 56 + 20 \sqrt{100 + 16 + 5} = 56 + 220 = 276.$
36. $(a + c)c = (4 + 5)5 = 45.$
37. $(a + e)(c + d) = (4 + 8)(5 + 7) = 144.$
38. $ab(a + b) = 4 \cdot 0(4 + 0) = 0.$
39. $acd(a + c + d) = 4 \cdot 5 \cdot 7(4 + 5 + 7) = 140 \cdot 16 = 2240.$
40. $ac(e - c + b + d) = 4 \cdot 5(8 - 5 + 0 + 7) = 20 \cdot 10 = 200.$
41. $(a + d)^2 = (4 + 7)^2 = 121.$
42. $(e - a)^2 = (8 - 4)^2 = 16.$
43. $(d - a)^3 = (7 - 4)^3 = 27.$
44. $\frac{3ad(3a - 2c)^2(e - c)^3}{6ed} = \frac{3 \cdot 4 \cdot 7(12 - 10)^2(8 - 5)^3}{3 \cdot 8 \cdot 7} = \frac{4 \cdot 27}{4} = 27.$
45. $\frac{ab(c + d)(a + b)}{(e - a)} = \frac{4 \cdot 0(5 + 7)(4 + 0)}{(8 - 4)} = 0.$
46. $c^3 - 3c^2a + 3ca^2 - a^3 = 125 - 3 \cdot 25 \cdot 4 + 3 \cdot 5 \cdot 16 - 64 = 1.$
47. If $x = 2$ and $y = 3$, $3x + 5y = 21$ becomes $3 \cdot 2 + 5 \cdot 3 = 21$. Yes.
48. If $x = 8$, $7x - 9 = 3x + 25$ becomes $7 \cdot 8 - 9 = 3 \cdot 8 + 25$, or $47 = 49$, which is false. No.
49. If $x = 2$, $x^2 + 5x + 6 = 0$ becomes $4 + 10 + 6 = 0$, which is false. No.
If $x = 3$, it becomes $9 + 15 + 6 = 0$, which is false. No.
If $x = 4$, it becomes $16 + 20 + 6 = 0$, which is false. No.
50. If $x = 5$, $3x^2 - 14x - 5 = 0$ becomes $3 \cdot 25 - 14 \cdot 5 - 5 = 0$. Yes.
If $x = \frac{1}{3}$, it becomes $3 \cdot \frac{1}{9} - \frac{14}{3} - 5 = 0$, which is false. No.
If $x = 6$, it becomes $3 \cdot 36 - 14 \cdot 6 - 5 = 0$, which is false. No.
If $x = 4$, it becomes $3 \cdot 16 - 14 \cdot 4 - 5 = 0$, which is false. No.

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|------------------|-------------------|--------------------|
| 1. $4 + 2 = 6.$ | 5. $-4 + 2 = -2.$ | 9. $4 - 6 = -2.$ |
| 2. $-2 + 4 = 2.$ | 6. $-7 + 5 = -2.$ | 10. $-2 - 4 = -6.$ |
| 3. $-3 + 5 = 2.$ | 7. $5 - 2 = 3.$ | 11. $-4 - 3 = -7.$ |
| 4. $-3 + 3 = 0.$ | 8. $2 - 5 = -3.$ | 12. $-3 - 2 = -5.$ |

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|--|---|
| 1. (a) $10^\circ - 5^\circ = 5^\circ$; (b) $10^\circ - 10^\circ = 0^\circ$; (c) $10^\circ - 18^\circ = -8^\circ.$ | |
| 2. (a) $-12^\circ + 7^\circ = -5^\circ$;
(b) $-12^\circ + 12^\circ = 0^\circ$;
(c) $-12^\circ + 25^\circ = +13^\circ.$ | 3. (a) $-12^\circ - 7^\circ = -19^\circ$;
(b) $-12^\circ - 12^\circ = -24^\circ$;
(c) $-12^\circ - 25^\circ = -37^\circ.$ |

4. $13^\circ + 7^\circ = 20^\circ$ or 69 miles $\cdot 20 = 1380$ miles.

5. $20^\circ - 4^\circ = 16^\circ$, at end of first day.

$20^\circ - 2 \cdot 4^\circ = 12^\circ$, at end of second day.

$20^\circ - 3 \cdot 4^\circ = 8^\circ$, at end of third day.

$20^\circ - 4 \cdot 4^\circ = 4^\circ$, at end of fourth day.

$20^\circ - 5 \cdot 4^\circ = 0^\circ$, at end of fifth day.

$20^\circ - 6 \cdot 4^\circ = -4^\circ$, at end of sixth day.

$$\frac{20 + 16}{4} = 9 \text{ days.}$$

6. (a) Property = + \$5200.

Debts = - \$2300.

(b) Standing = \$5200 - \$2300 = + \$2900.

7. (a) Property = + \$2300.

Debts = - \$5200.

(b) Standing = \$2300 - \$5200 = - \$2900.

8. $-12^\circ + 3^\circ \cdot 3 = -3^\circ$ at 9.00 A.M.

$-12^\circ + 3^\circ \cdot 4 = 0^\circ$ at 10.00 A.M.

$-12^\circ + 3^\circ \cdot 7 = +9^\circ$ at 1.00 P.M.

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- | | | |
|-----------------------|------------------------|------------------------|
| 1. $+3 + (+2) = +5$. | 5. $7 + (-4) = +3$. | 9. $4 + (-7) = -3$. |
| 2. $-3 + (-2) = -5$. | 6. $-7 + (+5) = -2$. | 10. $-3 + (-6) = -9$. |
| 3. $+3 + (-2) = +1$. | 7. $-6 + (-4) = -10$. | |
| 4. $-3 + (+2) = -1$. | 8. $6 + (-5) = +1$. | |

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|-------------------------------------|-------------------------------|
| 1. $+4 + (+7) = +11$. | 6. $-8 + (+5) = -3$. |
| 2. $-4 + (-7) = -11$. | 7. $-12 + (-9) = -21$. |
| 3. $+4 + (-7) = -3$. | 8. $-6 + 6 = 0$. |
| 4. $-4 + (+7) = +3$. | 9. $-6 + (-6) = -12$. |
| 5. $+8 + (-5) = +3$. | 10. $-4 + (+3) + (+6) = +5$. |
| 11. $3 + (-7) + (5) + (-4) = -3$. | |
| 12. $8 + (-2) + (-4) + (+6) = +8$. | |
| 13. $6 + (+3) = 9$. | 18. $-8 + (+2) = -6$. |
| 14. $6 + (-4) = 2$. | 19. $-10 + (-6) = -16$. |
| 15. $8 + (+4) = 12$. | 20. $-10 + (+17) = 7$. |
| 16. $8 + (-4) = 4$. | 21. $12 + (-8) = 4$. |
| 17. $-8 + (-2) = -10$. | 22. $-12 + (+16) = 4$. |

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Subtracting:

1. $8 - 5 = 3.$
2. $13 - 9 = 4.$
3. $5 - 8 = -3.$
4. $9 - 13 = -4.$
5. $-8 - (-5) = -3.$
6. $8 - (-5) = 13.$
7. $-10 - 5 = -15.$
8. $-4 - (-6) = 2.$
9. $-18 - 12 = -30.$
10. $+13 - 25 = -12.$

Changing signs and adding:

11. $8 + (-5) = 3.$
- $13 + (-9) = 4.$
- $5 + (-8) = -3.$
- $9 + (-13) = -4.$
- $-8 + (+5) = -3.$
- $8 + (+5) = 13.$
- $-10 + (-5) = -15.$
- $-4 + (+6) = 2.$
- $-18 + (-12) = -30.$
- $13 + (-25) = -12.$

Yes. The answers are the same as those obtained before.

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1. $8 - (+5) = 8 + (-5) = 3.$
2. $+8 - (-5) = 8 + (+5) = 13.$
3. $+5 - (-8) = 5 + (+8) = 13.$
4. $+5 - (+8) = 5 + (-8) = -3.$
5. $12 - (+9) = 12 + (-9) = 3.$
6. $12 - (-9) = 12 + (+9) = 21.$
7. $-12 - (9) = -12 + (-9) = -21.$
8. $-12 - (-9) = -12 + (9) = -3.$
9. $+6 - (+6) = 6 + (-6) = 0.$
10. $-6 - (-6) = -6 + (+6) = 0.$
11. $6 - (-6) = 6 + (6) = 12.$
12. $-6 - (+6) = -6 + (-6) = -12.$
13. $-14 - (-19) = -14 + (19) = 5.$
14. $0 - (+1) = 0 + (-1) = -1.$
15. $-0 - (-1) = +(+1) = 1.$
16. $1 - (-2) = 1 + (+2) = 3.$
17. $12 - (+3) - (+2) = 12 + (-3) + (-2) = 7.$
18. $-12 - (+3) - (+2) = -12 + (-3) + (-2) = -17.$
19. $-12 - (-3) - (-2) = -12 + (3) + (2) = -7.$
20. $18 - (-5) - (7) = 18 + (5) + (-7) = 16.$
21. $+6 + 4 = 10.$
22. $-6 + (-4) = -10.$
23. $-3 + 3 = 0.$
24. $+6 + (-6) = 0.$
25. $+6 + (-2) = 4.$
26. $-8 + 5 = -3.$
27. $-8 + 3 = -5.$
28. $-7 + 14 = 7.$
29. $9 - 6 = 3.$
30. $-7 - (-2) = -5.$
31. $+5 - 11 = -6.$
32. $-7 - (-11) = 4.$
33. $-5 - (-5) = 0.$
34. $2 - 2 = 0.$
35. $4 - (-14) = 18.$
36. $12 + (3) - (5) = 12 + (3) + (-5) = 10.$
37. $12 - (-3) + 6 = 12 + (3) + 6 = 21.$
38. $12 - (-4) + (-6) = 12 + (4) + (-6) = 10.$
39. $18 + (-6) - (+7) = 12 + (-7) = 5.$
40. $-16 + (-10) - (+11) = -26 + (-11) = -37.$
41. $-13 - (8) + (-14) = -27 + (-8) = -35.$

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- | | |
|----------------------------|-------------------------------------|
| 1. $+3 \cdot +4 = +12$. | 11. $+0 \cdot +4 = 0$. |
| 2. $+4 \cdot +12 = +48$. | 12. $-7 \cdot 0 = 0$. |
| 3. $-5 \cdot +4 = -20$. | 13. $+4 \cdot -5 \cdot +6 = -120$. |
| 4. $+6 \cdot -6 = -36$. | 14. $+4 \cdot -5 \cdot -6 = +120$. |
| 5. $-7 \cdot +8 = -56$. | 15. $-4 \cdot -5 \cdot -6 = -120$. |
| 6. $-7 \cdot -3 = +21$. | 16. $12 \cdot +0 \cdot -5 = 0$. |
| 7. $-12 \cdot +9 = -108$. | 17. $9 \cdot -10 \cdot -0 = 0$. |
| 8. $+6 \cdot -4 = -24$. | 18. $-4 \cdot +3 \cdot -6 = +72$. |
| 9. $-6 \cdot -6 = +36$. | 19. $-3 \cdot -2 \cdot -5 = -30$. |
| 10. $+5 \cdot -10 = -50$. | 20. $2 \cdot -3 \cdot +5 = -30$. |

Page 23 (First set)

- | | |
|----------------------------------|----------------------------------|
| 1. $+10 \div +2 = +5$. | 12. $+96 \div -6 \div +4 = -4$. |
| 2. $-10 \div -2 = +5$. | 13. $72 \div +9 \div -4 = -2$. |
| 3. $-15 \div +3 = -5$. | 14. $60 \div -5 \div -12 = 1$. |
| 4. $+14 \div -7 = -2$. | 15. $48 \div +3 \div -4 = -4$. |
| 5. $-18 \div -2 = +9$. | 16. $\frac{-10}{-5} = 2$. |
| 6. $-7 \div +7 = -1$. | 17. $\frac{-12}{6} = -2$. |
| 7. $0 \div +5 = 0$. | 18. $\frac{+16}{+2} = 8$. |
| 8. $0 \div -5 = 0$. | |
| 9. $+18 \div +3 \div -2 = -3$. | |
| 10. $+45 \div -5 \div -3 = 3$. | |
| 11. $-64 \div +8 \div -2 = +4$. | |

Page 23 (Second set)

- | | |
|--------------------------|---------------------------|
| 1. $(7) + (5) = 12$. | 16. $+7 - 0 = 7$. |
| 2. $(7) - (5) = 2$. | 17. $-0 - 3 = -3$. |
| 3. $(7) + (-5) = 2$. | 18. $(-3)(6) = -18$. |
| 4. $(7) - (-5) = 12$. | 19. $(-5)6 = -30$. |
| 5. $(-7) + (5) = -2$. | 20. $(7)(-5) = -35$. |
| 6. $-7 + 5 = -2$. | 21. $8(-3) = -24$. |
| 7. $(-9) - (4) = -13$. | 22. $(-5)(-12) = 60$. |
| 8. $-9 - 4 = -13$. | 23. $-3(-8) = 24$. |
| 9. $-11 + (-13) = -24$. | 24. $-5 \cdot 4 = -20$. |
| 10. $-6 - (-10) = 4$. | 25. $5 \cdot 0 = 0$. |
| 11. $-6 + 10 = 4$. | 26. $0 \cdot (-9) = 0$. |
| 12. $8 + (-10) = -2$. | 27. $4 \cdot 8 = 32$. |
| 13. $12 - 18 = -6$. | 28. $-3 \cdot 6 = -18$. |
| 14. $-18 - 12 = -30$. | 29. $12 \div (-2) = -6$. |
| 15. $15 - 14 = 1$. | 30. $-12 \div 2 = -6$. |

31. $-39 \div (-3) = 13.$

32. $45 \div (-15) = -3.$

33. $0 \div (-6) = 0.$

34. $0 \div 3 = 0.$

39. $2 - 3 + 4 - 5 - 6 = -8.$

40. $\begin{array}{r} 7 \\ -2 \\ 3 \\ -5 \\ \hline 3 \end{array}$

41. $\begin{array}{r} 6 \\ -2 \\ -3 \\ 4 \\ \hline 5 \end{array}$

42. $\begin{array}{r} -8 \\ 6 \\ 2 \\ -5 \\ \hline -5 \end{array}$

43. $\begin{array}{r} 4 \\ -9 \\ -3 \\ 6 \\ \hline -2 \end{array}$

44. $3 \cdot 6 \div 3 = 6.$

45. $-4(7) \div (-2) = 14.$

46. $3(-6) \div 2 = -9.$

47. $4 \cdot 6(-8) \div (-16) = 12.$

48. $18 \div (-3) \cdot 6 \div 4 = -9.$

54. $(-1)^2 + (-1)^3 + (-2)^2 + (-2)^3 = 1 - 1 + 4 - 8 = -4.$

55. $6 + 3 \cdot 2 + 18 \div (-3) = 6 + 6 - 6 = 6.$

56. $5^2 - 4 \div (-2) + 6(-3) = 25 + 2 - 18 = 9.$

57. $3 \cdot 6 \div 9 - 2 \cdot 6 \div 4 + (-3)^2 = 2 - 3 + 9 = 8.$

58. $6 + 6 \cdot 3^2 - 5^2 \cdot 2 \div 10 = 6 + 54 - 5 = 55.$

35. $-27 \div 9 = -3.$

36. $3 - 5 + 6 = 4.$

37. $-4 + 6 - 2 + 1 = 1.$

38. $-4 + 6 + 2 - 1 = 3.$

49. $3^2 = 9.$

50. $(-3)^2 = 9.$

51. $2^3 = 8.$

52. $(-2)^3 = -8.$

53. $(-4)^3 + 4^2 = -64 + 16 = -48.$

If $x = 3$ and $y = -2$:

59. $y^2 = 4.$

60. $y^3 = -8.$

61. $y^4 = 16.$

62. $y^5 = -32.$

63. $2y^2 = 8.$

64. $2y^3 = -16.$

65. $5x^2y^2 = 5 \cdot 9 \cdot 4 = 180.$

66. $4x^2y^4 = 4 \cdot 9 \cdot 16 = 576.$

67. $x^2 - y^2 = 9 - 4 = 5.$

68. $x^3 - y^3 = 27 - (-8) = 35.$

69. $x^2 + 2xy + y^2 = 9 + 2 \cdot 3(-2) + 4 = 1.$

70. $x^2 - 2xy + y^2 = 9 - 6(-2) + 4 = 25.$

71. $(x + y)(x - y) = 1 \cdot 5 = 5.$

72. $x^3 + 3x^2y + 3xy^2 + y^3 = 27 + 3 \cdot 9(-2) + 3 \cdot 3 \cdot 4 - 8 = 1.$

73. $y^3 - 3xy^2 + 3x^2y - x^3 = -8 - 3 \cdot 3 \cdot 4 + 3 \cdot 9(-2) - 27 = -125.$

74. If $x = 5$, $4x - 2 = 2x + 8$ becomes $4 \cdot 5 - 2 = 2 \cdot 5 + 8$, or $18 = 18$.
Yes.

75. If $x = -9$, $3x - 5 = 2x + 8$ becomes $-9 \cdot 3 - 5 = -9 \cdot 2 + 8$, but $-32 \neq -10$. No.

76. If $x = 4$, $x^2 - x - 12 = 0$ becomes $16 - 4 - 12 = 0$, or $0 = 0$. Yes.

If $x = -8$, it becomes $64 + 8 - 12 = 0$, but $60 \neq 0$. No.

If $x = -4$, it becomes $16 + 4 - 12 = 0$, but $8 \neq 0$. No.

77. If $x = \frac{2}{3}$, $3x^2 + 19x = 14$ becomes $3 \cdot \frac{4}{9} + \frac{38}{3} = 14$, or $14 = 14$. Yes.

If $x = 2$, it becomes $3 \cdot 4 + 2 \cdot 19 = 14$, but $50 \neq 14$. No.

If $x = -7$, it becomes $3 \cdot 49 - 7 \cdot 19 = 14$, or $14 = 14$. Yes.

78. $15^{\circ} - 3^{\circ} \cdot 2 = + 9^{\circ}$, at 9.00 A.M.

$15^{\circ} - 3^{\circ} \cdot 5 = 0^{\circ}$, at noon.

$15^{\circ} - 3^{\circ} \cdot 8 = - 9^{\circ}$, at 3.00 P.M.

79. Average height $= \frac{30 + (-7) + 18 + (-10) + 16}{5}$, or $9\frac{2}{5}$ feet.

The average height was $9\frac{2}{5}$ feet above the assumed level.

80. Mean height $= \frac{9\frac{2}{5} \cdot 5 + (-38)}{6}$, or $1\frac{1}{2}$ feet above the level.

81. Let $x =$ the height of the seventh point.

Then $\frac{1\frac{1}{2} \cdot 6 + x}{7} = 0$, or $x = -9$.

The last point was -9 feet high.

82. Euclid lived about -300 ; Sir Isaac Newton died in $+1727$.

83. The date -450 means 450 B.C.; the date $+1910$ means 1910 A.D.
The difference, $1910 - (-450)$, is 2360 years.

84. Rate upstream, $(8 - 1\frac{1}{2})$ miles per hour; rate downstream, $(8 + 1\frac{1}{2})$ miles per hour.

85. Rate toward bow, $(12 + 3)$ miles per hour; rate toward stern, $(12 - 3)$ miles per hour.

86. Balloon, -500 pounds; men, 1500 pounds; both, $[1500 + (-500)]$, or 1000 pounds.

87. (a) $(2 + 1\frac{1}{2})$ miles per hour; (b) $(2 - 1\frac{1}{2})$, or $-\frac{1}{2}$ mile per hour; i.e. he is being carried downstream $\frac{1}{2}$ mile per hour.

Page 28

1. $18 - 5 + 6 = 19$.
2. $18a - 5a + 6a = 19a$.
3. $12 - 7 + 3 + 4 = 12$.
4. $12a^2 - 7a^2 + 3a^2 + 4a^2 = 12a^2$.
5. $6 - 4 - 17 + 20 = 5$.
6. $6ab - 4ab - 17ab + 20ab = 5ab$.
7. $15 - 17 + 8 - 12 - 25 = -31$.
8. $8abc - 17abc - 4abc + 15abc - 12abc = -10abc$.
9. $11 + 5 - 9 - 3 + 16 - 25 = -5$.
10. $16ac - 9ac + 5ac - 2ac - 3ac + 11ac = 18ac$.
11. $7x + 4x - 15x - 8x + 3x = -9x$.
12. $12y - 17y + 10y + 20y - 25y = 0$.
13. $4xy - 8xy - 12xy + 13xy - xy = -4xy$.
14. $14x^2 - 13x^2 + x^2 - 5x^2 + 4x^2 = x^2$.
15. $5y^2 - 2y^2 - 11y^2 + y^2 - 7y^2 = -14y^2$.
16. $7a^2b - 5a^2b + 8a^2b - a^2b + 9a^2b = 18a^2b$.

Page 29

1. The sum of a , $3b$, and $-c = a + 3b - c$.
2. The sum of $4x$, $-2b$, $3y$, and $10 = 4x - 2b + 3y + 10$.
3. The sum of $3ab^2$, $2bx$, $-cy$, and $4a^2b = 3ab^2 + 2bx - cy + 4a^2b$.
4. The sum of $5x^3y$, $-5xy^3$, c^3y , and $-2cy^3$
 $= 5x^3y - 5xy^3 + c^3y - 2cy^3$.
5. The sum of $4x$, $-3a$, $2b$, $-5x$, and $3y = -x - 3a + 2b + 3y$.
6. The sum of $5a$, $-4b$, $+3c$, $6b$, and $-2c^2 = 5a + 2b + 3c - 2c^2$.
7. The sum of $3a^2$, $+2b^2$, $-5c^2$, $-4b^2$, and $5a^2 = 8a^2 - 2b^2 - 5c^2$.
8. The sum of $4a^3b$, $-4ab^3$, $-3a^2b$, $3ab^2$, $4a^3b$, and $2ab^3$
 $= 8a^3b - 2ab^3 - 3a^2b + 3ab^2$.
9. $-4a^2b + 6a^2b^2 - 15a^2b^2 + 3a^2b^2 + 0a^2b^2 = -4a^2b - 6a^2b^2$.
10. $-b^3 - 23b^3 + 17b^3 + b^3 - 0b^3 + 13b^3 = 7b^3$.
11. $12a^3b + 6a^3b - a^3b + 16a^3b - 13a^3b - 25a^3b = -5a^3b$.
12. $11\sqrt{a} - 14\sqrt{a} + 21\sqrt{a} - \sqrt{a} = 17\sqrt{a}$.
13. $3\sqrt{x-y} - \sqrt{x-y} + 9\sqrt{x-y} - 7\sqrt{x-y} = 4\sqrt{x-y}$.
14. $3(a+b) - 2(a+b) + 8(a+b) = 9(a+b)$.
15. $-7(a-2b) + (a-2b) + 12(a-2b) = 6(a-2b)$.
16. $8(a+b) + 3(a-b) - 4(a+b) - 2(a-b) = 4(a+b) + (a-b)$.
17. $4(x-y) - 3(x+3) - 6(x-y) + 5(x-3)$
 $= -2(x-y) - 3(x+3) + 5(x-3)$.
18. $(2x-y)^2 - 3(2x-y)^2 + 4(2x-y)^2 - 7(2x-y)^2 = -5(2x-y)^2$.

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- | | | |
|--|---|---|
| $\begin{array}{r} 1. \quad x + y + z \\ \quad x - 2y + 3z \\ \hline 3x + 4y - 7z \\ \hline 5x + 3y - 3z \end{array}$ | $\begin{array}{r} 5. \quad 4x - 3y - 5z \\ \quad \quad 6y - 2z \\ \hline 7x - 6y - 4z \\ \hline 11x - 3y - 11z \end{array}$ | $\begin{array}{r} 9. \quad 9ac - bc \\ \quad - 4ac \quad + 8ab \\ \hline \quad - ac \quad - 12ab \\ \hline \quad 4ac - bc - 4ab \end{array}$ |
| $\begin{array}{r} 2. \quad 2x + 5y - z \\ \quad 3x - 8y + 6z \\ \hline \quad x - y - z \\ \hline 6x - 4y + 4z \end{array}$ | $\begin{array}{r} 6. \quad x + 3y + 2z \\ \quad x + y - 3z \\ \hline -2x - 4y + z \\ \hline 0 \quad + 0 \quad + 0 \end{array}$ | $\begin{array}{r} 10. \quad a^2 - 4a + 10 \\ \quad - 6a^2 + 5a + 4 \\ \hline \quad 2a^2 + 3a - 16 \\ \hline -3a^2 + 4a - 2 \end{array}$ |
| $\begin{array}{r} 3. \quad 3x + 5y \\ \quad 4x - 7y + 6z \\ \hline 3x - 3y - 3z \\ \hline 10x - 5y + 3z \end{array}$ | $\begin{array}{r} 7. \quad 5x - y + 3z \\ \quad x + 2y - 11z \\ \hline \quad - 7y + 9z \\ \hline 6x - 6y + z \end{array}$ | $\begin{array}{r} 11. \quad -a^2 + a + 9 \\ \quad 6a^2 - 4a - 7 \\ \hline \quad 5a^2 - a \\ \hline 10a^2 - 4a + 2 \end{array}$ |
| $\begin{array}{r} 4. \quad 7x - y + 3z \\ \quad 5x \quad - 4z \\ \hline 2x + 6y - 5z \\ \hline 14x + 5y - 6z \end{array}$ | $\begin{array}{r} 8. \quad 8a - 7b - 6c \\ \quad - 4a - 3b + 5c \\ \hline \quad \quad 3b + 7c \\ \hline 4a - 7b + 6c \end{array}$ | $\begin{array}{r} 12. \quad \frac{1}{2}x - \frac{2}{3}y + \frac{2}{5}z \\ \quad x - y + 2z \\ \hline \frac{1}{8}x - \frac{1}{2}y + \frac{1}{10}z \\ \hline 1\frac{5}{6}x - 2\frac{1}{6}y + 2\frac{1}{2}z \end{array}$ |

$$\begin{array}{r}
 13. \quad \frac{1}{4}x \qquad \qquad - \frac{3}{20}z \\
 \frac{1}{8}x + \frac{4}{9}y \\
 \qquad - \frac{5}{3}y + \frac{3}{5}z \\
 \hline
 \frac{3}{8}x - 1\frac{2}{9}y + \frac{9}{20}z
 \end{array}$$

$$\begin{array}{r}
 14. \quad a - 3(x - y) + z \\
 - 10a + 4(x - y) \qquad + 5 \\
 \qquad - 2(x - y) \qquad + 6 \\
 \hline
 - 9a - (x - y) + z + 11
 \end{array}$$

$$\begin{array}{r}
 15. \quad a + 2(b - c) + d \\
 - 12a + 7(b - c) + 6d \\
 \qquad 11a - 5(b - c) \\
 \hline
 4(b - c) + 7d
 \end{array}$$

$$\begin{array}{r}
 16. \quad 7a^2 - 13b^2 + 12c^2 \\
 - a^2 + 15b^2 - 7c^2 \\
 \qquad 3b^2 + 5c^2 \\
 \hline
 6a^2 + 5b^2 + 10c^2
 \end{array}$$

$$\begin{array}{r}
 17. \quad x^2 - 2xy + y^2 \\
 - 4x^2 - 4xy - y^2 \\
 - 16x^2 - 8xy + y^2 + y^3 \\
 \hline
 - 19x^2 - 14xy + y^2 + y^3
 \end{array}$$

$$\begin{array}{r}
 18. \quad - a^2 + 5ab + b^2 \\
 5a^2 - 4ab - 9b^2 \\
 \qquad - 2ab + 2b^2 \\
 \hline
 4a^2 - ab - 6b^2
 \end{array}$$

$$\begin{array}{r}
 19. \quad 3x^2 - 6x + 11 \\
 \qquad - 4x - 8 \\
 - 3x^2 + 5x - 16 \\
 \qquad + 13x \\
 \hline
 8x - 13
 \end{array}$$

$$\begin{array}{r}
 20. \quad 8b^2 - 10bc + 12c^2 \\
 - 6b^2 - bc - c^2 \\
 \qquad - 11c^2 + c^3 \\
 \hline
 2b^2 - 11bc \qquad + c^3
 \end{array}$$

$$\begin{array}{r}
 21. \quad 4x^2y - xy + y^2 \\
 - 2x^2y - 3xy \qquad + 4xy^2 \\
 \qquad x^2y - 3xy + 4y^2 \\
 \hline
 3x^2y - 7xy + 5y^2 + 4xy^2
 \end{array}$$

$$\begin{array}{r}
 22. \quad \frac{1}{2}a + \frac{1}{3}b - \frac{1}{4}c \\
 - \frac{2}{3}a + \frac{1}{2}b - c \\
 \qquad - b + \frac{1}{2}c \\
 - \frac{3}{4}a \qquad \qquad + 7 \\
 \hline
 -\frac{1}{12}a - \frac{1}{6}b - \frac{3}{4}c + 7
 \end{array}$$

$$23. ax + bx + cx = (a + b + c)x.$$

$$24. 2ax - 3x + bx = (2a - 3 + b)x.$$

$$25. 3ax - 4cx + x = (3a - 4c + 1)x.$$

$$26. 3ax^2 - bx^2 - x^2 + a^2x^2 = (3a - b - 1 + a^2)x^2.$$

$$27. by - 4cy - y - 4by = -4cy - y - 3by = (-4c - 1 - 3b)y.$$

$$28. a(b + c) + 3(b + c) = (a + 3)(b + c).$$

$$29. 4(a - x) - 3b(a - x) = (4 - 3b)(a - x).$$

$$30. 8a(a + 3b) - 1(a + 3b) = (8a - 1)(a + 3b).$$

$$31. 7b(x^2 + y^2) - a(x^2 + y^2) + (x^2 + y^2) = (7b - a + 1)(x^2 + y^2).$$

Page 35 (First set)

$$1. \qquad \qquad \qquad x + 5 = 11.$$

$$\text{Subtracting 5 from each member,} \qquad x = 6.$$

Ans. II

$$2. \qquad \qquad \qquad x - 4 = 12.$$

$$\text{Adding 4 to each member,} \qquad x = 16.$$

Ans. I

$$3. \qquad \qquad \qquad 3x + 10 = 28.$$

$$\text{Subtracting 10 from each member,} \qquad 3x = 18.$$

Ans. II

$$\text{Dividing each member by 3,} \qquad x = 6.$$

Ans. IV

4. $5x - 6 = 19.$

Adding 6 to each member,

$$5x = 25.$$

Whence $x = 5.$

5. $9x - 12 = 6.$

Adding 12 to each member,

$$9x = 18.$$

Whence $x = 2.$

6. $4x = 12 + x.$

Whence $x = 4.$

7. $6x = 20 + 2x.$

Whence $x = 5.$

8. $9n = 40 - n.$

Whence $n = 4.$

9. $13n = -5n + 36.$

Whence $n = 2.$

10. $-4n = -13n + 27.$

Whence $n = 3.$

11. $3y + 2 = y + 8.$

Whence $y = 3.$

20. $4x - 3 + 8x - 17 = 40 + 6x - 54.$

Collecting, $12x - 20 = 6x - 14.$

Whence $x = 1.$

12. $5 + 4y = 3y + 20.$

Whence $y = 15.$

13. $2y - 3 = 17 - y.$

Whence $y = 6\frac{2}{3}.$

14. $8x - 15 = 6x - 15.$

Adding 15 to each member,

$$8x = 6x.$$

Whence $x = 0.$

15. $-7h + 19 = 25 - 9h.$

Whence $h = 3.$

16. $-7x + 18 = 4x + 18.$

Whence $x = 0.$

17. $5k - 4 = 3k + 18.$

Whence $k = 11.$

18. $8 - 6x + 12 + 10x = 26.$

Whence $x = 1\frac{1}{2}.$

19. $x + 2x + 18 + x + 2x + 18$
 $= 116.$

Whence $x = 13\frac{1}{3}.$

Page 35 (Second set)

1. Length, $3x$; perimeter, $2 \cdot x + 2 \cdot 3x$, or $8x$.

2. Length, $x + 10$ feet; perimeter, $2 \cdot x + 2(x + 10)$, or $4x + 20$ feet.

3. Width, $x - 18$ feet; perimeter, $2 \cdot x + 2(x - 18)$, or $4x - 36$ feet.

4. Width, x feet; length, $2x + 4$ feet; perimeter, $2x + 2(2x + 4)$, or $6x + 8$ feet.

5. Greater by 1. 6. $n + 1.$

7. $n + 1$ and $n + 2$; sum is $2n + 3.$

8. $n + 1, n + 2, n + 3$; the sum of the four $= 4n + 6.$

9. Greater by 2. 10. $n + 2; n - 2.$

11. $n, n + 2, n + 4$; their sum $= 3n + 6.$

12. $n, n + 2, n + 4, n + 6$; their sum $= 4n + 12.$

13. First, n ; second, $2n$; third, $6n$.
 14. First, n ; second, $n + 10$; third, $n + 10 - 7$, or $n + 3$.
 15. $x - 4$; $x + 6$.
 16. (a) B, x years; A, $2x$ years. (b) B, $x - 7$ years; A, $2x - 7$ years.
 (c) B, $x + 12$ years; A, $2x + 12$ years.
 17. $8 + x = 5$.
 18. $x = 10 - 2$.
 19. $x = 3 + 5$.
 20. $x = 5 + y$.
 21. $3x = 21$.
 22. $4a = 18 + 2$.
 23. $8 + x = 34 - x$.
 24. $2x - 9 = 14 + x$.
 25. $2x + 3x = 48 + x$.
 26. $3x - 12 = 50 + x$.

Page 37

1. Greater number + less number = 135.

If we represent the less number by n , $2n$ must represent the greater, and the above statement becomes

$$2n + n = 135.$$

Whence $n = 45$,
 and $2n = 90$.

2. Greater number + less number = 105.

If we represent the less number by n , $4n$ must represent the greater, and the above statement becomes

$$4n + n = 105.$$

Whence $n = 21$,
 and $4n = 84$.

3. Greater number - less number = 52.

If we let n represent the less number, $5n$ must represent the greater, and the above statement becomes

$$5n - n = 52.$$

Whence $n = 13$,
 and $5n = 65$.

4. Greater number + less number = 129.

If we let n represent the less number, $n + 5$ must represent the greater, and the above statement becomes

$$n + 5 + n = 129.$$

Whence $n = 62$,
 and $n + 5 = 67$.

5. Greater number + less number = 168.

If we represent the less number by n , $n + 18$ must represent the greater, and the above statement becomes

$$n + 18 + n = 168.$$

Whence $n = 75$,
and $n + 18 = 93$.

6. Twice the width + twice the length = 156 feet.

If we let w represent the width in feet, $5w$ must represent the length, and the above statement becomes

$$2 \cdot w + 2 \cdot 5w = 156.$$

Whence $w = 13$,
and $5w = 65$.

7. $2 \cdot \text{length} + 2 \cdot \text{width} = 96$ feet.

If we represent the width in feet by w , $12 + w$ must represent the length, and the above statement becomes

$$24 + 2w + 2w = 96.$$

Whence $w = 18$,
and $12 + w = 30$.

8. $2 \cdot \text{length} + 2 \cdot \text{width} = 98$ feet.

If we represent the width in feet by w , $2w + 4$ must represent the length, and the above statement becomes

$$4w + 8 + 2w = 98.$$

Whence $w = 15$,
and $2w + 4 = 34$.

9. $2 \cdot \text{length} + 2 \cdot \text{width} = 88$ feet.

If we let w represent the width in feet, $3w - 20$ must represent the length, and the above statement becomes

$$6w - 40 + 2w = 88.$$

Whence $w = 16$,
and $3w - 20 = 28$.

10. 1st number + 2d number + 3d number = 45.

If we represent the first number by n , $n + 1$ and $n + 2$ must represent the next two numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) = 45.$$

Whence $n = 14$,
 $n + 1 = 15$,
and $n + 2 = 16$.

11. 1st number + 2d number + 3d number + 4th number = 106.

If we represent the first number by n , $n + 1$, $n + 2$, and $n + 3$ must represent the three consecutive numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) + (n + 3) = 106.$$

Whence $n = 25$,

$$n + 1 = 26,$$

$$n + 2 = 27,$$

and $n + 3 = 28.$

12. The sum of five consecutive numbers = 85.

If we represent the first number by n , $n + 1$, $n + 2$, $n + 3$, $n + 4$ must represent the next four numbers greater, and the statement above becomes $(n) + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 85.$

Whence $n = 15,$

$$n + 1 = 16,$$

$$n + 2 = 17,$$

$$n + 3 = 18,$$

and $n + 4 = 19.$

13. 1st odd number + next odd number + next odd number = 291.

If we represent the first number by n , the others must be represented by $n + 2$ and $n + 4$ respectively, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 291.$$

Whence $n = 95,$

$$n + 2 = 97,$$

and $n + 4 = 99.$

14. 1st even number + next even number + next even number = 66.

If we represent the first number by n , $n + 2$ and $n + 4$ must represent the other numbers, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 66.$$

Whence $n = 20,$

$$n + 2 = 22,$$

and $n + 4 = 24.$

15. The sum of five consecutive odd numbers = 315.

Representing the first number by n , we must then represent the others by $n + 2$, $n + 4$, $n + 6$, $n + 8$ respectively, and the above statement becomes $(n) + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 315.$

Whence $n = 59,$

$$n + 2 = 61,$$

$$n + 4 = 63,$$

$$n + 6 = 65,$$

and $n + 8 = 67.$

16. A's age + B's age = 43 years.

If we represent B's age in years by x , $x - 2$ must represent A's age, and the above statement becomes

$$x - 2 + x = 43.$$

Whence $x = 22\frac{1}{2},$

and $x - 2 = 20\frac{1}{2}.$

17. A's age + 6 years + B's age + 6 years = 54 years.

If we represent A's age in years by x , $2x$ must represent B's age, and the above statement becomes

$$x + 6 + 2x + 6 = 54.$$

Whence $x = 14$,
and $2x = 28$.

18. Mileage of United States + mileage of Europe = 402,000 miles.

If we represent the mileage of Europe by m , $m + 52,000$ must represent that of United States, and the above statement becomes

$$m + 52,000 + m = 402,000.$$

Whence $m = 175,000$,
and $m + 52,000 = 227,000$.

19. Length of Nile + length of Amazon + length of Mississippi = 11,500 miles.

If we represent the length of the Amazon in miles by l , $l + 500$ must represent the length of the Nile, and $l + 500 + 300$, or $l + 800$, the length of the Mississippi. The above statement becomes

$$(l + 500) + (l) + (l + 800) = 11,500.$$

Whence $l = 3400$,
 $l + 500 = 3900$,
and $l + 800 = 4200$.

20. Height of Mount McKinley + height of Pike's Peak = 34,607 feet.

If we represent the height of Pike's Peak in feet by h , $h + 6313$ must represent the height of Mount McKinley, and the statement above becomes

$$(h + 6313) + (h) = 34,607.$$

Whence $h = 14,147$,
and $h + 6313 = 20,460$.

The height of Mount Everest = 34,607 feet - 5605 feet = 29,002 feet.

21. Horse power of a freight engine + that of a passenger engine + that of a tractor = 11,200 H.P.

If we represent the H.P. of the freight engine by x , $x - 1800$ must represent the H.P. of the tractor and $x + 1000$ that of the passenger engine. The above statement becomes

$$(x) + (x + 1000) + (x - 1800) = 11,200.$$

Whence $x = 4000$,
 $x + 1000 = 5000$,
and $x - 1800 = 2200$.

Page 40 (First set)

1. $3x - 2x = x.$
 $2x - 3x = -x.$
2. $3x - 4x = -x.$
 $4x - 3x = x.$
3. $-3x - (-2x) = -x.$
 $-2x - (-3x) = x.$
4. $-3x - (-5x) = 2x.$
 $-5x - (-3x) = -2x.$
5. $4x - (-x) = 5x.$
 $-x - 4x = -5x.$
6. $-3x - (-x) = -2x.$
 $-x - (-3x) = 2x.$
7. $5c - (-c) = 6c.$
 $-c - 5c = -6c.$
8. $-5ac - (-ac) = -4ac.$
 $-ac - (-5ac) = 4ac.$
9. $-11a^2c - 8a^2c = -19a^2c.$
 $8a^2c - (-11a^2c) = 19a^2c.$
10. $-6x^2y^2 - 6x^2y^2 = -12x^2y^2.$
 $6x^2y^2 - (-6x^2y^2) = 12x^2y^2.$
11. $0 - 3x = -3x.$
 $3x - 0 = 3x.$
12. $0 - (-4ab) = 4ab.$
 $-4ab - 0 = -4ab.$
13. $b - a.$
 $a - b.$
14. $2x - c.$
 $c - 2x.$
15. $-4y - x.$
 $x - (-4y) = x + 4y.$
16. $2b - (-3a) = 2b + 3a.$
 $-3a - (+2b) = -3a - 2b.$
17. $-5b - (-2a) = -5b + 2a.$
 $-2a - (-5b) = -2a + 5b.$
18. $-2a - (-2a) = 0.$
 $-2a - (-2a) = 0.$

Page 40 (Second set)

1. $\frac{a+3}{1} \quad \frac{a+2}{-1}$
2. $\frac{a-2}{2} \quad \frac{a-4}{-2}$
3. $\frac{3a+5}{a+8} \quad \frac{2a-3}{-a-8}$
4. $\frac{9-4a}{2-7a} \quad \frac{7+3a}{-2+7a}$
5. $\frac{4a-3}{-3} \quad \frac{4a}{3}$
6. $\frac{5c}{10c-2ab} \quad \frac{-5c+2ab}{-10c+2ab}$
7. $\frac{5xy}{2xy+a} \quad \frac{3xy-a}{-2xy-a}$
8. $\frac{x+3}{x+3} \quad \frac{0}{-x-3}$
9. $\frac{0}{-2x+3} \quad \frac{2x-3}{2x-3}$
10. $\frac{2x-2y+z}{x} \quad \frac{x-2y+3z}{-2z}$
11. $\frac{4x-5y+3z}{3y+5z} \quad \frac{4x-8y-2z}{3y+5z}$
12. $\frac{4a-b+2c}{a+b+2c} \quad \frac{3a-2b}{a+b+2c}$
13. $\frac{3a-5b}{-2a-b+3c} \quad \frac{5a-4b-3c}{-2a-b+3c}$
14. $\frac{b-x+y}{b-a} \quad \frac{a-x-y}{+2y}$
15. $\frac{4a-b+5c}{a+b+6c-6} \quad \frac{3a-2b-c+6}{a+b+6c-6}$

$$16. \quad \begin{array}{r} a - 3b \\ 2a - 2b - 2c + 4 \\ -a - b + 2c - 4 \end{array}$$

$$17. \quad \begin{array}{r} c - d + e \\ -c \qquad \qquad + a + b \\ 2c - d + e - a - b \end{array}$$

$$18. \quad \begin{array}{r} 2x + 5y - 3z \\ x - 2y + z \\ x + 7y - 4z \end{array}$$

$$19. \quad \begin{array}{r} 5x - 2y - 4z \\ 7x - 9y + 3z \\ -2x + 7y - 7z \end{array}$$

$$20. \quad \begin{array}{r} 6a + 3b \\ 5a - 4b - 6c \\ a + 7b + 6c \end{array}$$

$$21. \quad \begin{array}{r} 5 \\ + a + b - 2c \\ 5 - a - b + 2c \end{array}$$

$$22. \quad \begin{array}{r} 3xy + z \\ \qquad \qquad + 3ab + c \\ 3xy + z - 3ab - c \end{array}$$

$$30. \quad \begin{array}{r} 4a - 6b + 8y \\ \qquad \qquad \qquad + x - 12 \\ 4a - 6b + 8y - x + 12 \end{array}$$

$$31. \quad \begin{array}{ll} a^2 - 2ab + b^2 & (1) \\ a^2 - 12ab + 20 & (2) \end{array} \quad \begin{array}{ll} a^2 - 13a + 30 & (4) \\ 2a^2 + 20 - 14ab + b^2 & (3) \end{array}$$

$$(1) + (2), \quad 2a^2 - 14ab + b^2 + 20 \quad (3) \quad (4) - (3), \quad -a^2 - 13a + 10 + 14ab - b^2$$

$$32. \quad \begin{array}{ll} a - 3b + c & (1) \\ 4a + 5b - 6c + 4 & (2) \end{array} \quad \begin{array}{ll} a - b + c - x & (4) \\ 5a + 2b - 5c + 4 & (3) \end{array}$$

$$(1) + (2), \quad 5a + 2b - 5c + 4 \quad (3) \quad (4) - (3), \quad -4a - 3b + 6c - x - 4$$

$$33. \quad \begin{array}{ll} 5x + 3x^2y - 15xy^2 & (1) \\ -6x + 7x^2y - 12xy^2 & (2) \end{array}$$

$$(1) + (2), \quad -x + 10x^2y - 27xy^2 \quad (3)$$

$$\quad \quad \quad 11x - 5x^2y + 7xy^2 \quad (4)$$

$$(3) - (4), \quad -12x + 15x^2y - 34xy^2, \text{ or } 15x^2y - 12x - 34xy^2.$$

$$34. \quad \begin{array}{ll} 4abc^2 - 3ab^2c + 2a^2bc & (1) \\ 6abc^2 - 5ab^2c - 4a^2b^2c & (2) \end{array}$$

$$(1) + (2), \quad 10abc^2 - 8ab^2c + 2a^2bc - 4a^2b^2c \quad (3)$$

$$\quad \quad \quad 2abc^2 + 7ab^2c - 3a^2bc \quad (4)$$

$$(3) - (4), \quad 8abc^2 - 15ab^2c + 5a^2bc - 4a^2b^2c$$

$$23. \quad \begin{array}{r} 0 \\ 2x - 4y - z \\ -2x + 4y + z \end{array}$$

$$24. \quad \begin{array}{r} xy^2 + x^2y + z \\ -3xy^2 + 2x^2y \\ 4xy^2 - x^2y + z \end{array}$$

$$25. \quad \begin{array}{r} x - 2y + z \\ 3x + 2y - z \\ -2x - 4y + 2z \end{array}$$

$$26. \quad \begin{array}{r} x^2 - 7x + 10 \\ 3x^2 + 14x - 8 \\ -2x^2 - 21x + 18 \end{array}$$

$$27. \quad \begin{array}{r} x - y + z \\ 5x + 3y - 8z \\ -4x - 4y + 9z \end{array}$$

$$28. \quad \begin{array}{r} -8x^2 + 4 \\ x^2 - 5x + 6 \\ -9x^2 + 5x - 2 \end{array}$$

$$29. \quad \begin{array}{r} 3a - 5b + c \\ 0 \\ 3a - 5b + c \end{array}$$

$$\begin{array}{rcl}
 35. & 3x - 4xy - 2z & (1) \\
 & -3x + 7xy - 4z & (2) \\
 \hline
 & 3xy - 6z & (3) \quad (4) + (5), \\
 & 9x - 2xy + 4z - 7a^2bc & (6) \\
 (3) - (6), & -9x + 5xy - 10z + 7a^2bc &
 \end{array}$$

$$\begin{array}{rcl}
 36. & 4x - 3y + 6 & (1) \\
 & 3x + 5y - 10 & (2) \\
 (1) + (2), & 7x + 2y - 4 & \\
 37. & 7c + 5d - e & (1) \\
 & 4c + 5d - 9e & (2) \\
 (1) - (2), & 3c + 8e &
 \end{array}$$

$$\begin{array}{rcl}
 38. & x^2 + 2x + 5 & (1) \\
 & 2x^2 + x - 10 & (2) \\
 (1) + (2), & 3x^2 + 3x - 5 & (3) \\
 & x^2 - 5x + 3 & (4) \\
 (3) - (4), & 2x^2 + 8x - 8 &
 \end{array}$$

$$\begin{array}{rcl}
 39. & x + 3y - 2z & (1) \\
 & 4x - 5y + 3z & (2) \\
 (1) + (2), & 5x - 2y + z & (3) \\
 & 3x - 2y - 6z & (4) \\
 (3) - (4), & 2x + 7z &
 \end{array}$$

$$\begin{array}{rcl}
 40. & 5x + 3y - z & (1) \\
 & + 4y + 7z & (2) \\
 (1) + (2), & 5x + 7y + 6z & (3) \\
 & x - y + 3z & (4) \\
 (3) - (4), & 4x + 8y + 3z &
 \end{array}$$

$$\begin{array}{rcl}
 41. & 4x - 3y + 7 & (1) \\
 & 2x - 5y - 4 & (2) \\
 (1) - (2), & 2x + 2y + 11 & (3) \\
 & 4x - 8 & (4) \\
 (3) + (4), & 6x + 2y + 3 &
 \end{array}$$

$$\begin{array}{rcl}
 42. & 3c - 5d - e & (1) \\
 & + 6d + 11e + 5 & (2) \\
 (1) - (2), & 3c - 11d - 12e - 5 & (3) \\
 & 5c + 4e & (4) \\
 (3) - (4), & -2c - 11d - 16e - 5 &
 \end{array}$$

$$\begin{array}{rcl}
 43. & 3a + 3b - 4c & (1) \\
 & - 3b - 3c - 4 & (2) \\
 (1) - (2), & 3a + 6b - c + 4 & (3) \\
 & 4a - 8c + x & (4) \\
 (3) - (4), & -a + 6b + 7c + 4 - x &
 \end{array}$$

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$$\begin{array}{ll}
 1. & 8x - 2 = 6x + 6. \\
 \text{Transposing,} & 8x - 6x = 2 + 6. \\
 \text{Whence} & x = 4. \\
 2. & 4x - 5 = 2x + 10. \\
 \text{Transposing,} & \\
 & 4x - 2x = 5 + 10. \\
 \text{Whence} & x = 7\frac{1}{2}. \\
 3. & 6y - 5 = 9y + 2. \\
 \text{Transposing,} & 6y - 9y = 5 + 2. \\
 \text{Whence} & y = -2\frac{1}{3}. \\
 4. & 7y + 3 = 10 + 8y. \\
 \text{Transposing,} & \\
 & 7y - 8y = 10 - 3. \\
 \text{Whence} & y = -7. \\
 5. & 5n - 3 + 21 = 18 + 4n. \\
 \text{Combining,} & \\
 & 5n + 18 = 18 + 4n. \\
 \text{Transposing,} & \\
 & 5n - 4n = 18 - 18. \\
 \text{Whence} & n = 0.
 \end{array}$$

6. $6 + 4n - 15 = 15 - n.$

Transposing,

$$4n + n = 15 + 9.$$

Whence $n = 4\frac{4}{5}.$

7. $5n + 3 - 2n = 7 - 4.$

Combining, $3n + 3 = 3.$

Whence $n = 0.$

8. $3k + 9 + 5k + 31 = 0.$

Combining, $8k + 40 = 0.$

Whence $k = -5.$

9. $6k + 3 - 2k = 27.$

Whence $k = 6.$

10. $3x - 6 = 34 + 8x.$

Transposing,

$$-5x = 40.$$

Whence $x = -8.$

11. $2x - 14 - 5x + 4 = 0.$

Whence $x = -3\frac{1}{3}.$

12. $x + 12 - 11x = -15x + 22.$

Transposing,

$$15x - 10x = 22 - 12.$$

Whence $x = 2.$

20. $4x - 15 - 11x - 18 + 16x - 17 = 0.$

Whence $x = 5\frac{5}{9}.$

21. $5y - 6 + 3y + 18 - 2y - 25 + 1 = 0.$

Whence $y = 2.$

22. $0 = 9x - 3 - 4x + 27 + 16x + 18.$

Whence $x = -2.$

23. $7n - 5 - 4n + 8 = 3n + 18 - 2n - 3.$

Collecting, $3n + 3 = n + 15.$

Whence $n = 6.$

13. $5y + 3 = 17 + 3y + 8.$

Transposing,

$$5y - 3y = 25 - 3.$$

Whence $y = 11.$

14. $3y + 5 + 8y + \frac{1}{2} = 0.$

Whence $y = -\frac{1}{2}.$

15. $2 - 4h = 3 - 8h + 8.$

Transposing,

$$8h - 4h = 11 - 2.$$

Whence $h = 2\frac{1}{4}.$

16. $3 - 5h + 2 = 7h + 5.$

Whence $h = 0.$

17. $3h - 25 + 8h - 20 = 0.$

Whence $h = 4\frac{1}{11}.$

18. $14x - 6x = 22 + 17x - 11x.$

Transposing,

$$8x - 6x = 22.$$

Whence $x = 11.$

19. $7x - 13 + 8 = x - 27 - 5x.$

Transposing,

$$7x + 4x = 5 - 27.$$

Whence $x = -2.$

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1. $n + 5.$

6. $a + b + 3.$

11. $5n + a.$

16. $12 - y.$

2. $n + a.$

7. $a + b + c.$

12. $4n - 7.$

17. $x - a.$

3. $n - 3.$

8. $2a - b - 5.$

13. $3n + 8.$

18. $a - x.$

4. $n - b.$

9. $2a - b - c.$

14. $10 - 6, \text{ or } 4.$

19. $x + y - z.$

5. $4n.$

10. $2n - b.$

15. $x - 4.$

20. $18 - 7, \text{ or } 11.$

21. $18 - n$. 23. $n - 18$. 25. $a - 7$.
 22. $34 - 18$, or 16 . 24. $30 - b$. 26. $a - x$.
27. $6 - d$, if 6 is the greater number. $6 + d$, if 6 is the less number.
 28. $n - d$, if n is the greater number. $n + d$, if n is the less number.
 29. $10 - 4 = 6$; $10 - x$.
 30. $25 - 9 = 16$; $25 - y$; $16 - (a + b)$.
 31. $40 - 27 = 13$; $40 - 40 = 0$; $40 - a$; $a - b$.
 32. $36 - 22 = 14$; $a - 14$; $y - x$.
 33. $a + 6 - (a - 6) = 12$; $a + 6 - (b - 6) = a - b + 12$;
 $4x - 3 - (3x - 4) = x + 1$.
 34. $n + 4$ years; $n + x$ years; $n - 3$ years.
 35. $2n - 3 + 10$ years $= 2n + 7$ years; $2n - 3 + a$ years $= 2n - 3 + a$ years;
 $2n - 3 - 8$ years $= 2n - 11$ years; $2n - 3 - a$ years.
 36. A, $x - 4$ dollars; B, $x + 4$ dollars.
 37. A, $x + 50 + y$ dollars; B, $x + 50 - y$ dollars.
 38. (a) $x + 30 + 3x - 4 = 200$. (c) $x + 30 + 10 = 3x - 4$.
 (b) $x + 30 = 3x - 4$. (d) $x + 30 + 100 = 3x - 4 - 50$.
 39. (a) A, $x + 5$ years; B, $2x + 12$ years; C, $3x - 3$ years.
 (b) A, $x - 3$ years; B, $2x + 4$ years; C, $3x - 11$ years.
 (c) $x + 4 + 2x + 7 + 4 = 40$.
 (d) $(3x - 8 - 6) - (x - 6) = 24$, if C is older than A.
 (e) $x + 10 = 2x + 7$.
 (f) $3x - 8 - 4 = 2x + 7 + 10$.
 (g) $2x + 7 + x = 40$.
 (h) $x + 2 + 2x + 7 + 2 + 3x - 8 + 2 = 100$.
40. A number less 2 is 8.
 41. The sum of a number and 3 is 5.
 42. Three times a number is 27.
 43. Four times a number less 2 is 16.
 44. Subtracting a number from 18 gives the same result as subtracting 4 from the number.
 45. Subtracting 4 from three times a number gives the same result as adding 8 to twice the number.
 46. The sum of a number and a second number is 20.
 47. The result of subtracting a second number from a number is 2.
 48. Twice one number has the same value as the sum of a second number and 6.
 49. Three times a number less twice a second number is 8.
 50. The sum of a number and a second number is a .
 51. A number less a second number is b .
 52. The sum of a number and a is a second number.
 53. A number less b is equal to a second number.

54. Three times a number is equal to twice a second number.

55. A number is equal to twice a second number less 6.

56. The sum of three times a number and 4 is equal to the sum of twice a second number and 4.

57. Eighty less a certain number is equal to the sum of a second number and 30.

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- | | |
|---------|---------------------------|
| 1. Let | $n =$ the unknown number. |
| Then | $n + 22 = 50.$ |
| Whence | $n = 28.$ |
| 2. Let | $n =$ the unknown number. |
| Then | $n - 15 = 47.$ |
| Whence | $n = 62.$ |
| 3. Let | $n =$ the unknown number. |
| Then | $n + 9 = 28.$ |
| Whence | $n = 19.$ |
| 4. Let | $n =$ the unknown number. |
| Then | $n - 17 = 35.$ |
| Whence | $n = 52.$ |
| 5. Let | $n =$ the unknown number. |
| Then | $2n - 27 = 49.$ |
| Whence | $n = 38.$ |
| 6. Let | $n =$ the unknown number. |
| Then | $3n - 36 = 2n.$ |
| Whence | $n = 36.$ |
| 7. Let | $n =$ the unknown number. |
| Then | $3n - 17 = 2n - 1.$ |
| Whence | $n = 16.$ |
| 8. Let | $n =$ the unknown number. |
| Then | $5n + 6 = 2n + 15.$ |
| Whence | $n = 3.$ |
| 9. Let | $n =$ the unknown number. |
| Then | $4n + 9 = 7n - 33.$ |
| Whence | $n = 14.$ |
| 10. Let | $n =$ the unknown number. |
| Then | $n + 9 = 71 - n.$ |
| Whence | $n = 31.$ |

11. Let $n =$ the unknown number.
 Then $2n + 6 = 4n - 10$.
 Whence $n = 8$.

13. Let $n =$ the less number.
 Then $n + 12 =$ the greater number.
 Therefore $n + n + 12 = 74$.
 Whence $n = 31$,
 and $n + 12 = 43$.

14. Let $n =$ the less number.
 Then $n + 3 =$ the greater number.
 Therefore $n + n + 3 = 45$.
 Whence $n = 21$,
 and $n + 3 = 24$.

15. Let $n =$ the less number.
 Then $n + 8 =$ the greater number.
 Therefore $n + n + 8 = 44$.
 Whence $n = 18$,
 and $n + 8 = 26$.

16. Let $n =$ the first number.
 Then $n - 4 =$ the second number,
 and $n + 9 =$ the third number.
 Therefore $n + n - 4 + n + 9 = 83$.
 Whence $n = 26$,
 $n - 4 = 22$,
 and $n + 9 = 35$.

17. Let $n =$ the first number.
 Then $n - 3 =$ the second number,
 and $n - 3 + 18$, or $n + 15 =$ the third number.
 Therefore $n + n - 3 + n + 15 = 66$.
 Whence $n = 18$,
 $n - 3 = 15$,
 and $n + 15 = 33$.

18. Let $n =$ one number.
 Then $n + 1 =$ the other.
 Therefore $n + n + 1 = 37$.
 Whence $n = 18$,
 and $n + 1 = 19$.

19. Let $n =$ the first number.
 Then $n + 1$ and $n + 2 =$ the next two numbers.

Therefore $n + n + 1 + n + 2 = 39$.
 Whence $n = 12$,
 $n + 1 = 13$,
 and $n + 2 = 14$.

20. Let $n =$ the first number.
 Then $n + 1, n + 2$, and $n + 3 =$ the next three numbers.
 Therefore $n + n + 1 + n + 2 + n + 3 = 90$.
 Whence $n = 21$,
 $n + 1 = 22$,
 $n + 2 = 23$,
 and $n + 3 = 24$.

21. Let $n =$ the first even number.
 Then $n + 2 =$ the next even number.
 Therefore $n + n + 2 = 30$.
 Whence $n = 14$,
 and $n + 2 = 16$.

22. Let $n =$ the first odd number.
 Then $n + 2$ and $n + 4 =$ the next two odd numbers.
 Therefore $n + n + 2 + n + 4 = 87$.
 Whence $n = 27$,
 $n + 2 = 29$,
 and $n + 4 = 31$.

23. Let $n =$ the first even number.
 Then $n + 2, n + 4$, and $n + 6 =$ the next three even numbers.
 Therefore $n + n + 2 + n + 4 + n + 6 = 100$.
 Whence $n = 22$,
 $n + 2 = 24$,
 $n + 4 = 26$,
 and $n + 6 = 28$.

24. Let $w =$ the width in feet.
 Then $w + 3 =$ the length in feet.
 Therefore $2w + 2w + 6 = 38$.
 Whence $w = 8$,
 and $w + 3 = 11$.

25. Let $w =$ the width in feet.
 Then $w + 16 =$ the length in feet.
 Therefore $2w + 2w + 32 = 128$.
 Whence $w = 24$,
 and $w + 16 = 40$.

26. Let $w =$ the width in feet.
 Then $2w + 7 =$ the length in feet.
 Therefore $2w + 4w + 14 = 104.$
 Whence $w = 15,$
 and $2w + 7 = 37.$
27. Let $w =$ the width in feet.
 Then $4w + 5 =$ the length in feet.
 Therefore $2w + 8w + 10 = 90.$
 Whence $w = 8,$
 and $4w + 5 = 37.$
28. Let $x =$ the length of the night in hours.
 Then $x + 6\frac{1}{10} =$ the length of the day in hours.
 Therefore $x + x + 6\frac{1}{10} = 24.$
 Whence $x = 8\frac{1}{2}\frac{9}{10}$ hours, or 8 hours and 57 minutes,
 and $x + 6\frac{1}{10} = 15\frac{1}{2}\frac{1}{10}$ hours, or 15 hours and 3 minutes.
29. Let $x =$ A's age in years.
 Then $\frac{x}{2} =$ B's age in years,
 and $x + 7 =$ C's age in years.
 Therefore $x + \frac{x}{2} + x + 7 = 67.$
 Whence $x = 24,$
 $\frac{x}{2} = 12,$
 and $x + 7 = 31.$
30. Let $x =$ B's age in years.
 Then $3x =$ A's age in years,
 and $x + 10 =$ C's age in years.
 Therefore $3x + 5 + x + 5 + x + 10 + 5 = 60.$
 Whence $x = 7,$
 $3x = 21,$
 and $x + 10 = 17.$
31. Let $x =$ B's age in years.
 Then $x + 10 =$ A's age in years,
 and $x - 6 =$ C's age in years.
 Therefore $x + 10 - 4 + x - 4 + x - 6 - 4 = 46.$
 Whence $x = 18,$
 $x + 10 = 28,$
 and $x - 6 = 12.$

32. Let $x =$ B's age in years.
 Then $2x + 2 =$ A's age in years,
 and $2x - 5 =$ C's age in years.
 Therefore $2x + 2 + 6 + x + 6 + 2x - 5 + 6 = 70$.
 Whence $x = 11$,
 $2x + 2 = 24$,
 and $2x - 5 = 17$.
33. Let $x =$ the number of million bushels of oats.
 Then $x + 1838 =$ the number of million bushels of corn,
 and $x - 120 =$ the number of million bushels of wheat.
 Therefore $x + x + 1838 + x - 120 = 3981$.
 Whence $x = 754\frac{1}{3}$,
 $x + 1838 = 2592\frac{1}{3}$,
 and $x - 120 = 634\frac{1}{3}$.
34. Here $47 =$ the number of degrees in the width of the
 torrid zone,
 and 47 , or $23\frac{1}{2} =$ the number of degrees in the width of
 either frigid zone.
 Let $x =$ the number of degrees in the width of
 either temperate zone.
 Therefore $47 + 23\frac{1}{2} + 23\frac{1}{2} + x + x = 180$.
 Whence $x = 43$.
35. Let $x =$ the per cent in commerce.
 Then $x + 8 =$ the per cent in industries,
 and $x + 20 =$ the per cent in agriculture.
 Therefore $x + x + 8 + x + 20 = 100 - 24$.
 Whence $x = 16$,
 $x + 8 = 24$,
 and $x + 20 = 36$.
36. Let $x =$ the number of million pounds produced by Montana.
 Then $x - 110 =$ the number of million pounds produced by Michigan,
 and $x - 139 =$ the number of million pounds produced by Arizona.
 Therefore $x + x - 110 + x - 139 = 514$.
 Whence $x = 254\frac{1}{3}$,
 $x - 110 = 144\frac{1}{3}$,
 and $x - 139 = 115\frac{1}{3}$.
37. Let $x =$ the height in feet of the Washington Monument.
 Then $2x - 120 =$ the height in feet of the Eiffel Tower,
 $x - 105 =$ the height in feet of the Great Pyramid,
 and $x - 107 =$ the height in feet of St. Peter's.

Therefore $x + 2x - 120 + x - 105 + x - 107 = 2443$.

Whence

$$x = 555,$$

$$2x - 120 = 990,$$

$$x - 105 = 450,$$

and

$$x - 107 = 448.$$

38. Let $x =$ the area in square miles of the coal fields of all countries not named.

Then $2x + 38,400 =$ the area in square miles of the fields of the United States,

and $2x + 44,400 =$ the area in square miles of the fields of China and Japan.

Therefore $x + 2x + 38,400 + 2x + 44,400 = 471,800$.

Whence

$$x = 77,800,$$

$$2x + 38,400 = 194,000,$$

and

$$2x + 44,400 = 200,000.$$

39. Let $x =$ the output of Japan in tons.

Then $5x - 4615 =$ the output of other countries in tons,

$2x - 808 =$ the output of Spain and Portugal in tons,

and $5(2x - 808) - 5573$,

or $10x - 9613 =$ the output of the United States in tons.

Therefore $x + 5x - 4615 + 2x - 808 + 10x - 9613 = 486,084$.

Whence

$$x = 27,840,$$

$$5x - 4615 = 134,585,$$

$$2x - 808 = 54,872,$$

and

$$10x - 9613 = 268,787.$$

40. Let $x =$ the number of square miles in North America.

Then $2x + 982,000 =$ the number of square miles in Asia,

$x - 1,186,000 =$ the number of square miles in South America,

and $x - 4,383,000 =$ the area of Europe in square miles.

Therefore $x + 2x + 982,000 + x - 4,383,000 + x - 1,186,000 = 35,692,000$.

Whence

$$x = 8,055,800,$$

$$2x + 982,000 = 17,093,600,$$

$$x - 1,186,000 = 6,869,800,$$

and

$$x - 4,383,000 = 3,672,800.$$

41. Let $x =$ the number in the Revolution.

Then $9x - 15,621 =$ the number in the Civil War,

and $x + 266,841 =$ the number in the War of 1812.

Therefore $x + 9x - 15,621 + 266,841 + x = 3,658,811$.

- Whence $x = 309,781,$
 $9x - 15,621 = 2,772,408,$
 and $x + 266,841 = 576,622.$
42. Let $x =$ the capacity of St. Peter's.
 Then $x - 29,000 =$ the capacity of St. Paul's (London),
 $x - 17,000 =$ the capacity of Cathedral (Milan),
 and $x - 22,000 =$ the capacity of St. Paul's (Rome).
 Therefore $x + x - 29,000 + x - 17,000 + x - 22,000 = 148,000.$
 Whence $x = 54,000,$
 $x - 29,000 = 25,000,$
 $x - 17,000 = 37,000,$
 and $x - 22,000 = 32,000.$

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1. $14 - (6 - 3) - 5 = 14 - 3 - 5 = 6.$
2. $10 + (7 - 4) - (9 - 7) = 10 + 3 - 2 = 11.$
3. $(7 - 3 + 2) - (6 - 4) + 11 = 6 - 2 + 11 = 15.$
4. $11a - (4a - 9a) + (6a - a) = 11a - (-5a) + 5a = 21a.$
5. $(2a - 5a) - (4a - a - 7a) = -3a - (-4a) = a.$
6. $a - (b - c) + (2b - 3c) = a - b + c + 2b - 3c = a + b - c.$
7. $a - b - (c - d) + (a + b) - (b - c)$
 $= a - b - c + d + a + b - b + c = 2a - b + d.$
8. $(x - y) - (2y - 3x) + (x - 4y)$
 $= x - y - 2y + 3x + x - 4y = 5x - 7y.$
9. $x - (x - y + 2z) - (3z - y + 4) + (x - 6)$
 $= x - x + y - 2z - 3z + y - 4 + x - 6$
 $= x + 2y - 5z - 10.$
10. $7 - [8 - (3 - 10)] - (13 - 25) = 7 - [8 + 7] - (-12) = 19 - 15 = 4.$
11. $a + [2a - (3a - 2b)] + (3b - 2a)$
 $= a + [2b - a] + 3b - 2a = 5b - 2a.$
12. $(5x - 6y) - [-2x - (4z - y) - 2z]$
 $= 5x - 6y - [-2x - 6z + y] = 7x - 7y + 6z.$
13. $[3x - (2y - z)] - [- (3y - 2x) - 5x]$
 $= 3x - 2y + z - [-3y - 3x] = 6x + y + z.$
14. $[(a + 3) - (x - 5)] - [a + 3 + (x - 5)]$
 $= (a + 3 - x + 5) - (a + 3 + x - 5) = 10 - 2x.$
15. $7 - [-6 - \{-4 + (6 - 10)\} + 11] = 7 - [-6 - \{-8\} + 11]$
 $= 7 - [5 + 8] = -6.$
16. $-5x + [+10x - \{+11x - (2x - 7x + 4) - 3x\} - 22]$
 $= -5x + [+10x - \{+8x + 5x - 4\} - 22]$
 $= -5x + [-3x - 18] = -8x - 18.$

17. $\{4a - [2a - (3a - 2b) + 4a] - (4b - 6)\}$
 $= 4a - [6a - 3a + 2b] - 4b + 6$
 $= 4a - 3a - 2b - 4b + 6 = a - 6b + 6.$
18. $2x - 3y - [\{+ 3z - 7x - (y + 4z) - 9x\} + z]$
 $= 2x - 3y - [3z - 16x - y - 4z + z]$
 $= 2x - 3y + 16x + y = 18x - 2y.$
19. $(4y - 7x) - \{3x - [4x + (7y - 4x) - (3y - 3x)]\}$
 $= 4y - 7x - 3x + [4x + 7y - 4x - 3y + 3x]$
 $= 4y - 10x + [4y + 3x] = 8y - 7x.$
20. $[(a + b) + c] = [a + b + c].$
 $[(a + b) - c] = [a + b - c].$
21. $[4x + (3z - 5y)] = [4x + 3z - 5y].$
 $[4x - (3z - 5y)] = [4x - 3z + 5y].$
22. $[(a - 2b) + (3c - d)] = [a - 2b + 3c - d].$
 $[(a - 2b) - (3c - d)] = [a - 2b - 3c + d].$
23. $[(4x - 3) + (5y - 7)] = [4x - 3 + 5y - 7] = [4x + 5y - 10].$
 $[(4x - 3) - (5y - 7)] = [4x - 3 - 5y + 7] = [4x - 5y + 4].$
24. $[(x^2 - a^2) + (y^2 - 2a^2)] = [x^2 - a^2 + y^2 - 2a^2] = [x^2 + y^2 - 3a^2].$
 $[(x^2 - a^2) - (y^2 - 2a^2)] = [x^2 - a^2 - y^2 + 2a^2] = [x^2 - y^2 + a^2].$

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1. $x^2 - a^2 - 2ab - b^2 = (x^2) - (a^2 + 2ab + b^2).$
2. $12ab + x^2 - 9b^2 - 4a^2 = (x^2) - (4a^2 - 12ab + 9b^2).$
3. $y^2 - 4b^2 + 4ab - a^2 = (y^2) - (4b^2 - 4ab + a^2).$
4. $10ab + x^2 - a^2 - 25b^2 = (x^2) - (a^2 - 10ab + 25b^2).$
5. $x^2 - b^2 - 4a^2 + 4y^2 - 4ab - 4xy$
 $= (x^2 - 4xy + 4y^2) - (4a^2 + 4ab + b^2).$
6. $4ab + x^2 - 4b^2 + y^2 - a^2 - 2xy$
 $= (x^2 - 2xy + y^2) - (a^2 - 4ab + 4b^2).$
7. $16x^2 - a^2 - 16xy - b^2 + 2ab + 4y^2$
 $= (16x^2 - 16xy + 4y^2) - (a^2 - 2ab + b^2).$
8. $x^2 - b^2 - 10xy + 12ab - 36a^2 + 25y^2$
 $= (x^2 - 10xy + 25y^2) - (36a^2 - 12ab + b^2).$

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1. $(3)(-8) = -24.$
2. $(-2)(5) = -10.$
3. $(-7)(-3) = 21.$
4. $(-4x)(3) = -12x.$
5. $(-4x)^2 = (-4x)(-4x) = 16x^2.$
6. $(7)(-5a) = -35a.$
7. $(3a)(-6) = -18a.$
8. $(-2y)^2 = (-2y)(-2y) = 4y^2.$
9. $(-9a)(-10) = 90a.$
10. $(-3ax)^2 = (-3ax)(-3ax)$
 $= 9a^2x^2.$
11. $(4a)(-2a) = -8a^2.$

12. $(6abc)^2 = (6abc)(6abc)$
 $= 36a^2b^2c^2.$
13. $(-11x)(2x) = -22x^2.$
14. $(7x)(-3x) = -21x^2.$
15. $(-2a)^3 = (-2a)(-2a)(-2a)$
 $= -8a^3.$
16. $(-2a)(-3a^2) = 6a^3.$
17. $(5a^4)(7a^3) = 35a^7.$
18. $(-4x)^3 = -64x^3.$
19. $(a^8)(-20a) = -20a^9.$
20. $(-4a^5)(-6a^2) = 24a^7.$
21. $(+6y)^3 = 216y^3.$
22. $(4x)(5y) = 20xy.$
23. $(-3a^2x)^2 = 9a^4x^2.$
24. $(3x^2)(-y) = -3x^2y.$
25. $(5x^2y)(-2x^3) = -10x^5y.$
26. $(-6x^3y^2)^2 = 36x^6y^4.$
27. $(-x^4y)(-x^2y^4) = x^6y^5.$
28. $(2ax^2)^3 = (2ax^2)(2ax^2)(2ax^2)$
 $= 8a^3x^6.$
29. $(5a^3)(-4a^2)(-3a) = 60a^6.$
30. $(3ax)(-4a^2x)(-2ax^3) = 24a^4x^5.$

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1. $x + 3$
 $\frac{2x}{2x^2 + 6x}$
2. $7x^2 - 5$
 $\frac{3x^3}{21x^5 - 15x^3}$
3. $5x^2 - 2x$
 $\frac{-4x^2}{-20x^4 + 8x^3}$
4. $7xy - z$
 $\frac{3xy}{21x^2y^2 - 3xyz}$
5. $-4x^2 + 5x - 6$
 $\frac{6x^3}{-24x^5 + 30x^4 - 36x^3}$
6. $x^3 - 3x^2 + 4$
 $\frac{-5x^4}{-5x^7 + 15x^6 - 20x^4}$
7. $x^2 - 2xy + y^2$
 $\frac{-3xy}{-3x^3y + 6x^2y^2 - 3xy^3}$
8. $a^4 - a^2b^2 + b^4$
 $\frac{-a^2b^2}{-a^6b^2 + a^4b^4 - a^2b^6}$
9. $-a^2x^2 + 2ax - 7b^2$
 $\frac{-4abx}{4a^3bx^3 - 8a^2bx^2 + 28ab^3x}$
10. $7x^3 - 8x^2 - 12x + 6$
 $\frac{-\frac{3}{4}x^3}{-5\frac{1}{4}x^6 + 6x^5 + 9x^4 - 4\frac{1}{2}x^3}$
11. $-9a^2 - 12ax + 42x^2$
 $\frac{\frac{7}{3}ax^3}{-21a^3x^3 - 28a^2x^4 + 98ax^5}$
12. $4(2x - 3) = 8x - 12.$
13. $2x(x - y) = 2x^2 - 2xy.$
14. $-8(3x - 7) = -24x + 56.$
15. $-9(-4a + b) = 36a - 9b.$
16. $-3x(2x - 7) = -6x^2 + 21x.$
17. $-3(x^2 - 2x - 6) = -3x^2 + 6x + 18.$
18. $5xy(x^2 - 6x + 9) = 5x^3y - 30x^2y + 45xy.$
19. $-3x(ax - bx + 3cx^2) = -3ax^2 + 3bx^2 - 9cx^3.$
20. $-7ab(ax^2 + bx + c) = -7a^2bx^2 - 7ab^2x - 7abc.$

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$$\begin{array}{r}
 1. \quad x + 4 \\
 \quad x + 3 \\
 \hline
 \quad x^2 + 4x \\
 \quad + 3x + 12 \\
 (1) \cdot (2), \quad x^2 + 7x + 12
 \end{array}$$

$$\begin{array}{r}
 2. \quad 2x + 3 \\
 \quad x + 3 \\
 \hline
 \quad 2x^2 + 3x \\
 \quad + 6x + 9 \\
 (1) \cdot (2), \quad 2x^2 + 9x + 9
 \end{array}$$

$$\begin{array}{r}
 3. \quad 4x + 7 \\
 \quad 3x + 2 \\
 \hline
 \quad 12x^2 + 21x \\
 \quad + 8x + 14 \\
 (1) \cdot (2), \quad 12x^2 + 29x + 14
 \end{array}$$

$$\begin{array}{r}
 4. \quad 3x - 5 \\
 \quad 3x + 8 \\
 \hline
 \quad 9x^2 - 15x \\
 \quad + 24x - 40 \\
 (1) \cdot (2), \quad 9x^2 + 9x - 40
 \end{array}$$

$$\begin{array}{r}
 5. \quad 3x - 2 \\
 \quad 2x + 3 \\
 \hline
 \quad 6x^2 - 4x \\
 \quad + 9x - 6 \\
 (1) \cdot (2), \quad 6x^2 + 5x - 6
 \end{array}$$

$$\begin{array}{r}
 6. \quad -4a + 6 \\
 \quad 5a - 7 \\
 \hline
 \quad -20a^2 + 30a \\
 \quad + 28a - 42 \\
 (1) \cdot (2), \quad -20a^2 + 58a - 42
 \end{array}$$

$$\begin{array}{r}
 7. \quad 2x + y \\
 \quad x + 3y \\
 \hline
 \quad 2x^2 + xy \\
 \quad + 6xy + 3y^2 \\
 (1) \cdot (2), \quad 2x^2 + 7xy + 3y^2
 \end{array}$$

$$\begin{array}{r}
 8. \quad 2x - 3y \\
 \quad 3x - 2y \\
 \hline
 \quad 6x^2 - 9xy \\
 \quad - 4xy + 6y^2 \\
 (1) \cdot (2), \quad 6x^2 - 13xy + 6y^2
 \end{array}$$

$$\begin{array}{r}
 9. \quad 3x - \frac{1}{2} \\
 \quad 2x - \frac{1}{3} \\
 \hline
 \quad 6x^2 - x \\
 \quad - x + \frac{1}{6} \\
 (1) \cdot (2), \quad 6x^2 - 2x + \frac{1}{6}
 \end{array}$$

$$\begin{array}{r}
 10. \quad -3x + 11a \\
 \quad 5x - a \\
 \hline
 \quad -15x^2 + 55xa \\
 \quad + 3xa - 11a^2 \\
 (1) \cdot (2), \quad -15x^2 + 58xa - 11a^2
 \end{array}$$

$$\begin{array}{r}
 11. \quad ax - bx \\
 \quad cx + dx \\
 \hline
 \quad acx^2 - bcx^2 \\
 \quad + adx^2 - bdx^2 \\
 (1) \cdot (2), \quad acx^2 - bcx^2 + adx^2 - bdx^2
 \end{array}$$

$$\begin{array}{r}
 12. \quad -cx + d \\
 \quad bx - cx^2 \\
 \hline
 \quad -bcx^2 + bdx \\
 \quad + c^2x^3 - cdx^2 \\
 (1) \cdot (2), \quad -bcx^2 + bdx + c^2x^3 - cdx^2
 \end{array}$$

$$\begin{array}{r}
 13. \quad 4x - \frac{1}{3} \\
 \quad 6x + \frac{2}{5} \\
 \hline
 \quad 24x^2 - 2x \\
 \quad + 1\frac{3}{5}x - \frac{2}{15} \\
 (1) \cdot (2), \quad 24x^2 - \frac{2}{5}x - \frac{2}{15}
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^2 - 5x + 6 \\
 \quad x - 3 \\
 \hline
 \quad x^3 - 5x^2 + 6x \\
 \quad - 3x^2 + 15x - 18 \\
 (1) \cdot (2), \quad x^3 - 8x^2 + 21x - 18
 \end{array}$$

$$\begin{array}{rcl}
 15. & \begin{array}{r} 3x^2 - 3x - 7 \\ 2x + 4 \\ \hline 6x^3 - 6x^2 - 14x \\ + 12x^2 - 12x - 28 \\ \hline 6x^3 + 6x^2 - 26x - 28 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & 6x^3 + 6x^2 - 26x - 28 &
 \end{array}
 \qquad
 \begin{array}{rcl}
 17. & \begin{array}{r} a^2x^2 - 2a^2x + 4a^2 \\ ax + 2a \\ \hline a^3x^3 - 2a^3x^2 + 4a^3x \\ + 2a^3x^2 - 4a^3x + 8a^3 \\ \hline a^3x^3 + 8a^3 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & a^3x^3 + 8a^3 &
 \end{array}$$

$$\begin{array}{rcl}
 16. & \begin{array}{r} x^2 - xy + y^2 \\ x + y \\ \hline x^3 - x^2y + xy^2 \\ + x^2y - xy^2 + y^3 \\ \hline x^3 + y^3 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & x^3 + y^3 &
 \end{array}
 \qquad
 \begin{array}{rcl}
 18. & \begin{array}{r} 3x^3 - x^2 - 5x \\ 2x^3 - 5x^2 \\ \hline 6x^6 - 2x^5 - 10x^4 \\ - 15x^5 + 5x^4 + 25x^3 \\ \hline 6x^6 - 17x^5 - 5x^4 + 25x^3 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & 6x^6 - 17x^5 - 5x^4 + 25x^3 &
 \end{array}$$

$$\begin{array}{rcl}
 19. & \begin{array}{r} 2x^2 - 7x + 12 \\ x^2 - 3x - 5 \\ \hline 2x^4 - 7x^3 + 12x^2 \\ - 6x^3 + 21x^2 - 36x \\ - 10x^2 + 35x - 60 \\ \hline 2x^4 - 13x^3 + 23x^2 - x - 60 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & 2x^4 - 13x^3 + 23x^2 - x - 60 &
 \end{array}$$

$$\begin{array}{rcl}
 20. & \begin{array}{r} a^2 - \frac{1}{2}a + \frac{1}{4} \\ a^2 - a + \frac{1}{3} \\ \hline a^4 - \frac{1}{2}a^3 + \frac{1}{4}a^2 \\ - a^3 + \frac{1}{2}a^2 - \frac{1}{4}a \\ + \frac{1}{3}a^2 - \frac{1}{6}a + \frac{1}{12} \\ \hline a^4 - 1\frac{1}{2}a^3 + 1\frac{1}{12}a^2 - \frac{5}{12}a + \frac{1}{12} \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & a^4 - 1\frac{1}{2}a^3 + 1\frac{1}{12}a^2 - \frac{5}{12}a + \frac{1}{12} &
 \end{array}$$

$$\begin{array}{rcl}
 21. & \begin{array}{r} x^2 - xy + y^2 \\ x^2 + xy + y^2 \\ \hline x^4 - x^3y + x^2y^2 \\ + x^3y - x^2y^2 + xy^3 \\ + x^2y^2 - xy^3 + y^4 \\ \hline x^4 + x^2y^2 + y^4 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & x^4 + x^2y^2 + y^4 &
 \end{array}$$

$$\begin{array}{rcl}
 22. & \begin{array}{r} 3x^3 + 5x^2 - x + 2 \\ x^2 - 2x + 1 \\ \hline 3x^5 + 5x^4 - x^3 + 2x^2 \\ - 6x^4 - 10x^3 + 2x^2 - 4x \\ + 3x^3 + 5x^2 - x + 2 \\ \hline 3x^5 - x^4 - 8x^3 + 9x^2 - 5x + 2 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & 3x^5 - x^4 - 8x^3 + 9x^2 - 5x + 2 &
 \end{array}$$

$$\begin{array}{rcl}
 23. & \begin{array}{r} x^3 - x - 5 \\ 2x^2 - 3x - 4 \\ \hline 2x^5 - 2x^3 - 10x^2 \\ - 3x^4 + 3x^2 + 15x \\ - 4x^3 + 4x + 20 \\ \hline 2x^5 - 3x^4 - 6x^3 - 7x^2 + 19x + 20 \end{array} & \begin{array}{l} (1) \\ (2) \end{array} \\
 (1) \cdot (2), & 2x^5 - 3x^4 - 6x^3 - 7x^2 + 19x + 20 &
 \end{array}$$

$$24. \quad -x^3 + x^2 + 3x - 6 \quad (1)$$

$$-x^2 - 3x + 5 \quad (2)$$

$$\begin{array}{r} x^5 - x^4 - 3x^3 + 6x^2 \\ + 3x^4 - 3x^3 - 9x^2 + 18x \\ - 5x^3 + 5x^2 + 15x - 30 \end{array}$$

$$(1) \cdot (2), \quad x^5 + 2x^4 - 11x^3 + 2x^2 + 33x - 30$$

$$25. \quad a^3 - 5a^2 + 4a + 7 \quad (1)$$

$$a^3 + a^2 - a + 6 \quad (2)$$

$$\begin{array}{r} a^6 - 5a^5 + 4a^4 + 7a^3 \\ + a^5 - 5a^4 + 4a^3 + 7a^2 \\ - a^4 + 5a^3 - 4a^2 - 7a \\ + 6a^3 - 30a^2 + 24a + 42 \end{array}$$

$$(1) \cdot (2), \quad a^6 - 4a^5 - 2a^4 + 22a^3 - 27a^2 + 17a + 42$$

$$26. \quad 2x^3 - 5x^2 - 9x + 8 \quad (1)$$

$$8x^3 + 5x - 4 \quad (2)$$

$$\begin{array}{r} 16x^6 - 40x^5 - 72x^4 + 64x^3 \\ + 10x^4 - 25x^3 - 45x^2 + 40x \\ - 8x^3 + 20x^2 + 36x - 32 \end{array}$$

$$(1) \cdot (2), \quad 16x^6 - 40x^5 - 62x^4 + 31x^3 - 25x^2 + 76x - 32$$

$$27. \quad x^2y - y^2x \quad (1)$$

$$-5x^2y + 4xy \quad (2)$$

$$\begin{array}{r} -5x^4y^2 + 5x^3y^3 \\ + 4x^3y^2 - 4x^2y^3 \end{array}$$

$$(1) \cdot (2), \quad -5x^4y^2 + 5x^3y^3 + 4x^3y^2 - 4x^2y^3 \quad (3)$$

$$3x^2y - 7xy^2 \quad (4)$$

$$\begin{array}{r} -15x^6y^3 + 15x^5y^4 + 12x^5y^3 - 12x^4y^4 \\ + 35x^5y^4 - 28x^4y^4 - 35x^4y^5 + 28x^3y^5 \end{array}$$

$$(3) \cdot (4), \quad -15x^6y^3 + 50x^5y^4 + 12x^5y^3 - 40x^4y^4 - 35x^4y^5 + 28x^3y^5$$

$$28. \quad x^2 - xy + y^2 - xz - yz + z^2 \quad (1)$$

$$x + y + z \quad (2)$$

$$\begin{array}{r} x^3 - x^2y + xy^2 - x^2z - xyz + xz^2 \\ + x^2y - xy^2 - xyz + y^3 - y^2z + yz^2 \\ + x^2z - xyz - xz^2 + y^2z - yz^2 + z^3 \end{array}$$

$$(1) \cdot (2), \quad x^3 - 3xyz + y^3 + z^3$$

$$29. \quad a + b + c \quad (1)$$

$$a + b + c \quad (2)$$

$$\begin{array}{r} a^2 + ab + ac \\ + ab + b^2 + bc \\ + ac + bc + c^2 \end{array}$$

$$(1) \cdot (2), \quad a^2 + 2ab + b^2 + 2ac + 2bc + c^2$$

$$30. \quad c + d - \frac{1}{2} \quad (1)$$

$$\frac{c + d - \frac{1}{2}}{c^2 + cd - \frac{1}{2}c} \quad (2)$$

$$\begin{array}{r} + cd + d^2 - \frac{1}{2}d \\ - \frac{1}{2}c - \frac{1}{2}d + \frac{1}{4} \end{array}$$

$$(1) \cdot (2), \quad c^2 + 2cd + d^2 - c - d + \frac{1}{4}$$

$$31. \quad a - 2b + 3c - 4d \quad (1)$$

$$\frac{a - 2b + 3c - 4d}{a^2 - 2ab + 3ac - 4ad} \quad (2)$$

$$\begin{array}{r} - 2ab + 4b^2 - 6bc + 8bd \\ + 3ac - 6bc + 9c^2 - 12cd \\ - 4ad + 8bd - 12cd + 16d^2 \end{array}$$

$$(1) \cdot (2), \quad a^2 - 4ab + 6ac - 8ad + 4b^2 - 12bc + 16bd + 9c^2 - 24cd + 16d^2$$

$$32. \quad x + y + z \quad (1)$$

$$\frac{x + y + z}{x^2 + xy + xz} \quad (2)$$

$$\begin{array}{r} + xy + y^2 + yz \\ + xz + yz + z^2 \end{array}$$

$$(1) \cdot (2), \quad x^2 + 2xy + 2xz + y^2 + 2yz + z^2 \quad (3)$$

$$\frac{x + y + z}{x^3 + 2x^2y + 2x^2z + xy^2 + 2xyz + xz^2} \quad (4)$$

$$\begin{array}{r} + x^2y + 2xy^2 + 2xyz + y^3 + 2y^2z + yz^2 \\ + x^2z + 2xyz + 2xz^2 + y^2z + 2yz^2 + z^3 \end{array}$$

$$(3) \cdot (4), \quad x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3$$

$$33. \quad x + y \quad (1) \quad x - y \quad (6)$$

$$\frac{x + y}{x^2 + xy} \quad (2) \quad \frac{x - y}{x^2 - xy} \quad (7)$$

$$\frac{xy + y^2}{x^2 + 2xy + y^2} \quad (3) \quad \frac{-xy + y^2}{x^2 - 2xy + y^2} \quad (8)$$

$$(1) \cdot (2), \quad x^2 + 2xy + y^2 \quad (4) \quad \frac{x - y}{x^3 - 2x^2y + xy^2} \quad (9)$$

$$\frac{x + y}{x^3 + 2x^2y + xy^2} \quad (5) \quad \frac{-x^2y + 2xy^2 - y^3}{x^3 - 3x^2y + 3xy^2 - y^3} \quad (10)$$

$$(3) \cdot (4), \quad x^3 + 3x^2y + 3xy^2 + y^3 \quad (5) \quad (8) \cdot (9), \quad x^3 - 3x^2y + 3xy^2 - y^3 \quad (10)$$

$$\frac{x^3 - 3x^2y + 3xy^2 - y^3}{2x^3 + 6xy^2}$$

$$(5) + (10), \quad 2x^3 + 6xy^2$$

$$34. \quad 2a - b + 3 \quad (1)$$

$$\frac{2a - b + 3}{4a^2 - 2ab + 6a} \quad (2)$$

$$\begin{array}{r} - 2ab + b^2 - 3b \\ + 6a - 3b + 9 \end{array}$$

$$(1) \cdot (2), \quad 4a^2 - 4ab + b^2 + 12a - 6b + 9$$

$$35. \quad \frac{2x - 3ay}{2x - 3ay} \quad (1)$$

$$\frac{2x - 3ay}{4x^2 - 6axy} \quad (2)$$

$$\frac{-6axy + 9a^2y^2}{(1) \cdot (2), 4x^2 - 12axy + 9a^2y^2} \quad (3)$$

$$\frac{2x - 3ay}{8x^3 - 24ax^2y + 18a^2xy^2} \quad (4)$$

$$(3) \cdot (4), \frac{-12ax^2y + 36a^2xy^2 - 27a^3y^3}{8x^3 - 36ax^2y + 54a^2xy^2 - 27a^3y^3}$$

$$36. \quad \frac{x + 2y}{x + 2y} \quad (1) \quad \frac{x - 2y}{x - 2y} \quad (4)$$

$$\frac{x + 2y}{x^2 + 2xy} \quad (2) \quad \frac{x - 2y}{x^2 - 2xy} \quad (5)$$

$$\frac{+2xy + 4y^2}{(1) \cdot (2) \quad x^2 + 4xy + 4y^2 \quad (3) \quad (4) \cdot (5), \quad x^2 - 4xy + 4y^2 \quad (6)}$$

$$\frac{x^2 - 4xy + 4y^2}{(3) - (6), \quad 8xy}$$

$$37. \quad \frac{4x - 3y}{4x - 3y} \quad (1) \quad \frac{3x + 4y}{3x + 4y} \quad (4)$$

$$\frac{4x - 3y}{16x^2 - 12xy} \quad (2) \quad \frac{3x + 4y}{9x^2 + 12xy} \quad (5)$$

$$(1) \cdot (2), \frac{-12xy + 9y^2}{16x^2 - 24xy + 9y^2} \quad (3) \quad (4) \cdot (5), \frac{12xy + 16y^2}{9x^2 + 24xy + 16y^2} \quad (6)$$

$$(3) - (6), \frac{9x^2 + 24xy + 16y^2}{7x^2 - 48xy - 7y^2}$$

$$38. \quad \frac{x - 3}{x - 3} \quad (a), \quad \frac{2x - 1}{2x - 1} \quad (b)$$

$$\frac{x - 3}{x^2 - 3x} \quad \frac{2x - 1}{4x^2 - 2x}$$

$$\frac{-3x + 9}{x^2 - 6x + 9} \quad \frac{-2x + 1}{(b)^2, 4x^2 - 4x + 1} \quad (2)$$

$$\frac{x - 3}{x^3 - 6x^2 + 9x} \quad \frac{-3x^2 + 18x - 27}{(a)^3, x^3 - 9x^2 + 27x - 27} \quad (1)$$

$$(b)^2, \frac{+4x^2 - 4x + 1}{(1) - (2), x^3 - 13x^2 + 31x - 28} \quad (2)$$

$$39. \quad \frac{x^a - 3}{x^a + 4} \quad (1) \quad 40. \quad \frac{x^{2a} + 5}{x^{2a} + 5} \quad (1)$$

$$\frac{x^a - 3}{x^{2a} - 3x^a} \quad (2) \quad \frac{x^{2a} + 5}{x^{4a} + 5x^{2a}} \quad (2)$$

$$(1) \cdot (2), \frac{+4x^a - 12}{(1) \cdot (2), x^{2a} + x^a - 12} \quad \frac{+5x^{2a} + 25}{(1) \cdot (2), x^{4a} + 10x^{2a} + 25}$$

$$\begin{array}{r}
 41. \quad 2x^a - 3 \\
 \underline{2x^a - 3} \\
 4x^{2a} - 6x^a \\
 \quad - 6x^a + 9 \\
 \hline
 4x^{2a} - 12x^a + 9 \\
 \underline{2x^a - 3} \\
 8x^{3a} - 24x^{2a} + 18x^a \\
 \quad - 12x^{2a} + 36x^a - 27 \\
 \hline
 (a)^3, \quad 8x^{3a} - 36x^{2a} + 54x^a - 27
 \end{array}$$

$$\begin{array}{r}
 42. \quad 2x^{2a} - 3x \quad (1) \\
 \underline{2x^{2a} - 3x} \quad (2) \\
 4x^{4a} - 6x^{2a+1} \\
 \quad - 6x^{2a+1} + 9x^2 \\
 \hline
 (1) \cdot (2), \quad 4x^{4a} - 12x^{2a+1} + 9x^2
 \end{array}$$

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1. $5(x - 1) = 30.$

Removing parentheses,

$$5x - 5 = 30.$$

Whence $x = 7.$

2. $3 + 2(x - 3) = 1.$

$$3 + 2x - 6 = 1.$$

Whence $x = 2.$

3. $7(3x - 2) + 11 = 60.$

$$21x - 14 = 49.$$

Whence $x = 3.$

4. $4(2x - 5) + 15 = 3(x + 10).$

$$8x - 20 + 15 = 3x + 30.$$

Whence $x = 7.$

5. $12y - 2(4y - 7) - 16$
 $= 0.$

$$12y - 8y + 14 - 16 = 0.$$

Whence $y = \frac{1}{2}.$

6. $9y - 3(2y - 4)$
 $= 2(5 - 4y) + 2.$

$$9y - 6y + 12 = 10 - 8y + 2.$$

Whence $y = 0$

7. $4 - 2(4y - 3) = 3(y - 5).$

Simplifying,

$$4 - 8y + 6 = 3y - 15.$$

Whence $y = 2\frac{3}{11}.$

8.

$$7(y - 3) - 2(4 + y) = 9.$$

$$7y - 21 - 8 - 2y = 9.$$

Whence $y = 7\frac{2}{3}.$

9.

$$5(n - 7) + 24 + 4n = 0.$$

$$5n - 35 + 24 + 4n = 0.$$

Whence $n = 1\frac{2}{9}.$

10. $5n - 9(2n + 4) = 2(n - 9).$

$$5n - 18n - 36 = 2n - 18.$$

Whence $n = -1\frac{1}{3}.$

11. $7n - 12 - 2(n - 5) = n - 19.$

$$6n - 2n + 10 = -7.$$

Whence $n = -4\frac{1}{4}.$

12. $4(2n - 7) - 3(4n - 8) + 4$

$$= 2n - 3.$$

$$8n - 28 - 12n + 24 + 4 = 2n - 3.$$

Whence $n = \frac{1}{2}.$

13. $3h - 2(4h + 8) = 3h - 24.$

$$-8h - 16 = -24.$$

Whence $h = 1.$

14. $5(3h + 1) - 7h = 3(h - 7) + 4.$

$$15h + 5 - 7h = 3h - 21 + 4.$$

Whence $h = -4\frac{2}{5}.$

15.

Simplifying,

Whence

$$(h - 2)(h - 5) = (h + 3)(h + 2).$$

$$h^2 - 7h + 10 = h^2 + 5h + 6.$$

$$h = \frac{1}{3}.$$

$$16. \quad (h + 4)(h + 3) - (h + 2)(h + 1) - 42 = 0.$$

$$h^2 + 7h + 12 - h^2 - 3h - 2 - 42 = 0.$$

$$\text{Whence} \quad h = 8.$$

$$17. \quad (x + 4)(x + 6) = (x + 18)(x + 13).$$

$$x^2 + 10x + 24 = x^2 + 31x + 234.$$

$$\text{Whence} \quad x = -10.$$

$$18. \quad (k - 7)(5 + k) - (k - 5)(k + 7) + 5 = 0.$$

$$(k^2 - 2k - 35) - (k^2 + 2k - 35) + 5 = 0.$$

$$\text{Simplifying,} \quad -2k - 2k + 5 = 0.$$

$$\text{Whence} \quad k = 1\frac{1}{4}.$$

$$19. \quad (2x - 5)(4x - 7) = 8x^2 + 52.$$

$$\text{Simplifying,} \quad 8x^2 - 34x + 35 = 8x^2 + 52.$$

$$\text{Whence} \quad x = -\frac{1}{2}.$$

$$20. \quad (3y + 5)(4y + 7) - (2y + 3)(6y + 11) - 2 = 0.$$

$$\text{Simplifying,} \quad 12y^2 + 41y + 35 - (12y^2 + 40y + 33) - 2 = 0.$$

$$\text{Whence} \quad y = 0.$$

$$21. \quad (n + 3)(6n + 5) - (2n + 4)(3n - 8) = 38.$$

$$\text{Simplifying,} \quad 6n^2 + 23n + 15 - (6n^2 - 4n - 32) = 38.$$

$$\text{Whence} \quad n = -\frac{1}{3}.$$

$$22. \quad (x + 3)^2 - (x + 5)^2 = -40.$$

$$\text{Expanding,} \quad x^2 + 6x + 9 - x^2 - 10x - 25 = -40.$$

$$\text{Whence} \quad x = 6.$$

$$23. \quad (x + 2)^2 - (x - 4)^2 + 48 = 0.$$

$$\text{Expanding,} \quad x^2 + 4x + 4 - x^2 + 8x - 16 + 48 = 0.$$

$$\text{Whence} \quad x = -3.$$

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$$1. \text{ Area, } a \cdot b = ab; \text{ perimeter, } 2a + 2b.$$

$$2. \text{ Area, } 3(x - 4) = 3x - 12; \text{ perimeter, } 2(x - 4) + 2 \cdot 3 = 2x - 2.$$

$$3. \text{ Area, } (2x - 4)(x + 2) = 2x^2 - 8;$$

$$\text{perimeter, } 2(2x - 4) + 2(x + 2) = 6x - 4.$$

$$4. 4 \cdot \$100 = \$400.$$

$$5. n \cdot \$80 = \$80n.$$

$$6. a \cdot b \text{ cents} = ab \text{ cents.}$$

$$7. (xa + yb) \text{ cents.}$$

$$8. x(b + 10) \text{ dollars} = (bx + 10x) \text{ dollars.}$$

$$9. x(b + 2) \text{ dollars} + y(c - 3) \text{ dollars} = (bx + 2x + cy - 3y) \text{ dollars.}$$

$$10. 5\% \text{ of } 16 = .8; \text{ of } x = .05x.$$

11. 3% of $(x + 120) = .03x + 3.6$; of $(12x - 300a) = .36x - 9a$.
12. $3(n + 4)$ years $= (3n + 12)$ years.
13. (a) $.04x = 180$.
 (b) $.03x = .05(1000 - x)$.
 (c) $.05x = .04(1000 - x) - 20$.
14. Width, $10 + 2 + 2 = 14$ inches; length, $12 + 2 + 2 = 16$ inches.
15. Width, $10 + x + x = (10 + 2x)$ inches;
 length, $12 + x + x = (12 + 2x)$ inches;
 area, $(10 + 2x)(12 + 2x) = (120 + 44x + 4x^2)$ square inches;
 area of picture $= 12 \cdot 10 = 120$ square inches;
 area of frame $= 120 + 44x + 4x^2 - 120$
 $= (44x + 4x^2)$ square inches.

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2. Let $n =$ the less number.
 Then $49 - n =$ the greater number.
 Therefore $2(49 - n) - 13 = 5n$.
 Whence $n = 12\frac{1}{7}$,
 and $49 - n = 36\frac{6}{7}$.
3. Let $n =$ the less number.
 Then $143 - n =$ the greater number.
 Therefore $10n + 5(143 - n) = 950$.
 Whence $n = 47$,
 and $143 - n = 96$.
4. Let $n =$ the greater number.
 Then $45 - n =$ the less number.
 Therefore $5n + 4(45 - n) = 207$.
 Whence $n = 27$,
 and $45 - n = 18$.
5. Let $n =$ the less number.
 Then $88 - n =$ the greater number.
 Therefore $3(88 - n) = 5n + 29$.
 Whence $n = 29\frac{3}{8}$,
 and $88 - n = 58\frac{5}{8}$.
6. Let $n =$ the greater number.
 Then $93 - n =$ the less number.
 Therefore $7(93 - n) - 7 = 6n$.
 Whence $n = 49\frac{7}{13}$,
 and $93 - n = 43\frac{6}{13}$.

7. Let $n =$ the less number.
 Then $48 - n =$ the greater number.
 Therefore $2(48 - n) - 7 = 3n - 5$.
 Whence $n = 18\frac{4}{5}$,
 and $48 - n = 29\frac{1}{5}$.
8. Let $n =$ one number.
 Then $12\frac{1}{2} - n =$ the other number.
 Therefore $7n - 10(12\frac{1}{2} - n) = 45$.
 Whence $n = 10$,
 and $12\frac{1}{2} - n = 2\frac{1}{2}$.
9. Let n and $121 - n =$ the two parts of 121.
 Therefore $4n + 8 = 3(121 - n)$.
 Whence $n = 50\frac{5}{7}$, one part,
 and $121 - n = 70\frac{2}{7}$, the other.
10. Let n and $15 - n =$ the numbers.
 Therefore $2n - 5(15 - n) = 240$.
 Whence $n = 45$, one number,
 and $15 - n = -30$, the other number.
11. Let $n =$ one number.
 Then $14 - n =$ the other.
 Therefore $9n - 11(14 - n) = 0$.
 Whence $n = 7\frac{7}{10}$,
 and $14 - n = 6\frac{3}{10}$.
12. Let $n =$ the first number.
 Then $n + 1 =$ the next consecutive number.
 Therefore $n^2 + n^2 + 2n + 1 = 17 + 2n^2$.
 Whence $n = 8$,
 and $n + 1 = 9$.
13. Let $n =$ one number.
 Then $n + 1 =$ the next number.
 Therefore $(n^2 + 2n + 1) - n^2 = 75$.
 Whence $n = 37$,
 and $n + 1 = 38$.
14. Let n and $n + 1 =$ the numbers.
 Therefore $(n^2 + 2n + 1) - n^2 = 23$.
 Whence $n = 11$, the first number,
 and $n + 1 = 12$, the next number.

15. Let $n =$ one odd number.
 Then $n + 2 =$ the next odd number.
 Therefore $(n^2 + 4n + 4) - n^2 = 104$.
 Whence $n = 25$,
 and $n + 2 = 27$.
16. Let $n =$ one odd number.
 Then $n + 2 =$ the next odd number.
 Therefore $(n^2 + 4n + 4) - n^2 = 40$.
 Whence $n = 9$,
 and $n + 2 = 11$.
17. Let $n =$ one even number.
 Then $n + 2 =$ the next even number.
 Therefore $n(n + 2) = (n^2 + 4n + 4) - 56$.
 Whence $n = 26$,
 and $n + 2 = 28$.
18. Let n and $n + 2 =$ two consecutive odd numbers.
 Therefore $n(n + 2) = n^2 + 46$.
 Whence $n = 23$,
 and $n + 2 = 25$.
20. Let $s =$ the side of the square field in rods.
 Then $s + 30 =$ the length of the rectangular field in rods,
 and $s - 20 =$ the breadth of the rectangular field in rods.
 Therefore $s^2 = (s + 30)(s - 20)$.
 Whence $s = 60$,
 $s + 30 = 90$,
 and $s - 20 = 40$.
 Area of square $= 60 \cdot 60 = 3600$ square rods, or $22\frac{1}{2}$ acres.
 Area of rectangle $= 90 \cdot 40 = 3600$ square rods, or $22\frac{1}{2}$ acres.
21. Let $x =$ the width of the court in feet.
 Then $2x + 24 =$ the length of the court in feet.
 Therefore $2x + 2(2x + 24) = 210$.
 Whence $x = 27$,
 and $2x + 24 = 78$.
22. Let $x =$ the width of the court in feet.
 Then $2x + 6 =$ the length of the court in feet.
 Therefore $2x + 2(2x + 6) = 228$.
 Whence $x = 36$,
 and $2x + 6 = 78$.

23. Let l = the length of the court in feet.
 Then $l - 20$ = the breadth of the court in feet.
 Therefore $2l + 2(l - 20) = 80 \cdot 3$.
 Whence $l = 70$,
 and $l - 20 = 50$.
24. Let n = the breadth of the field in yards.
 Then $3n - 50$ = the length of the field in yards.
 Therefore $2n + 2(3n - 50) = \frac{780}{3}$.
 Whence $n = 45$,
 and $3n - 50 = 85$.
26. Let d = the number of dimes.
 Then $38 - d$ = the number of quarters,
 and $10d$ = the value of the dimes in cents.
 Also $(38 - d)25$ = the value of the quarters in cents.
 Therefore $10d + (38 - d)25 = 530$.
 Solving, $d = 28$,
 and $38 - d = 10$.
27. Let d = the number of dimes.
 Then $d + 5$ = the number of nickels,
 and $2d + 5$ = the number of quarters.
 Also $10d$ = the value of the dimes in cents,
 $5(d + 5)$ = the value of the nickels in cents,
 and $25(2d + 5)$ = the value of the quarters in cents.
 Therefore $10d + 5(d + 5) + 25(2d + 5) = 605$.
 Solving, $d = 7$,
 $d + 5 = 12$,
 and $2d + 5 = 19$.
28. Let n = the number of nickels.
 Then $40 - n$ = the number of dimes.
 Also $5n$ = the value of the nickels in cents,
 and $10(40 - n)$ = the value of the dimes in cents.
 Therefore $5n + 10(40 - n) = 290$.
 Solving, $n = 22$,
 and $40 - n = 18$.
29. Let b = B's age now in years.
 Then $b + 20$ = A's age now in years.
 Therefore $b + 20 + 10 = 2(b + 10)$.
 Whence $b = 10$,
 and $b + 20 = 30$.

35. Let x = the number of dollars invested at 5%.
 Then $1240 - x$ = the number of dollars invested at 6%.
 Hence $.05x$ = the income from the 5% investment,
 and $.06(1240 - x)$ = the income from the 6% investment.
 Therefore $.06(1240 - x) = 15 + .05x$.
 Solving, $x = 540$,
 and $1240 - x = 700$.
 Hence \$540 is invested at 5%, and \$700 at 6%.

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1. $-10 \div 2 = -5.$
2. $12 \div (-3) = -4.$
3. $-16 \div (-4) = 4.$
4. $8a^3 \div a^2 = 8a.$
5. $-4a^6 \div 2a^2 = -2a^4.$
6. $6x^2 \div (-3x) = -2x.$
7. $-18x^7 \div (-6x^4) = 3x^3.$
8. $-25ax^3 \div 5ax = -5x^2.$
9. $12ax^3 \div (-3bx^3) = -\frac{4a}{b}.$
10. $-28ay^4 \div (-7cy^3) = \frac{4ay}{c}.$
11. $70x^4y^9 \div (-10x^2y^8) = -7x^2y.$
12. $48ax^5 \div (-16bx^4) = -\frac{3ax}{b}.$
13. $-36x^6y^6 \div (-6x^2y^3) = 6x^4y^3.$
14. $63c^3d^5 \div (-9bcd^3) = \frac{-7c^2d^2}{b}.$
15. $64a^5b^7 \div (-16ab^7) = -4a^4.$
16. $-28a^6b^{12} \div (-7a^4b^8) = 4a^2b^4.$
17. $\frac{15x^2y^4z^5}{75xy^4z} = \frac{xz^4}{5}.$
18. $\frac{42x^{24}y^{56}}{-6x^8y^7} = -7x^{16}y^{49}.$
19. $\frac{x^a}{x^2} = x^{a-2}.$
20. $\frac{-x^{3a}}{x^a} = -x^{2a}.$
21. $\frac{-17a^9b^8c^7}{51a^7b^8c} = \frac{-a^2c^6}{3}.$
22. $\frac{39x^{13}y^{26}z^{39}}{-13x^{13}y^{13}z^{13}} = -3y^{13}z^{26}.$
23. $\frac{-11a^2b^{11}c^{13}}{66ac^{12}} = \frac{-ab^{11}c}{6}.$
24. $\frac{-121a^{11}b^{22}c^{33}}{-11a^{11}b^{11}c^{11}} = 11b^{11}c^{22}.$
25. $\frac{x^{a+b}}{x^b} = x^a.$
26. $\frac{x^c}{x} = x^{c-1}.$
27. $\frac{3a^3x^3a}{a^2x^a} = 3ax^{2a}.$
28. $\frac{6x^{2a+3}}{-2x^4} = -3x^{2a-1}.$

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1. $\frac{6x^2 - 4x}{2x} = 3x - 2.$
2. $\frac{9x - 18x^4}{-3x} = \frac{9x}{-3x} + \frac{-18x^4}{-3x} = -3 + 6x^3.$
3. $\frac{4xy - 12x^2}{-4x} = \frac{4xy}{-4x} + \frac{-12x^2}{-4x} = -y + 3x.$
4. $\frac{9ax^3 - 12x^5}{-3ax^2} = \frac{9ax^3}{-3ax^2} + \frac{-12x^5}{-3ax^2} = -3x + \frac{4x^3}{a}.$
5. $\frac{25x^2y + 30xy^5}{-5xy} = -5x - 6y^4.$
6. $\frac{16bx^4 - 36x^2}{4bx^2} = 4x^2 - \frac{9}{b}.$
7. $\frac{14x^3y^4 - 28x^5y^6}{7x^2y^3} = 2xy - 4x^3y^3.$

8. $\frac{4x^4y - 8x^6y^2 + 12x^8y^4}{4x^4y} = 1 - 2x^2y + 3x^4y^3.$
9. $\frac{a^3cd^2 - a^2c^3}{a^2cd} = ad - \frac{c^2}{d}.$
10. $\frac{ax^4 - bx^3 + cx^2}{-x^2} = -ax^2 + bx - c.$
11. $\frac{15a^2b^2 + 9a^4b^2 - 30a^6b^2}{-3a^2b^2} = -5 - 3a^2 + 10a^4.$
12. $\frac{16a^4b^5c^6 - 24a^5b^6c^7 - 48a^6b^7c^8}{8a^3b^2c} = 2ab^3c^5 - 3a^2b^4c^6 - 6a^3b^5c^7.$
13. $\frac{85xyz - 51x^2yz^2 + 102x^3yz^3 - 170x^5y^5z}{-17xyz} = -5 + 3xz - 6x^2z^2 + 10x^4y^4.$
14. $\frac{4(x-3) + a(x-3)}{x-3} = 4 + a.$
15. $\frac{3x(3x+4) - 4y(3x+4)}{3x+4} = 3x - 4y.$
16. $\frac{5a(2x^2 - y) - 3b(2x^2 - y)}{2x^2 - y} = 5a - 3b.$
17. $\frac{(a+b)^4 - 3(a+b)^3}{(a+b)^2} = (a+b)^2 - 3(a+b).$
18. $\frac{21(x-y)^7 - 35(x-y)^5}{-7(x-y)^5} = -3(x-y)^2 + 5.$
19. $\frac{16(3x-4)^4 - 24(3x-4)^5 - 48(3x-4)^7}{-8(3x-4)^4} = -2 + 3(3x-4) + 6(3x-4)^3.$
20. $\frac{-5(ac^2 - 2d)^3 + x(ac^2 - 2d)}{5(ac^2 - 2d)} = -(ac^2 - 2d)^2 + \frac{x}{5}.$
21. $\frac{4x^4 - 8x^{3a} - 6x^{2a-2}}{2x^3} = 2x - 4x^{3a-3} - 3x^{2a-5}.$
22. $\frac{3x^a - 2x^{a+1} - x^{a+2} + x^2}{x^2} = 3x^{a-2} - 2x^{a-1} - x^a + 1.$
23. $\frac{6x^{2a-3} - 12x^{4a+4} - 18x^{3a+5}}{-3x^{2a}} = -2x^{-3} + 4x^{2a+4} + 6x^{a+5}.$

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$$\begin{array}{r|l}
 1. & x^2 + 7x + 12 \\
 & x^2 + 3x \\
 \hline
 & 4x + 12 \\
 & \underline{4x + 12}
 \end{array}
 \begin{array}{l}
 x + 3 \\
 x + 4
 \end{array}$$

$$\begin{array}{r|l}
 2. & x^2 - 2x - 15 \\
 & x^2 - 5x \\
 \hline
 & 3x - 15 \\
 & \underline{3x - 15}
 \end{array}
 \begin{array}{l}
 x - 5 \\
 x + 3
 \end{array}$$

$$\begin{array}{r|l}
 3. \quad x^2 + 5x + 6 & x + 3 \\
 x^2 + 3x & \underline{x + 2} \\
 \hline
 2x + 6 & \\
 2x + 6 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 4. \quad x^2 - 7x + 6 & x - 1 \\
 x^2 - x & \underline{x - 6} \\
 \hline
 -6x + 6 & \\
 -6x + 6 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 5. \quad 6x^2 - 13x + 6 & 2x - 3 \\
 6x^2 - 9x & \underline{3x - 2} \\
 \hline
 -4x + 6 & \\
 -4x + 6 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 6. \quad 25x^4 + 30x^2 - 7 & 5x^2 + 7 \\
 25x^4 + 35x^2 & \underline{5x^2 - 1} \\
 \hline
 -5x^2 - 7 & \\
 -5x^2 - 7 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 7. \quad 12a^2 + 19a - 21 & 4a - 3 \\
 12a^2 - 9a & \underline{3a + 7} \\
 \hline
 28a - 21 & \\
 28a - 21 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 8. \quad x^3 - 2x^2 + 4x - 8 & x - 2 \\
 x^3 - 2x^2 & \underline{x^2 + 4} \\
 \hline
 4x - 8 & \\
 4x - 8 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 9. \quad a^3 + 3a^2b + 3ab^2 + b^3 & a + b \\
 a^3 + a^2b & \underline{a^2 + 2ab + b^2} \\
 \hline
 2a^2b + 3ab^2 & \\
 2a^2b + 2ab^2 & \\
 \hline
 ab^2 + b^3 & \\
 ab^2 + b^3 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 10. \quad x^3 - 15x^2 + 65x - 63 & x - 7 \\
 x^3 - 7x^2 & \underline{x^2 - 8x + 9} \\
 \hline
 -8x^2 + 65x & \\
 -8x^2 + 56x & \\
 \hline
 9x - 63 & \\
 9x - 63 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 11. \quad -25x^3 + 5x^2 + 5x - 1 & 5x^2 - 1 \\
 -25x^3 & -5x + 1 \\
 \hline
 & 5x^2 - 1 \\
 & 5x^2 - 1 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 12. \quad 2x^3 - 14x^2 + 14x + 12 & 2x - 4 \\
 2x^3 - 4x^2 & x^2 - 5x - 3 \\
 \hline
 & -10x^2 + 14x \\
 & -10x^2 + 20x \\
 \hline
 & -6x + 12 \\
 & -6x + 12 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 13. \quad 3a^3 + 28a^2 + 89a - 140 & 3a - 5 \\
 3a^3 - 5a^2 & a^2 + 11a + 48 + \frac{100}{3a - 5} \\
 \hline
 & 33a^2 + 89a \\
 & 33a^2 - 55a \\
 \hline
 & 144a - 140 \\
 & 144a - 240 \\
 \hline
 & 100
 \end{array}$$

$$\begin{array}{r|l}
 14. \quad 6x^3 - 23x^2 + 37x - 24 & 2x - 3 \\
 6x^3 - 9x^2 & 3x^2 - 7x + 8 \\
 \hline
 & -14x^2 + 37x \\
 & -14x^2 + 21x \\
 \hline
 & 16x - 24 \\
 & 16x - 24 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 15. \quad 12a^3 - 53a^2 + 53a + 8 & 4a^2 - 7a - 1 \\
 12a^3 - 21a^2 - 3a & 3a - 8 \\
 \hline
 & -32a^2 + 56a + 8 \\
 & -32a^2 + 56a + 8 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 16. \quad 15a^3 - 56a^2 + 99a - 70 & 3a^2 - 7a + 10 \\
 15a^3 - 35a^2 + 50a & 5a - 7 \\
 \hline
 & -21a^2 + 49a - 70 \\
 & -21a^2 + 49a - 70 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 17. \quad a^4 + 11a^3 + 23a^2 - 55a - 140 & a^2 - 5 \\
 a^4 - 5a^2 & a^2 + 11a + 28 \\
 \hline
 & 11a^3 + 28a^2 \\
 & 11a^3 - 55a \\
 \hline
 & 28a^2 - 140 \\
 & 28a^2 - 140 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 18. & a^4 + 4a^3 + 6a^2 + 4a + 1 \quad | \quad a^2 + 2a + 1 \\
 & \underline{a^4 + 2a^3 + a^2} \quad \quad \quad \underline{a^2 + 2a + 1} \\
 & 2a^3 + 5a^2 + 4a \\
 & \underline{2a^3 + 4a^2 + 2a} \\
 & a^2 + 2a + 1 \\
 & \underline{a^2 + 2a + 1}
 \end{array}$$

$$\begin{array}{r|l}
 19. & a^4 - 8a^3 + 24a^2 - 32a + 16 \quad | \quad a^2 - 4a + 4 \\
 & \underline{a^4 - 4a^3 + 4a^2} \quad \quad \quad \underline{a^2 - 4a + 4} \\
 & -4a^3 + 20a^2 - 32a \\
 & \underline{-4a^3 + 16a^2 - 16a} \\
 & 4a^2 - 16a + 16 \\
 & \underline{4a^2 - 16a + 16}
 \end{array}$$

$$\begin{array}{r|l}
 20. & x^4 + 4x^3 - 31x^2 + 40x + 21 \quad | \quad x^2 - 7x - 3 \\
 & \underline{x^4 - 7x^3 - 3x^2} \quad \quad \quad \boxed{x^2 + 11x + 49 + \frac{416x + 168}{x^2 - 7x - 3}} \\
 & 11x^3 - 28x^2 + 40x \\
 & \underline{11x^3 - 77x^2 - 33x} \\
 & 49x^2 + 73x + 21 \\
 & \underline{49x^2 - 343x - 147} \\
 & 416x + 168
 \end{array}$$

$$\begin{array}{r|l}
 21. & 10x^4 - 27x^3 - 42x^2 + 11x - 36 \quad | \quad 2x^2 - 5x + 9 \\
 & \underline{10x^4 - 25x^3 + 45x^2} \quad \quad \quad \boxed{5x^2 - x - 46 + \frac{378 - 210x}{2x^2 - 5x + 9}} \\
 & -2x^3 - 87x^2 + 11x \\
 & \underline{-2x^3 + 5x^2 - 9x} \\
 & -92x^2 + 20x - 36 \\
 & \underline{-92x^2 + 230x - 414} \\
 & -210x + 378
 \end{array}$$

$$\begin{array}{r|l}
 22. & x^4 - 3a^2x^2 + a^4 \quad \quad \quad | \quad x^2 - ax + a^2 \\
 & \underline{x^4 - ax^3 + a^2x^2} \quad \quad \quad \boxed{x^2 + ax - 3a^2 + \frac{4a^4 - 4a^3x}{x^2 - ax + a^2}} \\
 & ax^3 - 4a^2x^2 + a^4 \\
 & \underline{ax^3 - a^2x^2 + a^3x} \\
 & -3a^2x^2 - a^3x + a^4 \\
 & \underline{-3a^2x^2 + 3a^3x - 3a^4} \\
 & -4a^3x + 4a^4
 \end{array}$$

$$\begin{array}{r|l}
 23. & a^4 + 3a^2b^2 + 4b^4 \quad \quad \quad | \quad a^2 - ab + 2b^2 \\
 & \underline{a^4 - a^3b + 2a^2b^2} \quad \quad \quad \underline{a^2 + ab + 2b^2} \\
 & a^3b + a^2b^2 + 4b^4 \\
 & \underline{a^3b - a^2b^2 + 2ab^3} \\
 & 2a^2b^2 - 2ab^3 + 4b^4 \\
 & \underline{2a^2b^2 - 2ab^3 + 4b^4}
 \end{array}$$

$$\begin{array}{r|l}
 24. \quad 16x^4 - 60x^2y^2 + 25y^4 & 4x^2 - 10xy - 5y^2 \\
 \hline
 16x^4 - 40x^3y - 20x^2y^2 & 4x^2 + 10xy + 15y^2 + \frac{200xy^3 + 100y^4}{4x^2 - 10xy - 5y^2} \\
 \hline
 40x^3y - 40x^2y^2 + 25y^4 & \\
 40x^3y - 100x^2y^2 - 50xy^3 & \\
 \hline
 & 60x^2y^2 + 50xy^3 + 25y^4 \\
 & 60x^2y^2 - 150xy^3 - 75y^4 \\
 & \hline
 & 200xy^3 + 100y^4
 \end{array}$$

$$\begin{array}{r|l}
 25. \quad 9a^4 + 26a^2b^2 + 49b^4 & 3a^2 + 4ab + 7b^2 \\
 9a^4 + 12a^3b + 21a^2b^2 & 3a^2 - 4ab + 7b^2 \\
 \hline
 -12a^3b + 5a^2b^2 + 49b^4 & \\
 -12a^3b - 16a^2b^2 - 28ab^3 & \\
 \hline
 & 21a^2b^2 + 28ab^3 + 49b^4 \\
 & 21a^2b^2 + 28ab^3 + 49b^4 \\
 & \hline
 &
 \end{array}$$

$$\begin{array}{r|l}
 26. \quad 4a^8 - 44a^4b^4 + 100b^8 & 2a^4 + 2a^2b^2 - 10b^4 \\
 4a^8 + 4a^6b^2 - 20a^4b^4 & 2a^4 - 2a^2b^2 - 10b^4 \\
 \hline
 -4a^6b^2 - 24a^4b^4 + 100b^8 & \\
 -4a^6b^2 - 4a^4b^4 + 20a^2b^6 & \\
 \hline
 & -20a^4b^4 - 20a^2b^6 + 100b^8 \\
 & -20a^4b^4 - 20a^2b^6 + 100b^8 \\
 & \hline
 &
 \end{array}$$

$$\begin{array}{r|l}
 27. \quad 25x^3 - 10x^2 + 40x - 18 & 5x - 6 \\
 25x^3 - 30x^2 & \\
 \hline
 20x^2 + 40x & 5x^2 + 4x + 12\frac{4}{5} + \frac{58\frac{4}{5}}{5x - 6} \\
 20x^2 - 24x & \\
 \hline
 & 64x - 18 \\
 & 64x - 76\frac{4}{5} \\
 & \hline
 & 58\frac{4}{5}
 \end{array}$$

$$\begin{array}{r|l}
 28. \quad x^3 - y^3 & x - y \\
 x^3 - x^2y & x^2 + xy + y^2 \\
 \hline
 x^2y & \\
 x^2y - xy^2 & \\
 \hline
 xy^2 - y^3 & \\
 xy^2 - y^3 & \\
 \hline
 &
 \end{array}$$

$$\begin{array}{r|l}
 29. \quad a^3 - 125b^3 & a - 5b \\
 a^3 - 5a^2b & a^2 + 5ab + 25b^2 \\
 \hline
 5a^2b & \\
 5a^2b - 25ab^2 & \\
 \hline
 & 25ab^2 - 125b^3 \\
 & 25ab^2 - 125b^3 \\
 & \hline
 &
 \end{array}$$

$$\begin{array}{r|l}
 30. \quad a^6 + 343b^3 & a^2 + 7b \\
 a^6 + 7a^4b & a^4 - 7a^2b + 49b^2 \\
 \hline
 -7a^4b & \\
 -7a^4b - 49a^2b^2 & \\
 \hline
 & 49a^2b^2 + 343b^3 \\
 & 49a^2b^2 + 343b^3 \\
 & \hline
 &
 \end{array}$$

$$\begin{array}{r|l}
 31. \quad x^4 - 16 & x + 2 \\
 \hline
 x^4 + 2x^3 & x^3 - 2x^2 + 4x - 8 \\
 - 2x^3 & \\
 \hline
 - 2x^3 - 4x^2 & \\
 \hline
 4x^2 & \\
 4x^2 + 8x & \\
 \hline
 - 8x - 16 & \\
 - 8x - 16 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 32. \quad y^5 - 5y^2 - 3000 & y - 5 \\
 \hline
 y^5 - 5y^4 & y^4 + 5y^3 + 25y^2 + 120y + 600 \\
 5y^4 & \\
 5y^4 - 25y^3 & \\
 \hline
 25y^3 - 5y^2 & \\
 25y^3 - 125y^2 & \\
 \hline
 120y^2 & \\
 120y^2 - 600y & \\
 \hline
 600y - 3000 & \\
 600y - 3000 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 33. \quad x^4 + y^4 & x - y \\
 \hline
 x^4 - x^3y & x^3 + x^2y + xy^2 + y^3 + \frac{2y^4}{x - y} \\
 x^3y & \\
 \hline
 x^3y - x^2y^2 & \\
 x^2y^2 & \\
 x^2y^2 - xy^3 & \\
 \hline
 xy^3 + y^4 & \\
 xy^3 - y^4 & \\
 \hline
 2y^4 &
 \end{array}$$

$$\begin{array}{r|l}
 34. \quad x^4 + y^4 & x + y \\
 \hline
 x^4 + x^3y & x^3 - x^2y + xy^2 - y^3 + \frac{2y^4}{x + y} \\
 - x^3y & \\
 \hline
 - x^3y - x^2y^2 & \\
 x^2y^2 & \\
 x^2y^2 + xy^3 & \\
 \hline
 - xy^3 + y^4 & \\
 - xy^3 - y^4 & \\
 \hline
 2y^4 &
 \end{array}$$

$$\begin{array}{r}
 35. \quad x^4 - y^4 \quad | \quad x + y \\
 \hline
 x^4 + x^3y \quad | \quad x^3 - x^2y + xy^2 - y^3 \\
 - x^3y \\
 \hline
 - x^3y - x^2y^2 \\
 \hline
 x^2y^2 \\
 x^2y^2 + xy^3 \\
 - xy^3 - y^4 \\
 - xy^3 - y^4 \\
 \hline
 y^4
 \end{array}$$

$$\begin{array}{r}
 36. \quad x^4 - y^4 \quad | \quad x - y \\
 \hline
 x^4 - x^3y \quad | \quad x^3 + x^2y + xy^2 + y^3 \\
 x^3y \\
 \hline
 x^3y - x^2y^2 \\
 \hline
 x^2y^2 \\
 x^2y^2 - xy^3 \\
 \hline
 xy^3 - y^4 \\
 xy^3 - y^4 \\
 \hline
 y^4
 \end{array}$$

$$\begin{array}{r}
 37. \quad x^5 + y^5 \quad | \quad x + y \\
 \hline
 x^5 + x^4y \quad | \quad x^4 - x^3y + x^2y^2 - xy^3 + y^4 \\
 - x^4y \\
 \hline
 - x^4y - x^3y^2 \\
 \hline
 x^3y^2 \\
 x^3y^2 + x^2y^3 \\
 - x^2y^3 \\
 - x^2y^3 - xy^4 \\
 \hline
 xy^4 + y^5 \\
 xy^4 + y^5
 \end{array}$$

$$\begin{array}{r}
 38. \quad x^5 + y^5 \quad | \quad x - y \\
 \hline
 x^5 - x^4y \quad | \quad x^4 + x^3y + x^2y^2 + xy^3 + y^4 + \frac{2y^5}{x - y} \\
 x^4y \\
 \hline
 x^4y - x^3y^2 \\
 x^3y^2 \\
 x^3y^2 - x^2y^3 \\
 x^2y^3 \\
 x^2y^3 - xy^4 \\
 \hline
 xy^4 + y^5 \\
 xy^4 - y^5 \\
 \hline
 2y^5
 \end{array}$$

$$\begin{array}{r}
 39. \quad x^5 - y^5 \quad | \quad x + y \\
 \hline
 x^5 + x^4y \quad | \quad x^4 - x^3y + x^2y^2 - xy^3 + y^4 - \frac{2y^5}{x + y} \\
 - x^4y \\
 \hline
 - x^4y - x^3y^2 \\
 \hline
 x^3y^2 \\
 x^3y^2 + x^2y^3 \\
 - x^2y^3 \\
 - x^2y^3 - xy^4 \\
 \hline
 xy^4 - y^5 \\
 xy^4 + y^5 \\
 \hline
 - 2y^5
 \end{array}$$

$$\begin{array}{r}
 40. \quad x^5 - y^5 \quad | \quad x - y \\
 \hline
 x^5 - x^4y \quad | \quad x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\
 \hline
 x^4y \\
 x^4y - x^3y^2 \\
 \hline
 x^3y^2 \\
 x^3y^2 - x^2y^3 \\
 \hline
 x^2y^3 \\
 x^2y^3 - xy^4 \\
 \hline
 xy^4 - y^5 \\
 \hline
 xy^4 - y^5
 \end{array}$$

$$\begin{array}{r}
 41. \quad x^{2a} - 5x^a + 6 \quad | \quad x^a - 3 \\
 \hline
 x^{2a} - 3x^a \quad | \quad x^a - 2 \\
 \hline
 -2x^a + 6 \\
 -2x^a + 6 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 42. \quad x^{6a} - 7x^{3a} + 12 \quad | \quad x^{3a} - 4 \\
 \hline
 x^{6a} - 4x^{3a} \quad | \quad x^{3a} - 3 \\
 \hline
 -3x^{3a} + 12 \\
 -3x^{3a} + 12 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 43. \quad x^{2a} + 3x^a + 2x^{a+1} + 3x \quad | \quad x^a + 2x \\
 \hline
 x^{2a} \quad + 2x^{a+1} \quad | \quad x^a + 3 - \frac{3x}{x^a + 2x} \\
 \hline
 3x^a \quad + 3x \\
 3x^a \quad + 6x \\
 \hline
 -3x
 \end{array}$$

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$$\begin{array}{ll}
 1. & x + 2a = 6a. \\
 \text{Whence} & x = 4a.
 \end{array}$$

$$\begin{array}{ll}
 2. & x + a = b. \\
 \text{Whence} & x = b - a.
 \end{array}$$

$$\begin{array}{ll}
 3. & bx + b = 4b. \\
 \text{Dividing by } b, & \\
 & x + 1 = 4. \\
 \text{Whence} & x = 3.
 \end{array}$$

$$\begin{array}{ll}
 4. & cx + c^2 = 6c^3. \\
 & x + c = 6c^2. \\
 \text{Whence} & x = 6c^2 - c.
 \end{array}$$

$$\begin{array}{ll}
 5. & 5(b - x) = 10b. \\
 & b - x = 2b. \\
 \text{Whence} & x = -b.
 \end{array}$$

$$\begin{array}{ll}
 6. & bx - (b + c) = 5b - c. \\
 & bx - b - c = 5b - c. \\
 \text{Whence} & x = 6.
 \end{array}$$

$$\begin{array}{ll}
 7. & 3ax - ab = 2ax - ac. \\
 \text{Dividing by } a, & \\
 & 3x - b = 2x - c. \\
 \text{Whence} & x = b - c.
 \end{array}$$

$$\begin{array}{ll}
 8. & 4bx - 7a^2b = 6ab^2 + 3bx. \\
 \text{Dividing by } b, & \\
 & x - 7a^2 = 6ab. \\
 \text{Whence} & x = 7a^2 + 6ab.
 \end{array}$$

$$\begin{array}{ll}
 9. & ax + bx = ac + bc. \\
 \text{Grouping,} & \\
 & x(a + b) = c(a + b). \\
 \text{Whence} & x = c.
 \end{array}$$

$$\begin{array}{ll}
 10. & a^2x + 1 - a^4 - x = 0. \\
 \text{Grouping,} & x(a^2 - 1) = a^4 - 1. \\
 \text{Whence} & x = a^2 + 1.
 \end{array}$$

11. $ax + 2ab = 2a^2 + bx.$
 Grouping, $ax - bx = 2a^2 - 2ab.$
 Whence $x = 2a.$
12. $ax - a^3 - 4 = 3a - x.$
 Arranging, $ax + x = a^3 + 3a + 4.$
 Dividing by $a + 1,$ $x = a^2 - a + 4.$
13. $4b^2c^2 + (a + bx)c = (a - bx)c.$
 $4b^2c + a + bx = a - bx.$
 Whence $x = -2bc.$
14. $ax - ac + bc = 2ac - 5bc + 2bx.$
 Arranging, $ax - 2bx = 3ac - 6bc.$
 Whence $x = 3c.$
15. $(x + a)(x + b) = x^2 + 2a^2 + 3ab.$
 $x^2 + ax + bx + ab = x^2 + 2a^2 + 3ab.$
 Whence $x = 2a.$
16. $15(x - a) - 6(x + a) = 3(5a - 3x).$
 Simplifying, $5x - 5a - 2x - 2a = 5a - 3x.$
 Whence $x = 2a.$
17. $4x - cx - 8 + 2a + 6c = 6a - 3ac + 2cx.$
 Arranging, $4x - 3cx = 4a - 3ac + 8 - 6c.$
 Whence $x = a + 2.$
18. $9ab + (x - 3a)(x - 3b) = (x + 3a)(x - 3a) - 9a^2.$
 Expanding,
 $9ab + x^2 - 3ax - 3bx + 9ab = x^2 - 9a^2 - 9a^2.$
 Whence $x = 6a.$
19. $(5a - 4b)x - 5(b^2 + 4a^2 + 6ab) = 10b^2 - 3(2a^2 + 3bx) - a(2x - b).$
 Expanding,
 $5ax - 4bx - 5b^2 - 20a^2 - 30ab = 10b^2 - 6a^2 - 9bx - 2ax + ab.$
 $7ax + 5bx = 14a^2 + 31ab + 15b^2.$
 Whence $x = 2a + 3b.$
20. $a^2x + 3ax + 10a = a^3 + x + 3.$
 Arranging, $a^2x + 3ax - x = a^3 - 10a + 3.$
 Whence $x = a - 3.$

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1. Let x = the number of hours which must elapse
 before A and B are 120 miles apart.
 Then $8x + 10x = 120.$
 Whence $x = 6\frac{2}{3}.$

Let $y =$ the number of hours which must elapse
before A and B are 180 miles apart.

Then $8y + 10y = 180.$

Whence $y = 10.$

2. Let $r =$ the rate of B in miles per hour.

Then $2r =$ the rate of A in miles per hour.

Therefore $5 \cdot 2r + 5r = 135.$

Whence $r = 9,$

and $2r = 18.$

3. Let $r =$ the rate of B in miles per hour.

Then $r + 2 =$ the rate of A in miles per hour.

Therefore $8(r + 2) + 8r = 96.$

Whence $r = 5,$

and $r + 2 = 7.$

4. Let $r =$ the rate of B in miles per hour.

Then $r + 4 =$ the rate of A in miles per hour.

Therefore $6(r + 4) + 6r = 168.$

Whence $r = 12,$

and $r + 4 = 16.$

5. Let $r =$ the rate of A in miles per hour.

Then $r - 3 =$ the rate of B in miles per hour.

Therefore $r - 3 = \frac{3r}{4}.$

Whence $r = 12,$

and $r - 3 = 9.$

Let $h =$ the required number of hours.

Therefore $12h + 9h = 168.$

Whence $h = 8.$

6. Let $r =$ the rate of B in miles per hour.

Then $2r =$ the rate of A in miles per hour.

Therefore $3 \cdot 2r + 5r = 77.$

Whence $r = 7,$

and $2r = 14.$

A travels $3 \cdot 14$, or 42 miles, and B travels $5 \cdot 7$, or 35 miles.

7. Let $r =$ the rate of A in miles per hour.

Then $r - 4 =$ the rate of B in miles per hour.

Therefore $6 \cdot r = 9(r - 4).$

Whence $r = 12,$

and $r - 4 = 8.$

$6 \cdot 12 + 9 \cdot 8 = 144$ miles, the total distance traveled.

8. Let $x =$ the rate of each in miles per hour.
Therefore $8x + 8x = 144.$
Whence $x = 9.$

9. Let $x =$ the rate of A in miles per hour.
Then $x + 2 =$ the rate of B in miles per hour.
Therefore $9x + 9(x + 2) = 144.$
Whence $x = 7,$
and $x + 2 = 9.$

10. Let $r =$ the rate of A in miles per hour.
Then $3r =$ the rate of B in miles per hour.
Therefore $12 \cdot r + 12 \cdot 3r = 144.$
Whence $r = 3,$
and $3r = 9.$

11. Let $r =$ the rate of A in miles per hour.
Then $r + 4 =$ the rate of B in miles per hour.
Therefore $6 \cdot r + 6(r + 4) = 144.$
Whence $r = 10,$
and $r + 4 = 14.$

12. Let $r =$ the rate of A in miles per hour.
Then $r + 2 =$ the rate of B in miles per hour.
Therefore $8r + 8(r + 2) = 144.$
Whence $r = 8,$
and $r + 2 = 10.$

13. Let $r =$ the rate of B in miles per hour.
Then $r + 4 =$ the rate of A in miles per hour.
Therefore $9(r + 4) + 9r = 144.$
Whence $r = 6,$
and $r + 4 = 10.$

14. Let $r =$ the rate of A in miles per hour.
Then $r + 6 =$ the rate of B in miles per hour.
Therefore $r + 6 = 2r.$
Whence $r = 6,$
and $r + 6 = 12.$

Let $t =$ the required time in hours.
Therefore $t \cdot 6 + t \cdot 12 = 144.$
Whence $t = 8.$

15. Let $h =$ the required number of hours.
Therefore $6 \cdot h + 9(h - 4) = 144.$
Whence $h = 12.$

16. Let h = the required number of hours.

Therefore $7(h - 3) + 9(h - 5) = 144$.

Whence $h = 13\frac{1}{8}$ hours.

17. Let t = the required number of hours.

Therefore $3t + 4t = 42$.

Whence $t = 6$.

18. Let t = the required number of hours.

Therefore $8t + 10t = 72$.

Whence $t = 4$.

19. Let t = the required number of hours.

Therefore $37t + 38t = 285$.

Whence $t = 3\frac{4}{5}$.

20. Let x = the number of hours that A travels.

Then $x - 4$ = the number of hours that B travels.

Therefore $8x = 10(x - 4)$.

Whence $x = 20$.

21. Let h = the number of hours traveled by the first.

Then $h - 2$ = the number of hours traveled by the second.

Therefore $10h + 12(h - 2) = 108$.

Whence $h = 6$.

A has traveled $10 \cdot 6$, or 60 miles, and B $12 \cdot 4$, or 48 miles.

22. Let x = the time in hours of the passenger train.

Then $x + 2$ = the time in hours of the freight train.

Therefore $42x + 24(x + 2) = 246$.

Whence $x = 3$.

23. Let x = the rate of the second messenger in miles per hour.

Therefore $8 \cdot 8 = 6 \cdot x$.

Whence $x = 10\frac{2}{3}$.

24. Let h = the required number of hours.

Therefore $45h - 18h = 144$.

Whence $h = 5\frac{1}{3}$.

25. $\frac{216}{9 + 15} = 9$ hours. $9 \cdot 9 = 81$ miles, A's distance;
 $9 \cdot 15 = 135$ miles, B's distance.

26. $\frac{5}{6} \cdot 216 = 180$ miles, B's distance; $2\frac{1}{6} \cdot 36 = 36$ miles, A's distance.

27. $\frac{216 - 56}{2} = 80$ miles, B's distance; $80 + 56 = 136$ miles, A's distance.

28. $108 - 24 = 84$ miles, A's distance; $108 + 24 = 132$ miles, B's distance.

29. $\frac{108 + 54}{6} = 27$ miles per hour, A's rate; $\frac{54}{9} = 6$ miles per hour, B's rate.

30. $\frac{108 + 36}{6} = 24$ miles per hour, A's rate; $\frac{72}{6} = 12$ miles per hour, B's rate.

31. $\frac{108 + 12}{6} = 20$ miles per hour, A's rate; $\frac{108 - 12}{6} = 16$ miles per hour, B's rate.

32. $\frac{216 - 96}{6} = 20$ miles per hour, A's rate; $\frac{96}{6} = 16$ miles per hour, B's rate.

33. Let $x =$ the rate of B in miles per hour.

Then $x + 4 =$ the rate of A in miles per hour.

Therefore $12 \cdot (x + 4) + 12 \cdot x = 216.$

Whence $x = 7,$

and $x + 4 = 11.$

34. Let $r =$ the rate of B in miles per hour.

Then $r + 2 =$ the rate of A in miles per hour.

Therefore $6(r + 2) + 6r = 216.$

Whence $r = 17,$

and $r + 2 = 19.$

Then $6 \cdot 17 = 102$ miles, B's distance,

and $6 \cdot 19 = 114$ miles, A's distance.

35. Let $r =$ the rate of B in miles per hour.

Then $r + 4 =$ the rate of A in miles per hour.

Therefore $9(r + 4) + 9r = 216.$

Whence $r = 10,$

and $r + 4 = 14.$

Then $9 \cdot 10 = 90$ miles, B's distance,

and $9 \cdot 14 = 126$ miles, A's distance.

36. Let $v =$ the velocity of the bullet in feet per second.

Therefore $\frac{1650}{v} + \frac{1650}{1100} = \frac{5}{2}.$

Whence $v = 1650.$

37. Let $v =$ the velocity of the bullet in feet per second.

Therefore $\frac{825}{v} + \frac{825}{1100} = \frac{7}{4}.$

Whence $v = 825.$

38. Let $x =$ the distance in feet from the boy to the target.

Therefore $\frac{3000 - x}{1100} = \frac{3000}{1650} + \frac{x}{1100}.$

Whence $x = 500.$

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$$1. \quad 5a - 7x + 3b - 10c - 14a + 12y - 8x + 12a - 11x + 9b \\ = 3a - 26x + 12b - 10c + 12y.$$

$$2. \quad 2cd - a^2b + 7cd^2 - 12ab + 17ad - cd + 4c^2d \\ = cd + 7cd^2 + 4c^2d - a^2b - 12ab + 17ad.$$

$$3. \quad \begin{array}{r} 10x - 9 + 4ax - 2cd \\ - 15x \quad \quad - 6ax + 12cd \\ \quad \quad 10 - \quad ax + 5cd \\ 11x - 19 - 7ax - 6cd \\ \hline 6x - 18 - 10ax + 9cd \end{array}$$

$$4. \quad \begin{array}{r} a^3 + 4a^2 - a - 3 \\ - 5a^3 + 3a^2 - a \\ 6a^3 - 3a^2 - 2a + 8 \\ - 2a^3 - 5a^2 - a + 5 \\ 2a^3 - 3a^2 + 4a - 5 \\ \hline 2a^3 - 4a^2 - a + 5 \end{array}$$

$$5. \quad \begin{array}{r} Ax^3 + \quad Bx^2 \quad + \quad Cx \\ \quad \quad Ax^2 \quad + \quad Bx \quad + C \\ \hline Ax^3 + (A + B)x^2 + (B + C)x + C \end{array}$$

$$6. \quad \begin{array}{r} 7(a - b) - 10(c - d) + 8(x - y) \\ - 4(a - b) - 6(c - d) - 7(x - y) \\ 7(a - b) - 15(c - d) + 12(x - y) \\ \hline 10(a - b) - 31(c - d) + 13(x - y) \end{array}$$

$$7. \quad \begin{array}{r} - 4(a + b) - 2(x + y) + 3(x - y) \\ 4(a + b) - 9(x + y) + 8(x - y) \\ - (a + b) - 10(x + y) + (x - y) \\ \hline - (a + b) - 21(x + y) + 12(x - y) \end{array}$$

$$8. \quad \begin{array}{r} \frac{1}{3}ax + \frac{2}{3}ax^2 - \frac{5}{6}a^2x^2 + a \\ - ax + \frac{1}{6}ax^2 + 2a^2x^2 - 9a \\ \quad \quad \frac{1}{2}ax^2 + a^2x^2 - \frac{1}{4}a \\ \hline - \frac{2}{3}ax + \frac{4}{3}ax^2 + 2\frac{1}{6}a^2x^2 - 8\frac{1}{4}a \end{array}$$

$$9. \quad \begin{array}{r} x^2 + 10xy - 5y^2 - 9 \\ 2x^2 - 3xy + y^2 - 1 \\ \hline - x^2 + 13xy - 6y^2 - 8 \end{array}$$

$$10. \quad \begin{array}{r} - a^2y + 12ay^2 - ay - 15 \\ 3a^2y - 7ay^2 - 5ay \quad + z \\ \hline - 4a^2y + 19ay^2 + 4ay - 15 - z \end{array}$$

$$11. \quad \begin{array}{r} 3ab^2c^2d^3 - 12 \\ - 4ab^2c^2d^3 \quad \quad - 17a^2bcd^2 + 3abcd + ax \\ \hline 7ab^2c^2d^3 - 12 + 17a^2bcd^2 - 3abcd - ax \end{array}$$

12. 0

$$\frac{27xy + 13z - 12 + ab}{-27xy - 13z + 12 - ab}$$

13. $(a + b) - (a - b) + 4a - (a + 3b) + c$
 $= a + b - a + b + 4a - a - 3b + c$
 $= 3a - b + c.$

14. $15x - 2(12x - 16y) + xy + 3(3x - 4xy) - x$
 $= 14x - 24x + 32y + xy + 9x - 12xy$
 $= -x - 11xy + 32y.$

15. $ab - (7x + 4y - 2z) + 42 + 5(5ab - x - y)$
 $= ab - 7x - 4y + 2z + 42 + 25ab - 5x - 5y$
 $= 26ab - 12x - 9y + 2z + 42.$

16. $[x - (-4x + 8y + 2z) + b - 4(3x - z)] - 5b$
 $= x + 4x - 8y - 2z + b - 12x + 4z - 5b$
 $= -7x - 8y + 2z - 4b.$

17. $[ab - \{-5ac + 7(d - a) + 4ca + 7d\} - 6ba] + 13a$
 $= ab - \{-ac + 14d - 7a\} - 6ba + 13a$
 $= -5ab + ac - 14d + 7a + 13a$
 $= 20a - 5ab + ac - 14d.$

18. $\{5x^2 - 2[-4 - (2x^2 - 3x)] + 5\} - (x^2 + 4x - 2)$
 $= 5x^2 - 2(-4 - 2x^2 + 3x) + 5 - x^2 - 4x + 2$
 $= 4x^2 + 8 + 4x^2 - 6x + 7 - 4x$
 $= 8x^2 - 10x + 15.$

19. $x^4 - 4x^2y^2 + 4y^4$ (1)

$x^4 + 4x^2y^2 + 4y^4$ (2)

$$\begin{array}{r} x^8 - 4x^6y^2 + 4x^4y^4 \\ + 4x^6y^2 - 16x^4y^4 + 16x^2y^6 \\ + 4x^4y^4 - 16x^2y^6 + 16y^8 \end{array}$$

(1) · (2), x^8 $- 8x^4y^4$ $+ 16y^8$

20. $4x^4 - x^2 + 5$ (1)

$3x^3 - x + 7$ (2)

$$\begin{array}{r} 12x^7 - 3x^5 \qquad \qquad + 15x^3 \\ - 4x^5 \qquad \qquad + x^3 \qquad \qquad - 5x \\ + 28x^4 \qquad \qquad - 7x^2 \qquad \qquad + 35 \end{array}$$

(1) · (2), $12x^7 - 7x^5 + 28x^4 + 16x^3 - 7x^2 - 5x + 35$

21.
$$\frac{a^2 + b^2 + c^2 - ab - ac - bc}{a + b + c} \quad (1)$$
- $$\frac{a^3 + ab^2 + ac^2 - a^2b - a^2c - abc}{-ab^2 + a^2b - abc + b^3 + bc^2 - b^2c} \quad (2)$$
- $$\frac{-ac^2 + a^2c - abc - bc^2 + b^2c + c^3}{(1) \cdot (2), a^3 - 3abc + b^3 + c^3}$$
22.
$$\frac{x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6}{x + y} \quad (1)$$
- $$\frac{x^7 - x^6y + x^5y^2 - x^4y^3 + x^3y^4 - x^2y^5 + xy^6}{+ x^6y - x^5y^2 + x^4y^3 - x^3y^4 + x^2y^5 - xy^6 + y^7} \quad (2)$$
- $$(1) \cdot (2), x^7 + y^7$$
23.
$$\frac{-9x^5 - 4x^4 - 3x^3 + x^2 + 5x + 10}{2x^4 - x^3 - x^2 + 3x} \quad (1)$$
- $$\frac{-18x^9 - 8x^8 - 6x^7 + 2x^6 + 10x^5 + 20x^4}{+ 9x^8 + 4x^7 + 3x^6 - x^5 - 5x^4 - 10x^3} \quad (2)$$
- $$\frac{+ 9x^7 + 4x^6 + 3x^5 - x^4 - 5x^3 - 10x^2}{- 27x^6 - 12x^5 - 9x^4 + 3x^3 + 15x^2 + 30x}{(1) \cdot (2), -18x^9 + x^8 + 7x^7 - 18x^6 + 5x^4 - 12x^3 + 5x^2 + 30x}$$
24.
$$\frac{3x^2 - 4y}{3x^2 + 4y} \quad (1) \quad \frac{9x^4 - 16y^2}{9x^4 + 16y^2} \quad (3)$$
- $$\frac{9x^4 - 12x^2y}{+ 12x^2y - 16y^2} \quad (2) \quad \frac{81x^8 - 144x^4y^2}{+ 144x^4y^2 - 256y^4} \quad (4)$$
- $$(1) \cdot (2), 9x^4 - 16y^2 \quad (3) \cdot (4) 81x^8 - 256y^4$$
25.
$$\frac{a - 3b + 2c}{a - 3b + 2c} \quad (1)$$
- $$\frac{a^2 - 3ab + 2ac}{-3ab + 9b^2 - 6bc} \quad (2)$$
- $$\frac{+ 2ac - 6bc + 4c^2}{(1) \cdot (2), a^2 - 6ab + 4ac + 9b^2 - 12bc + 4c^2}$$
26.
$$\frac{x - y + z}{x - y + z} \quad (1)$$
- $$\frac{x^2 - xy + xz}{-xy + y^2 - yz} \quad (2)$$
- $$\frac{+ xz - yz + z^2}{(1) \cdot (2), x^2 - 2xy + 2xz + y^2 - 2yz + z^2} \quad (3)$$
- $$\frac{x - y + z}{x^3 - 2x^2y + 2x^2z + xy^2 - 2xyz + xz^2} \quad (4)$$
- $$\frac{-x^2y + 2xy^2 - 2xyz - y^3 + 2y^2z - yz^2}{+ x^2z - 2xyz + 2xz^2 + y^2z - 2yz^2 + z^3}{(3) \cdot (4), x^8 - 3x^2y + 3x^2z + 3xy^2 - 6xyz + 3xz^2 - y^3 + 3y^2z - 3yz^2 + z^3}$$

$$27. \quad \begin{array}{r} x^a - y^b \\ x^a + y^b \\ \hline x^{2a} - x^a y^b \end{array} \quad \begin{array}{l} (1) \\ (2) \end{array}$$

$$(1) \cdot (2), \quad \begin{array}{r} + x^a y^b - y^{2b} \\ \hline x^{2a} - y^{2b} \end{array}$$

$$28. \quad \begin{array}{r} x^a + 2y^b + 3z^c \\ x^a - 3y^b \\ \hline x^{2a} + 2x^a y^b + 3x^a z^c \end{array} \quad \begin{array}{l} (1) \\ (2) \end{array}$$

$$(1) \cdot (2), \quad \begin{array}{r} - 3x^a y^b - 6y^{2b} - 9y^b z^c \\ \hline x^{2a} - x^a y^b + 3x^a z^c - 6y^{2b} - 9y^b z^c \end{array}$$

$$29. \quad \begin{array}{r} 6a^5 - 13a^4 + 4a^3 + 3a^2 \\ 6a^5 - 6a^4 \\ \hline - 7a^4 + 4a^3 \\ - 7a^4 + 7a^3 \\ \hline - 3a^3 + 3a^2 \\ - 3a^3 + 3a^2 \\ \hline \end{array} \quad \begin{array}{r} - a^2 + a \\ - 6a^3 + 7a^2 + 3a \\ \hline \end{array}$$

$$30. \quad \begin{array}{r} - 30a^4 - 11a^3 + 82a^2 + 12a - 48 \\ - 30a^4 - 20a^3 + 40a^2 \\ \hline 9a^3 + 42a^2 + 12a \\ 9a^3 + 6a^2 - 12a \\ \hline 36a^2 + 24a - 48 \\ 36a^2 + 24a - 48 \\ \hline \end{array} \quad \begin{array}{r} 3a^2 + 2a - 4 \\ - 10a^2 + 3a + 12 \\ \hline \end{array}$$

$$31. \quad \begin{array}{r} 10x^6 - 11x^5 - 3x^4 + 20x^3 + 10x^2 + 2 \\ 10x^6 - 6x^5 + 4x^4 - 4x^3 \\ \hline - 5x^5 - 7x^4 + 24x^3 + 10x^2 \\ - 5x^5 + 3x^4 - 2x^3 + 2x^2 \\ \hline - 10x^4 + 26x^3 + 8x^2 + 2 \\ - 10x^4 + 6x^3 - 4x^2 + 4x \\ \hline 20x^3 + 12x^2 - 4x + 2 \\ 20x^3 - 12x^2 + 8x - 8 \\ \hline 24x^2 - 12x + 10 \end{array} \quad \begin{array}{r} 5x^3 - 3x^2 + 2x - 2 \\ 2x^3 - x^2 - 2x + 4 \\ \hline + \frac{24x^2 - 12x + 10}{5x^3 - 3x^2 + 2x - 2} \end{array}$$

$$32. \quad \begin{array}{r} a^8 - 3a^6 + a^4 - 7a^2 + 3 \\ a^8 - 2a^6 + a^4 \\ \hline - a^6 - 7a^2 + 3 \\ - a^6 + 2a^4 - a^2 \\ \hline - 2a^4 - 6a^2 + 3 \\ - 2a^4 + 4a^2 - 2 \\ \hline - 10a^2 + 5 \end{array} \quad \begin{array}{r} a^4 - 2a^2 + 1 \\ a^4 - a^2 - 2 + \frac{5 - 10a^2}{a^4 - 2a^2 + 1} \\ \hline a^2 - 1 \\ a^2 - 1 \\ \hline a^4 - a^2 \\ - a^2 + 1 \\ \hline a^4 - 2a^2 + 1 \end{array}$$

$$\begin{array}{r|l}
 33. \quad a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 7ab^4 - b^5 & a^2 - 2ab + b^2 \\
 \hline
 a^5 - 2a^4b + \quad a^3b^2 & a^3 - 3a^2b + 3ab^2 - b^3 \\
 \hline
 - 3a^4b + 9a^3b^2 - 10a^2b^3 & + \frac{2ab^4}{a^2 - 2ab + b^2} \\
 \hline
 - 3a^4b + 6a^3b^2 - 3a^2b^3 & \\
 \hline
 3a^3b^2 - 7a^2b^3 + 7ab^4 & a - b \\
 3a^3b^2 - 6a^2b^3 + 3ab^4 & a - b \\
 \hline
 - a^2b^3 + 4ab^4 - b^5 & a^2 - ab \\
 - a^2b^3 + 2ab^4 - b^5 & - ab + b^2 \\
 \hline
 2ab^4 & a^2 - 2ab + b^2
 \end{array}$$

$$\begin{array}{r|l}
 34. \quad 6a^2 + 5a - 6 & 3a - 2 \\
 2a^2 - 13a + 20 & 2a - 5 \\
 \hline
 12a^4 + 10a^3 - 12a^2 & 6a^2 - 4a \\
 - 78a^3 - 65a^2 + 78a & - 15a + 10 \\
 + 120a^2 + 100a - 120 & 6a^2 - 19a + 10 \\
 \hline
 12a^4 - 68a^3 + 43a^2 + 178a - 120 & 6a^2 - 19a + 10 \\
 12a^4 - 38a^3 + 20a^2 & 2a^2 - 5a - 12 \\
 \hline
 - 30a^3 + 23a^2 + 178a & \\
 - 30a^3 + 95a^2 - 50a & \\
 \hline
 - 72a^2 + 228a - 120 & \\
 - 72a^2 + 228a - 120 &
 \end{array}$$

$$\begin{array}{r|l}
 35. \quad x^3 + 36xy - 27y^3 + 64 & x - 3y + 4 \\
 x^2 - 3x^2y + 4x^2 & x^2 + 3xy - 4x + 9y^2 + 12y + 16 \\
 \hline
 3x^2y - 4x^2 + 36xy & \\
 3x^2y - 9xy^2 + 12xy & \\
 \hline
 - 4x^2 + 9xy^2 + 24xy & \\
 - 4x^2 + 12xy - 16x & \\
 \hline
 9xy^2 + 12xy + 16x - 27y^3 + 64 & \\
 9xy^2 - 27y^3 + 36y^2 & \\
 \hline
 12xy + 16x + 64 - 36y^2 & \\
 12xy - 36y^2 + 48y & \\
 \hline
 16x + 64 - 48y & \\
 16x + 64 - 48y &
 \end{array}$$

36. $6y - 20 + 5y - 18 = 36y - 4 - 40y + 9.$
 Collecting, $11y - 38 = -4y + 5.$
 Whence $y = 2\frac{1}{5}.$

37. $4y - 3(6 - y) + 2 = 6(y - 2) - 13y + 24.$
 Simplifying, $4y - 18 + 3y = 6y - 12 - 13y + 22.$
 Whence $y = 2.$

38. $6y - 4(1 - 2y) + 10 = 4(2y - 5) - 4y + 1.$

Simplifying, $6y - 4 + 8y + 9 = 8y - 20 - 4y.$

Whence $y = -2\frac{1}{2}.$

39. $(y + 7)(y - 11) = (y + 8)(y - 5) - 2.$

Expanding, $y^2 - 4y - 77 = y^2 + 3y - 40 - 2.$

Whence $y = -5.$

40. $(y - 3)^3 = (y^2 - 6y + 9)(5 + y) - 8y^2.$

Expanding, $y^3 - 9y^2 + 27y - 27 = y^3 - y^2 - 21y + 45 - 8y^2.$

Whence $y = \frac{3}{2}.$

41. $y^2 + a^2 - (y + 1)^2 - (a + 1)^2 = 0.$

Expanding,

$y^2 + a^2 - y^2 - 2y - 1 - a^2 - 2a - 1 = 0.$

Whence $y = -a - 1.$

42. $3ay + 6ac + 4b^2 = 3ab + 4by + 8bc.$

Arranging, $3ay - 4by = 3ab - 4b^2 - 6ac + 8bc.$

Whence $y = b - 2c.$

43. $x = \frac{-10 + \sqrt{100 + 44}}{2} = 1.$

44. $x = \frac{6 + \sqrt{36 + 160}}{10} = \frac{6 + 14}{10} = 2.$

45. $x = \frac{-5 - \sqrt{25 + 96}}{6} = \frac{-5 - 11}{6} = -2\frac{2}{3}.$

46. $x = \frac{-7 - \sqrt{49 + 176}}{4} = \frac{-7 - 15}{4} = -5\frac{1}{2}.$

47. $12s = \frac{1}{2} \cdot 18(20)^2.$

Whence $s = 300.$

48. $e = \frac{1}{2} \cdot 7(32.2 \times 40)^2 = \frac{1}{2} \cdot 7(1288)^2 = \frac{1}{2} \cdot 7(1,658,944) = 5,806,304.$

49. $s = \frac{3 + 4 + 5}{2} = 6.$

$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 6.$

50. Let $n =$ the number.

Then $n + 1$ and $n + 2 =$ the next two numbers.

Therefore $n^2 + 38 = n^2 + 3n + 2.$

Whence $n = 12,$

$n + 1 = 13,$

$n + 2 = 14.$

and

51. Let $n =$ a certain odd number.
 Then $n + 2$ and $n + 4 =$ the next two odd numbers.
 Therefore $n^2 + 98 = n^2 + 6n + 8$.
 Whence $n = 15$, the first odd number,
 $n + 2 = 17$, the second odd number,
 and $n + 4 = 19$, the third odd number.

52. Let $n =$ an odd number.
 Then $n + 1$ and $n + 3 =$ the next two even numbers.
 Therefore $n^2 + 47 = n^2 + 4n + 3$.
 Whence $n = 11$, the odd number,
 $n + 1 = 12$, the next even number,
 and $n + 3 = 14$, the second even number.

53. Let $n =$ the first number.
 Then $n + 1$, $n + 2$, and $n + 3 =$ the next three numbers.
 Therefore $n^2 + n + 42 = n^2 + 5n + 6$.
 Whence $n = 9$, the first number,
 $n + 1 = 10$, the second number,
 $n + 2 = 11$, the third number,
 and $n + 3 = 12$, the fourth number.

54. Let $x =$ the number of years sought.
 Therefore $42 + x = 2(15 + x)$.
 Whence $x = 12$.

55. Let $x =$ the number of years sought.
 Therefore $36 - x = 3(16 - x)$.
 Whence $x = 6$.

56. Let $x =$ the number of years sought.
 Therefore $52 + x = 2(21 + x)$.
 Whence $x = 10$.

57. Let $x =$ the number of years sought.
 Therefore $87 - x = 4(42 - x)$.
 Whence $x = 27$.

58. Let $q =$ the number of quarters.
 Then $2q =$ the number of nickels,
 and $32 - 3q =$ the number of dimes.
 Then $25q =$ the value of quarters in cents,
 $10q =$ the value of nickels in cents,
 and $320 - 30q =$ the value of dimes in cents.
 Therefore $25q + 10q + 320 - 30q = 360$.

Whence $q = 8,$
 $2q = 16,$
 and $32 - 3q = 8.$

59. Let $d =$ the distance to the target in feet.

Therefore $\frac{d}{1280} + \frac{d}{1120} = 6.$

Whence $d = 3584.$

60. Let $x =$ age of each.

$(x + 12)3 - 36 + x =$ the result.

$4x =$ the result.

Therefore he divided each result by 4.

61. Let $x =$ the per cent over 60 years.

Then $9x =$ the per cent between 20 and 60,

and $9x + 5 =$ the per cent under 20.

Therefore $x + 9x + 9x + 5 = 100.$

Whence $x = 5,$

$9x = 45,$

and $9x + 5 = 50.$

62. Let $x =$ the length in feet of the Hoosac tunnel.

Then $2x - 1320 =$ the length in feet of the St. Gothard tunnel,

and $2x - 10,320 =$ the length in feet of the Mount Ceniz tunnel.

Therefore $x + 2x - 1320 + 2x - 10,320 = 113,760.$

Whence $x = 25,080,$

$2x - 1320 = 48,840,$

and $2x - 10,320 = 39,840.$

63. Let $x =$ the height in feet of Niagara Falls.

Then $2x + 32 =$ the height in feet of falls of Zambesi,

$8x + 188 =$ the height in feet of falls of Yosemite,

and $8x - 332 =$ the height in feet of Staubbach Falls.

Therefore $x + 2x + 32 + 8x + 188 + 8x - 332 = 3004.$

Whence $x = 164,$

$2x + 32 = 360,$

$8x + 188 = 1500,$

and $8x - 332 = 980.$

64. Let $x =$ the weight in pounds of cubic foot of water.

Then $x - 11.5 =$ the weight in pounds of cubic foot of alcohol,

and $16x - 151.3 =$ the weight in pounds of cubic foot of mercury.

Therefore $x + x - 11.5 + 16x - 151.3 = 962.2.$

Whence

$$x = 62.5,$$

$$x - 11.5 = 51,$$

and

$$16x - 151.3 = 848.7.$$

65. Let

 x = the rainfall of San Diego in inches.

Then

 $5x - 1$ = the rainfall of Boston in inches, $5x - 2$ = the rainfall of St. Louis in inches,

and

 $15x - 12$ = the rainfall of Neahbay in inches.Therefore $x + 5x - 1 + 5x - 2 + 15x - 12 = 219$.

Whence

$$x = 9,$$

$$5x - 1 = 44,$$

$$5x - 2 = 43,$$

and

$$15x - 12 = 123.$$

66. Let

 x = the number of years in the expectation of one at 65.Then $2x - .94$ = the number of years in the expectation of one at 45, $2x + 14.37$ = the number of years in the expectation of one at 10,and $2x + 8.98$ = the number of years in the expectation of one at 21.Therefore $x + 2x - .94 + 2x + 14.37 + 2x + 8.98 = 109.42$.

Whence

$$x = 12.43,$$

$$2x - .94 = 23.92,$$

$$2x + 14.37 = 39.23,$$

and

$$2x + 8.98 = 33.84.$$

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$$3. 15^2 = (10 + 5)^2 = 100 + 25 + 2 \cdot 10 \cdot 5 = 225.$$

$$4. 17^2 = (20 - 3)^2 = 400 + 9 - 2 \cdot 20 \cdot 3 = 289.$$

$$5. 18^2 = (20 - 2)^2 = 400 + 4 - 2 \cdot 20 \cdot 2 = 324.$$

$$6. 19^2 = (20 - 1)^2 = 400 + 1 - 2 \cdot 20 \cdot 1 = 361.$$

$$7. (x + y)^2 = x^2 + y^2 + 2xy.$$

$$14. (3x - y)^2 = 9x^2 - 6xy + y^2.$$

$$8. (x - c)^2 = x^2 + c^2 - 2xc.$$

$$15. (x + 2y)^2 = x^2 + 4xy + 4y^2.$$

$$9. (x + 1)^2 = x^2 + 1 + 2x.$$

$$16. (a - 4b)^2 = a^2 - 8ab + 16b^2.$$

$$10. (a + 2)^2 = a^2 + 4a + 4.$$

$$17. (y^5 - 2)^2 = y^{10} - 4y^5 + 4.$$

$$11. (b^2 + 3)^2 = b^4 + 6b^2 + 9.$$

$$18. (2a + 1)^2 = 4a^2 + 4a + 1.$$

$$12. (c - 4)^2 = c^2 - 8c + 16.$$

$$19. (3a^2 + 2)^2 = 9a^4 + 12a^2 + 4.$$

$$13. (2c^2 + d)^2 = 4c^4 + 4c^2d + d^2.$$

$$20. (4a + 3)^2 = 16a^2 + 24a + 9.$$

$$21. (2a^2 - 3b)^2 = 4a^4 - 12a^2b + 9b^2.$$

$$22. (3c^2 - 4a^2)^2 = 9c^4 - 24c^2a^2 + 16a^4.$$

$$23. (2x^3 - 6)^2 = 4x^6 - 24x^3 + 36.$$

$$24. (3xy + y^2)^2 = 9x^2y^2 + 6xy^3 + y^4.$$

$$25. (7x^2 - 3y^4)^2 = 49x^4 - 42x^2y^4 + 9y^8.$$

$$26. (8y^3 + 2bx)^2 = 64y^6 + 32y^3bx + 4b^2x^2.$$

$$27. (6xy - 2a^2)^2 = 36x^2y^2 - 24xya^2 + 4a^4.$$

$$28. (4xy - 2x^2)^2 = 16x^2y^2 - 16x^3y + 4x^4.$$

$$29. (3x^4 - 5x^6)^2 = 9x^8 - 30x^{10} + 25x^{12}.$$

$$30. (2x^2 - 6x^3y)^2 = 4x^4 - 24x^5y + 36x^6y^2.$$

$$31. 41^2 = (40 + 1)^2 = 1681.$$

$$32. 92^2 = (90 + 2)^2 = 8464.$$

$$33. 101^2 = (100 + 1)^2 = 10,201.$$

$$34. 202^2 = (200 + 2)^2 = 40,804.$$

$$35. 1001^2 = (1000 + 1)^2 = 1,002,001.$$

$$36. \frac{a^2 + 2ab + b^2}{a + b} = a + b.$$

$$37. \frac{a^2 - 2ac + c^2}{a - c} = a - c.$$

$$38. \frac{a^2 - 4a + 4}{a - 2} = a - 2.$$

$$39. \frac{9x^2 - 6x + 1}{3x - 1} = 3x - 1.$$

$$40. \frac{y^2 + 40y + 400}{20 + y} = 20 + y.$$

$$41. \frac{9x^2 - 12xy + 4y^2}{2y - 3x} = 2y - 3x.$$

$$42. \frac{4a^2 - 20a + 25}{-5 + 2a} = -5 + 2a.$$

$$43. \frac{4x^2 - 12xy + 9y^2}{3y - 2x} = 3y - 2x.$$

$$44. (x^2 - 2xy + y^2) \div (x - y) = x - y.$$

$$45. (a^2 + 2a + 1) \div (a + 1) = a + 1.$$

$$46. (a^2 + 4a + 4) \div (a + 2) = a + 2.$$

$$47. (4x^2 + 4x + 1) \div (2x + 1) = 2x + 1.$$

$$48. (16x^2 - 8x + 1) \div (4x - 1) = 4x - 1.$$

$$49. (16x^2 - 8xy + y^2) \div (4x - y) = 4x - y.$$

$$50. (9x^4 - 6x^2 + 1) \div (3x^2 - 1) = 3x^2 - 1.$$

$$51. (16y^6 - 40y^3 + 25) \div (4y^3 - 5) = 4y^3 - 5.$$

$$52. c^2 + 2cd + d^2 = (c + d)(c + d).$$

$$53. a^2 - 2a + 1 = (a - 1)(a - 1).$$

$$54. x^2 - 10x + 25 = (x - 5)(x - 5).$$

$$55. 4a^2 - 4a + 1 = (2a - 1)(2a - 1).$$

$$56. 4x^2 + 12x + 9 = (2x + 3)(2x + 3).$$

$$57. 9a^2b^2 - 6ab + 1 = (3ab - 1)(3ab - 1).$$

$$58. 16x^4y^2 - 24x^2ya + 9a^2 = (4x^2y - 3a)(4x^2y - 3a).$$

$$59. 25a^4b^6 - 60a^2b^3c^2 + 36c^4 = (5a^2b^3 - 6c^2)(5a^2b^3 - 6c^2).$$

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$$\begin{aligned} 1. (a + b + c)^2 &= [(a + b) + c][(a + b) + c] \\ &= (a + b)^2 + 2c(a + b) + c^2 \\ &= a^2 + 2ab + b^2 + 2ac + 2bc + c^2. \end{aligned}$$

$$\begin{aligned} 2. (a + b - c)^2 &= [(a + b) - c]^2 \\ &= (a + b)^2 - 2c(a + b) + c^2 \\ &= a^2 + 2ab + b^2 - 2ac - 2bc + c^2. \end{aligned}$$

$$\begin{aligned} 3. (a + b - 1)^2 &= [(a + b) - 1]^2 \\ &= (a + b)^2 - 2(a + b) + 1 \\ &= a^2 + 2ab + b^2 - 2a - 2b + 1. \end{aligned}$$

4. $(a + b + 1)^2 = [(a + b) + 1]^2$
 $= (a + b)^2 + 2(a + b) + 1$
 $= a^2 + 2ab + b^2 + 2a + 2b + 1.$
5. $(a - b + c)^2 = [(a - b) + c]^2$
 $= (a - b)^2 + 2c(a - b) + c^2$
 $= a^2 - 2ab + b^2 + 2ac - 2bc + c^2.$
6. $(a - b - c)^2 = [(a - b) - c]^2$
 $= (a - b)^2 - 2c(a - b) + c^2$
 $= a^2 - 2ab + b^2 - 2ac + 2bc + c^2.$
7. $(a + 2b - 3)^2 = [(a + 2b) - 3]^2$
 $= (a + 2b)^2 - 2 \cdot 3(a + 2b) + 9$
 $= a^2 + 4ab + 4b^2 - 6a - 12b + 9.$
8. $(a - 2b + c)^2 = [(a - 2b) + c]^2$
 $= (a - 2b)^2 + 2c(a - 2b) + c^2$
 $= a^2 - 4ab + 4b^2 + 2ac - 4bc + c^2.$
9. $(2a + b - 4c)^2 = [(2a + b) - 4c]^2$
 $= (2a + b)^2 - 8c(2a + b) + 16c^2$
 $= 4a^2 + 4ab + b^2 - 16ac - 8bc + 16c^2.$
10. $(3a - 2b + 5c)^2 = [(3a - 2b) + 5c]^2$
 $= (3a - 2b)^2 + 10c(3a - 2b) + 25c^2$
 $= 9a^2 - 12ab + 4b^2 + 30ac - 20bc + 25c^2.$
11. $(2a - 3b - 4c)^2 = [(2a - 3b) - 4c]^2$
 $= (2a - 3b)^2 - 2 \cdot 4c(2a - 3b) + 16c^2$
 $= 4a^2 - 12ab + 9b^2 - 16ac + 24bc + 16c^2.$
12. $(4c - 1 + 2ab)^2 = [(4c - 1) + 2ab]^2$
 $= (4c - 1)^2 + 2 \cdot 2ab(4c - 1) + 4a^2b^2$
 $= 16c^2 - 8c + 1 + 16abc - 4ab + 4a^2b^2.$

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1. $(6 + 3)(6 - 3) = 36 - 9 = 27.$
2. $(8 + 4)(8 - 4) = 64 - 16 = 48.$
3. $(9 - 2)(9 + 2) = 81 - 4 = 77.$
4. $(10 - 4)(10 + 4) = 100 - 16 = 84.$
5. $(7 + 2)(7 - 2) = 49 - 4 = 45.$
6. $(12 - 3)(12 + 3) = 144 - 9 = 135.$
8. $35 \cdot 25 = (30 + 5)(30 - 5) = 900 - 25 = 875.$
9. $33 \cdot 27 = (30 + 3)(30 - 3) = 900 - 9 = 891.$
10. $36 \cdot 44 = (40 - 4)(40 + 4) = 1600 - 16 = 1584.$
11. $35 \cdot 45 = (40 - 5)(40 + 5) = 1600 - 25 = 1575.$
12. $52 \cdot 48 = (50 + 2)(50 - 2) = 2500 - 4 = 2496.$
13. $65 \cdot 75 = (70 - 5)(70 + 5) = 4900 - 25 = 4875.$
14. $72 \cdot 68 = (70 + 2)(70 - 2) = 4900 - 4 = 4896.$
15. $75 \cdot 85 = (80 - 5)(80 + 5) = 6400 - 25 = 6375.$
16. $97 \cdot 103 = (100 - 3)(100 + 3) = 10,000 - 9 = 9991.$

17. $(x + 3)(x - 3) = x^2 - 9$.
 18. $(a + 5)(a - 5) = a^2 - 25$.
 19. $(2x + 4)(2x - 4) = 4x^2 - 16$.
 20. $(3n + 5)(3n - 5) = 9n^2 - 25$.
 21. $(x + y)(x - y) = x^2 - y^2$.
 22. $(x - a)(x + a) = x^2 - a^2$.
 23. $(x - 1)(x + 1) = x^2 - 1$.
 24. $(a + 2)(a - 2) = a^2 - 4$.
 25. $(a^2 + 3)(a^2 - 3) = a^4 - 9$.
 26. $(4 + y^2)(4 - y^2) = 16 - y^4$.
 27. $(4 - x)(x + 4) = 16 - x^2$.
 28. $(2c + a)(2c - a) = 4c^2 - a^2$.
 29. $(3a + b)(b - 3a) = b^2 - 9a^2$.
 30. $(4b + 2c)(4b - 2c) = 16b^2 - 4c^2$.
 31. $(3xy - 2)(3xy + 2) = 9x^2y^2 - 4$.
 32. $(4ab - 3)(4ab + 3) = 16a^2b^2 - 9$.
 33. $(a^4 - b^3)(a^4 + b^3) = a^8 - b^6$.
 34. $(a^6 + a^3)(-a^3 + a^6) = a^{12} - a^6$.
 35. $(x^3 - 2y)(x^3 + 2y) = x^6 - 4y^2$.
 36. $(4ab - a^2)(4ab + a^2) = 16a^2b^2 - a^4$.
 37. $(3cd^2 + 2d)(-2d + 3cd^2) = 9c^2d^4 - 4d^2$.
 38. $(6cd + 3)(-3 + 6cd) = 36c^2d^2 - 9$.
 39. $(4xy + 2y)(2y - 4xy) = 4y^2 - 16x^2y^2$.
 40. $(3abc - 2bc)(2bc + 3abc) = 9a^2b^2c^2 - 4b^2c^2$.
 41. $(a^2 - b^2) \div (a + b) = a - b$.
 42. $(c^2 - d^2) \div (c - d) = c + d$.
 43. $(9 - b^2) \div (3 + b) = 3 - b$.
 44. $(36 - a^2) \div (6 - a) = 6 + a$.
 45. $(9x^2 - 16) \div (3x + 4) = 3x - 4$.
 46. $(x^4 - 1) \div (x^2 - 1) = x^2 + 1$.
 47. $(x^2 - y^2) \div (x - y) = x + y$.
 48. $(x^2 - 1) \div (x + 1) = x - 1$.
 49. $(4x^2 - 9) \div (2x - 3) = 2x + 3$.
 50. $(25 - 16x^2) \div (5 + 4x) = 5 - 4x$.
 51. $(16 - x^8) \div (4 - x^4) = 4 + x^4$.
 52. $(y^6 - 4) \div (y^3 - 2) = y^3 + 2$.
 53. $(c^2 - d^2) = (c - d)(c + d)$.
 54. $(n^2 - 4) = (n + 2)(n - 2)$.
 55. $n^2 - 16 = (n + 4)(n - 4)$.
 56. $9 - 4a^2 = (3 - 2a)(3 + 2a)$.
 57. $36b^2 - 1 = (6b + 1)(6b - 1)$.
 58. $a^2 - 9 = (a + 3)(a - 3)$.
 59. $25 - 4n^2 = (5 - 2n)(5 + 2n)$.
 60. $100 - 9x^2 = (10 - 3x)(10 + 3x)$.
 61. $[(x + y) + 1][(x + y) - 1] = (x + y)^2 - 1 = x^2 + 2xy + y^2 - 1$.
 62. $(x + a + 3)(x + a - 3) = [(x + a) + 3][(x + a) - 3]$
 $= x^2 + 2xa + a^2 - 9$.
 63. $[(x - a) + 3][(x - a) - 3] = (x - a)^2 - 9$
 $= x^2 - 2ax + a^2 - 9$.
 64. $(x + 4 + c)(x + 4 - c) = [(x + 4) + c][(x + 4) - c]$
 $= (x + 4)^2 - c^2$
 $= x^2 + 8x + 16 - c^2$.
 65. $(2a - b + c)(2a - b - c) = [(2a - b) + c][(2a - b) - c]$
 $= (2a - b)^2 - c^2$
 $= 4a^2 - 4ab + b^2 - c^2$.
 66. $[x + (b + c)][x - (b + c)] = x^2 - (b + c)^2 = x^2 - b^2 - 2bc - c^2$.
 67. $[x + (b - c)][x - (b - c)] = x^2 - (b - c)^2 = x^2 - b^2 + 2bc - c^2$.
 68. $[3 + (x - y)][3 - (x - y)] = 9 - (x - y)^2 = 9 - x^2 + 2xy - y^2$.
 69. $[4x + (2y - x)][4x - (2y - x)] = 16x^2 - (2y - x)^2$
 $= 16x^2 - 4y^2 + 4xy - x^2$
 $= 15x^2 - 4y^2 + 4xy$.

70. $[10 - (a - 5)][10 + (a - 5)] = 100 - (a - 5)^2$
 $= 100 - a^2 + 10a - 25$
 $= 75 + 10a - a^2.$
71. $49x^2 - 1 = (7x + 1)(7x - 1).$
72. $64x^2 - 25 = (8x - 5)(8x + 5).$
73. $(a + b)^2 - 1 = (a + b + 1)(a + b - 1).$
74. $(x - y)^2 - 4 = (x - y - 2)(x - y + 2).$
75. $(2x - 1)^2 - a^2 = (2x - 1 + a)(2x - 1 - a).$
76. $(3y - z)^2 - b^2 = (3y - z + b)(3y - z - b).$
77. $b^2 - (x + y)^2 = [b + (x + y)][b - (x + y)] = (b + x + y)(b - x - y).$
78. $b^2 - (x - y)^2 = [b + (x - y)][b - (x - y)].$
79. $c^2 - (x + c)^2 = [c + (x + c)][c - (x + c)].$
80. $4 - (x - 2)^2 = [2 + (x - 2)][2 - (x - 2)].$

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1. $(x + 1)(x + 2) = x^2 + 3x + 2.$ 2. $(x + 2)(x + 3) = x^2 + 5x + 6.$
3. $(x + 3)(x + 4) = x^2 + 7x + 12.$
4. $(x + 4)(x + 5) = x^2 + 9x + 20.$
5. $(x + 5)(x + 6) = x^2 + 11x + 30.$
6. $(n + 1)(n - 2) = n^2 - n - 2.$
7. $(n - 2)(n + 3) = n^2 + n - 6.$
8. $(n - 3)(n + 4) = n^2 + n - 12.$
9. $(n - 4)(n + 5) = n^2 + n - 20.$
10. $(n - 5)(n + 6) = n^2 + n - 30.$
11. $(a - 1)(a - 2) = a^2 - 3a + 2.$
12. $(a - 2)(a - 3) = a^2 - 5a + 6.$
13. $(a - 3)(a - 4) = a^2 - 7a + 12.$
14. $(a - 4)(a - 5) = a^2 - 9a + 20.$
15. $(a - 4)(a - 6) = a^2 - 10a + 24.$
16. $(2y + 3)(2y + 4) = 4y^2 + 14y + 12.$
17. $(2y + 2)(2y + 3) = 4y^2 + 10y + 6.$
18. $(3a + 1)(3a + 4) = 9a^2 + 15a + 4.$
19. $(2n + 3)(2n + 5) = 4n^2 + 16n + 15.$
20. $(2n + 3)(2n - 5) = 4n^2 - 4n - 15.$
21. $(2n + 3)(2n - 4) = 4n^2 - 2n - 12.$
22. $(2a + 1)(2a - 5) = 4a^2 - 8a - 5.$
23. $(3a - 5)(3a + 1) = 9a^2 - 12a - 5.$
24. $(4a + 3)(4a - 5) = 16a^2 - 8a - 15.$
25. $(4a - 5)(4a - 6) = 16a^2 - 44a + 30.$
26. $(4ab + 1)(4ab - 6) = 16a^2b^2 - 20ab - 6.$
27. $(3x - 2)(3x + 5) = 9x^2 + 9x - 10.$

$$28. (4a - 3b)(4a + 5b) = 16a^2 + 8ab - 15b^2.$$

$$29. (5a - 6b)(5a - 7b) = 25a^2 - 65ab + 42b^2.$$

$$30. \frac{x^2 + 3x + 2}{x + 1} = x + 2.$$

$$31. \frac{x^2 + 5x + 6}{x + 2} = x + 3.$$

$$32. \frac{x^2 + 4x + 3}{x + 3} = x + 1.$$

$$33. \frac{x^2 + 6x + 5}{x + 1} = x + 5.$$

$$34. \frac{x^2 + 7x + 12}{x + 3} = x + 4.$$

$$35. \frac{x^2 - 7x + 12}{x - 3} = x - 4.$$

$$36. \frac{x^2 - 5x + 6}{x - 3} = x - 2.$$

$$37. \frac{x^2 - 5x + 4}{x - 1} = x - 4.$$

$$38. \frac{x^2 - 6x + 5}{x - 5} = x - 1.$$

$$39. \frac{x^2 - 7x + 6}{x - 6} = x - 1.$$

$$40. \frac{x^2 - 7x + 12}{x - 4} = x - 3.$$

$$41. x^2 + 3x + 2 = (x + 1)(x + 2).$$

$$42. x^2 + 5x + 6 = (x + 3)(x + 2).$$

$$43. x^2 + 7x + 12 = (x + 4)(x + 3).$$

$$44. x^2 + 6x + 8 = (x + 4)(x + 2).$$

$$45. x^2 + 7x + 10 = (x + 2)(x + 5).$$

$$46. x^2 + 8x + 15 = (x + 3)(x + 5).$$

$$47. x^2 - 3x + 2 = (x - 2)(x - 1).$$

$$48. x^2 - 5x + 6 = (x - 3)(x - 2).$$

$$49. x^2 - 6x + 8 = (x - 4)(x - 2).$$

$$50. x^2 - 8x + 15 = (x - 5)(x - 3).$$

$$51. x^2 - 8x + 12 = (x - 6)(x - 2).$$

$$52. x^2 - 9x + 14 = (x - 7)(x - 2).$$

$$53. x^2 + 8x + 7 = (x + 1)(x + 7).$$

$$54. x^2 + 9x + 8 = (x + 1)(x + 8).$$

$$55. x^2 - 10x + 9 = (x - 9)(x - 1).$$

$$56. x^2 - 11x + 10 = (x - 10)(x - 1).$$

$$57. x^2 - 10x + 16 = (x - 2)(x - 8).$$

$$58. x^2 + 10x + 21 = (x + 3)(x + 7).$$

$$59. x^2 + 9x + 20 = (x + 4)(x + 5).$$

$$60. x^2 - 9x + 18 = (x - 6)(x - 3).$$

$$61. x^2 + 9x + 14 = (x + 2)(x + 7).$$

$$62. x^2 - 12x + 32 = (x - 4)(x - 8).$$

$$63. x^2 + 11x + 10 = (x + 10)(x + 1).$$

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$$1. (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc.$$

$$2. (a + b + 1)^2 = a^2 + b^2 + 1 + 2ab + 2a + 2b.$$

$$3. (a + b + 2)^2 = a^2 + b^2 + 4 + 2ab + 4a + 4b.$$

$$4. (2a + b - c)^2 = 4a^2 + b^2 + c^2 + 4ab - 4ac - 2bc.$$

$$5. (a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc.$$

$$6. (2x + y - 1)^2 = 4x^2 + y^2 + 1 + 4xy - 4x - 2y.$$

$$7. (4x + y - 2)^2 = 16x^2 + y^2 + 4 + 8xy - 16x - 4y.$$

$$8. (a - bc + d)^2 = a^2 + b^2c^2 + d^2 - 2abc + 2ad - 2bcd.$$

$$9. (3a - 3b + 1)^2 = 9a^2 + 9b^2 + 1 - 18ab + 6a - 6b.$$

$$10. (a - b + c - d)^2 \\ = a^2 + b^2 + c^2 + d^2 - 2ab + 2ac - 2ad - 2bc + 2bd - 2cd.$$

$$11. (x + y - a + 1)^2 \\ = x^2 + y^2 + a^2 + 1 + 2xy - 2ax + 2x - 2ay + 2y - 2a.$$

$$12. (a - y + b - 3)^2 \\ = a^2 + y^2 + b^2 + 9 - 2ay + 2ab - 6a - 2by + 6y - 6b.$$

Page 100 (Second set)

1. $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$
2. $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3.$
3. $(x + 1)^3 = x^3 + 3x^2 + 3x + 1.$
4. $(x - 1)^3 = x^3 - 3x^2 + 3x - 1.$
5. $(x + 2)^3 = x^3 + 6x^2 + 12x + 8.$
6. $(x - 2)^3 = x^3 - 6x^2 + 12x - 8.$
7. $(x - 3)^3 = x^3 - 9x^2 + 27x - 27.$
8. $(x + 3)^3 = x^3 + 9x^2 + 27x + 27.$
9. $(2x + y)^3 = 8x^3 + 12x^2y + 6xy^2 + y^3.$
10. $(x - 2y)^3 = x^3 - 6x^2y + 12xy^2 - 8y^3.$
11. $(2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1.$
12. $(3x + 2)^3 = 27x^3 + 54x^2 + 36x + 8.$
13. $(3x - 2y)^3 = 27x^3 - 54x^2y + 36xy^2 - 8y^3.$
14. $(4x + 3y)^3 = 64x^3 + 144x^2y + 108xy^2 + 27y^3.$
15. $(5x - 2y)^3 = 125x^3 - 150x^2y + 60xy^2 - 8y^3.$

16. The cube of the binomial $a + b$ is equal to the cube of a plus three times the square of a multiplied by b , plus three times the product of a and the square of b , plus the cube of b .

17. The cube of the binomial $a - b$ is equal to the cube of a minus three times the square of a multiplied by b , plus three times the product of a and the square of b , minus the cube of b .

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1. $\sqrt{4a^2} = 2a.$
2. $\sqrt{9a^4} = 3a^2.$
7. $\sqrt{49x^{14}y^{10}} = 7x^7y^5.$
8. $\sqrt{81x^{12}d^{20}} = 9x^6d^{10}.$
9. $\sqrt{121c^{22}d^6} = 11c^{11}d^3.$
10. $\sqrt{169x^6} = 13x^3.$
11. $\sqrt{196y^{10}} = 14y^5.$
17. $\sqrt{x^{2n}} = x^n.$
18. $\sqrt{x^{4n}} = x^{2n}.$
19. $\sqrt{x^{6n}} = x^{3n}.$
20. $\sqrt{a^{4n}b^{8n}} = a^{2n}b^{4n}.$
21. $\sqrt[3]{8} = 2.$
32. $\sqrt[3]{-729a^3b^9} = -9ab^3.$
33. $\sqrt[3]{1000a^6b^{12}} = 10a^2b^4.$
3. $\sqrt{25a^6x^4} = 5a^3x^2.$
4. $\sqrt{4x^4} = 2x^2.$
22. $\sqrt[3]{-8a^3} = -2a.$
23. $\sqrt[3]{27} = 3.$
24. $\sqrt[3]{64} = 4.$
25. $\sqrt[3]{-125} = -5.$
26. $\sqrt[3]{216} = 6.$
34. $\sqrt[3]{-27a^3b^6c^{12}} = -3ab^2c^4.$
35. $\sqrt[3]{8a^{12}b^3c^9} = 2a^4bc^3.$
5. $\sqrt{9x^6} = 3x^3.$
6. $\sqrt{16x^{16}} = 4x^8.$
12. $\sqrt{225y^4z^6} = 15y^2z^3.$
13. $\sqrt{256c^{12}} = 16c^6.$
14. $\sqrt{400a^2b^2c^4} = 20abc^2.$
15. $\sqrt{324b^8} = 18b^4.$
16. $\sqrt{625b^2y^{18}z^6} = 25by^9z^3.$
27. $\sqrt[3]{-a^3} = -a.$
28. $\sqrt[3]{-a^6} = -a^2.$
29. $\sqrt[3]{a^9b^3} = a^3b.$
30. $\sqrt[3]{343y^3} = 7y.$
31. $\sqrt[3]{512x^{15}} = 8x^5.$

36. $\sqrt[3]{-125 a^6 b^6 z^6} = -5 a^2 b^2 z^2$.
 37. $\sqrt[3]{x^{3n}} = x^n$.
 38. $\sqrt[3]{x^{6n}} = x^{2n}$.
 39. $\sqrt[3]{-x^{9n}} = -x^{3n}$.
 40. $\sqrt[3]{x^{12n} y^{6n}} = x^{4n} y^{2n}$.
 41. $\sqrt[5]{x^{15}} = x^3$.
 42. $\sqrt[5]{32 x^{10}} = 2 x^2$.

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1. $3x + 6 = 3(x + 2)$.
 2. $x^3 - x^2 = x^2(x - 1)$.
 3. $8x - 2x^4 = 2x(4 - x^3)$.
 4. $xy + y^2 = y(x + y)$.
 5. $5b^3 - 15b^6 = 5b^3(1 - 3b^3)$.
 6. $c^3 - c^2y = c^2(c - y)$.
 7. $x^3 - x + x^2 = x(x^2 - 1 + x)$.
 8. $c^5 - c^3 + c^2 = c^2(c^3 - c + 1)$.
 9. $3y - 9y^4 + 12 = 3(y - 3y^4 + 4)$.
 10. $c^2 + 2bc - c = c(c + 2b - 1)$.
 11. $3y^2 - 15y + 6y^3 = 3y(y - 5 + 2y^2)$.
 12. $10ab - 14bc - 8b^2 = 2b(5a - 7c - 4b)$.
 13. $y^5 + 3y^4z + 6y^3z^2 = y^3(y^2 + 3yz + 6z^2)$.
 14. $4a^2x^2 - 6a^3y^3 + 12a^4x^2y^2 = 2a^2(2x^2 - 3ay^3 + 6a^2x^2y^2)$.
 15. $12z^3 + 30a^4z^7 - 18c^3z^5 = 6z^3(2 + 5a^4z^4 - 3c^3z^2)$.
 16. $6c^6 + 10c^{10} - 20c^2 = 2c^2(3c^4 + 5c^8 - 10)$.
 17. $8y^8 - 4y^4 - 12y^{12} = 4y^4(2y^4 - 1 - 3y^8)$.
 18. $x^4 + x^3 - x^2 + x = x(x^3 + x^2 - x + 1)$.
 19. $-8y^6 - 4y^6 + 12y^3 + 6y^5 = 6y^3(-2y^3 + 2 + y^2)$.
 20. $14b^5 - 49b^7 + 21b^2 - 7b^3 = 7b^2(2b^3 - 7b^5 + 3 - b)$.
 21. $-a^2x^2y^3 - 3a^4x^3y^2 - 2a^3x^5y^6z^3 + 5a^4x^4y^4z$
 $= a^2x^2y^2(-y - 3a^2x - 2ax^3y^4z^3 + 5a^2x^2y^2z)$.
 22. $-18a^3b^3c^4 - 45a^4cy^2 - 36a^3c^2xz - 63a^4cxy$
 $= -9a^3c(2b^3c^3 + 5ay^2 + 4cxz + 7axy)$.
 23. $32c^7x^2 + 80c^{10}bx - 112c^6b + 48c^{12}x$
 $= 16c^6(2cx^2 + 5c^4bx - 7b + 3c^6x)$.
 24. $56a^4b^3x^2 - 28a^3x + 112a^5x^2y^2 - 196a^6x^3$
 $= 28a^3x(2ab^3x - 1 + 4a^2xy^2 - 7a^3x^2)$.

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1. $2(x + y) + a(x + y) = (x + y)(2 + a)$.
 2. $b(x + 2y) + (x + 2y) = (x + 2y)(b + 1)$.
 3. $c(c - a) - a(c - a) = (c - a)(c - a)$.
 4. $2c(3b - d) + 3a(3b - d) = (3b - d)(2c + 3a)$.
 5. $6d(c + x) - (c + x) + 4k(c + x) = (c + x)(6d - 1 + 4k)$.
 6. $-5(2a - 3b) + 7a(2a - 3b) - 3b(2a - 3b)$
 $= (2a - 3b)(7a - 5 - 3b)$.
 7. $k(a - b) + 3(b - a) = k(a - b) - 3(a - b) = (a - b)(k - 3)$.

8. $h(x - y) + k(y - x) = h(x - y) - k(x - y)$
 $= (x - y)(h - k).$
9. $5x(c - 2d) - 6(2d - c) = 5x(c - 2d) + 6(c - 2d)$
 $= (c - 2d)(5x + 6).$
10. $3y(k - 4h) + (4h - k) = 3y(k - 4h) - (k - 4h)$
 $= (k - 4h)(3y - 1).$
11. $2c(4k - 3c) - 5h(3c - 4k) = 2c(4k - 3c) + 5h(4k - 3c)$
 $= (4k - 3c)(2c + 5h).$
12. $3y(5x - b) - 7(b - 5x) + 6z(5x - b)$
 $= 3y(5x - b) + 7(5x - b) + 6z(5x - b)$
 $= (5x - b)(3y + 7 + 6z).$
16. $ax + bx + ay + by = (ax + bx) + (ay + by)$
 $= x(a + b) + y(a + b)$
 $= (a + b)(x + y).$
17. $ch + h^2 + cx + hx = (ch + h^2) + (cx + hx)$
 $= h(c + h) + x(c + h)$
 $= (c + h)(h + x).$
18. $bh - ch + bk - ck = (bh - ch) + (bk - ck)$
 $= h(b - c) + k(b - c)$
 $= (b - c)(h + k).$
19. $bx + xz - ab - az = (bx + xz) + (-ab - az)$
 $= x(b + z) - a(b + z)$
 $= (b + z)(x - a).$
20. $ax + 3a - bx - 3b = (ax + 3a) + (-bx - 3b)$
 $= a(x + 3) - b(x + 3)$
 $= (x + 3)(a - b).$
21. $2x^2 + 10xy - 4x - 20y = 2(x^2 + 5xy - 2x - 10y)$
 $= 2[x(x + 5y) - 2(x + 5y)]$
 $= 2(x - 2)(x + 5y).$
22. $6ab - 2ac + 3b - c = (6ab - 2ac) + (3b - c)$
 $= 2a(3b - c) + (3b - c)$
 $= (3b - c)(2a + 1).$
23. $36ax + 45ac - 4x - 5c = 9a(4x + 5c) - (4x + 5c)$
 $= (4x + 5c)(9a - 1).$
24. $6a^3 + 14ab - 15a^2y - 35by = 2a(3a^2 + 7b) - 5y(3a^2 + 7b)$
 $= (3a^2 + 7b)(2a - 5y).$
25. $3ax - ay - 6bx + 2by = a(3x - y) - 2b(3x - y)$
 $= (3x - y)(a - 2b).$
26. $12ax - 6ay - 50cx + 25cy = 6a(2x - y) - 25c(2x - y)$
 $= (2x - y)(6a - 25c).$
27. $2h^3 - 3h^2k - 10h + 15k = h^2(2h - 3k) - 5(2h - 3k)$
 $= (2h - 3k)(h^2 - 5).$

$$\begin{aligned} 28. \quad 10 dh - 45 hk - 22 cd + 99 ck &= 5 h (2 d - 9 k) - 11 c (2 d - 9 k) \\ &= (2 d - 9 k) (5 h - 11 c). \end{aligned}$$

$$\begin{aligned} 29. \quad 28 hx + 9 ky - 21 hy - 12 kx &= 28 hx - 21 hy - 12 kx + 9 ky \\ &= 7 h (4 x - 3 y) - 3 k (4 x - 3 y) \\ &= (4 x - 3 y) (7 h - 3 k). \end{aligned}$$

$$\begin{aligned} 30. \quad -2 ax + 7 a^2 + 16 bx - 56 ab &= -a (2 x - 7 a) + 8 b (2 x - 7 a) \\ &= (2 x - 7 a) (8 b - a). \end{aligned}$$

$$\begin{aligned} 31. \quad 6 hk + 15 xy - 10 ky - 9 hx &= 6 hk - 9 hx - 10 ky + 15 xy \\ &= 3 h (2 k - 3 x) - 5 y (2 k - 3 x) \\ &= (2 k - 3 x) (3 h - 5 y). \end{aligned}$$

$$\begin{aligned} 32. \quad -21 ax + 12 cx - 4 cd + 7 ad &= 12 cx - 21 ax - 4 cd + 7 ad \\ &= 3 x (4 c - 7 a) - d (4 c - 7 a) \\ &= (4 c - 7 a) (3 x - d). \end{aligned}$$

$$\begin{aligned} 33. \quad 7 gy - 77 hy - cg + 11 ch &= 7 y (g - 11 h) - c (g - 11 h) \\ &= (g - 11 h) (7 y - c). \end{aligned}$$

$$\begin{aligned} 34. \quad 5 a^3 + 10 a - 5 a^2 - 10 &= 5 [a (a^2 + 2) - (a^2 + 2)] \\ &= 5 (a^2 + 2) (a - 1). \end{aligned}$$

$$\begin{aligned} 35. \quad 3 a - 5 ax^3 - 6 ax + 10 ax^4 &= a [(3 - 5 x^3) - 2 x (3 - 5 x^3)] \\ &= a (3 - 5 x^3) (1 - 2 x). \end{aligned}$$

$$\begin{aligned} 36. \quad 4 abxy - 24 dxy - 3 abgh + 18 dgh &= 4 xy (ab - 6 d) - 3 gh (ab - 6 d) \\ &= (ab - 6 d) (4 xy - 3 gh). \end{aligned}$$

$$\begin{aligned} 37. \quad 8 acxy - 20 bx^2y - 6 abc^2 + 15 b^2cx &= 4 xy (2 ac - 5 bx) - 3 bc (2 ac - 5 bx) \\ &= (2 ac - 5 bx) (4 xy - 3 bc). \end{aligned}$$

$$\begin{aligned} 38. \quad ax + bx + cx + ay + by + cy &= x (a + b + c) + y (a + b + c) \\ &= (a + b + c) (x + y). \end{aligned}$$

$$\begin{aligned} 39. \quad cg + 2 ch + ck - gx - 2 hx - kx &= c (g + 2 h + k) - x (g + 2 h + k) \\ &= (g + 2 h + k) (c - x). \end{aligned}$$

$$\begin{aligned} 40. \quad 3 ab - bx + 2 by - 3 ac + cx - 2 cy &= b (3 a - x + 2 y) - c (3 a - x + 2 y) \\ &= (3 a - x + 2 y) (b - c). \end{aligned}$$

$$\begin{aligned} 41. \quad 4 ax - ac + 2 ay + 12 bx - 3 bc + 6 by &= a (4 x - c + 2 y) + 3 b (4 x - c + 2 y) \\ &= (4 x - c + 2 y) (a + 3 b). \end{aligned}$$

$$\begin{aligned} 42. \quad ax - 6 cy - 2 bx + 4 by + 3 cx - 2 ay &= ax - 2 bx + 3 cx - 2 ay + 4 by - 6 cy \\ &= x (a - 2 b + 3 c) - 2 y (a - 2 b + 3 c) \\ &= (a - 2 b + 3 c) (x - 2 y). \end{aligned}$$

$$\begin{aligned} 43. \quad -2 xz + 6 ay + cz + 4 xy - 3 az - 2 cy &= 6 ay + 4 xy - 2 cy - 3 az - 2 xz + cz \\ &= 2 y (3 a + 2 x - c) - z (3 a + 2 x - c) \\ &= (3 a + 2 x - c) (2 y - z). \end{aligned}$$

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1. $c^2 + 2cd + d^2$.
2. $a^2 + 4a + 4$.
3. $4a^2 + 4a + 1$.
4. $x^6 + 6x^3 + 9$.
5. $4x^2 + 20x + 25$.
6. $9x^6 + 12x^3y + 4y^2$.
7. $100x^2 + 120xy + 36y^2$.
8. $49 + 112y^4 + 64y^8$.
9. $x^2 + 2x + 1$.
10. $y^2 - 6y + 9$.
11. $4y^2 - 40y + 100$.
12. $9y^2 + 36y + 36$.
13. $16z^4 - 16xz^2 + 4x^2$.
14. $121y^6 - 88y^3z + 16z^2$.
15. $100a^8 + 240a^4x^5 + 144x^{10}$.
16. $y^4 - 6y^2 + 9$.
17. $y^8 - 10y^4 + 25$.
18. $16k^4 + 16k^2 + 4$.
19. $25k^4 - 80k^2h^2 + 64h^4$.
20. $16a^{16} + 104a^8k + 169k^2$.

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1. $c^2 + 2cd + d^2 = (c + d)^2$.
2. $x^2 + 2xy + y^2 = (x + y)^2$.
3. $x^2 + 2x + 1 = (x + 1)^2$.
4. $a^2 - 10a + 25 = (a - 5)^2$.
5. $a^2 + 8a + 16 = (a + 4)^2$.
6. $4h^2 + 4h + 1 = (2h + 1)^2$.
7. $9 - 6x + x^2 = (3 - x)^2$.
8. $16k^2 - 8k + 1 = (4k - 1)^2$.
9. $4h^2 - 12h + 9 = (2h - 3)^2$.
10. $x^4 + 1 + 2x^2 = (x^2 + 1)^2$.
11. $x^6 - 6x^3 + 9 = (x^3 - 3)^2$.
12. $25h^2 + 4k^2 - 20hk = (5h - 2k)^2$.
13. $9h^2 - 24hk + 16k^2 = (3h - 4k)^2$.
14. $x^2 - 24xz^2 + 144z^4 = (x - 12z^2)^2$.
15. $12x^4z^3 + 9z^6 + 4x^8 = (3z^3 + 2x^4)^2$.
16. $c^6d^{10} + 2c^3d^5 + 1 = (c^3d^5 + 1)^2$.
17. $24x^2y^2 + 16y^4 + 9x^4 = (4y^2 + 3x^2)^2$.
18. $x^4 + 4y^4 + 4x^2y^2 = (x^2 + 2y^2)^2$.
19. $4x^2y^8 - 4xy^4 + 1 = (2xy^4 - 1)^2$.
20. $25h^6k^2 - 30h^3kz + 9z^2 = (5h^3k - 3z)^2$.
21. $121c^6d^4 - 220c^3d^2g^2 + 100g^4 = (11c^3d^2 - 10g^2)^2$.
22. $169a^4 - 156a^2x^4y^3 + 36x^8y^6 = (13a^2 - 6x^4y^3)^2$.

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1. $x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)(x + 1)$.
2. $x^5 - 4x^4 + 4x^3 = x^3(x^2 - 4x + 4) = x^3(x - 2)(x - 2)$.
3. $2a^4 + 12a^2 + 18 = 2(a^4 + 6a^2 + 9) = 2(a^2 + 3)(a^2 + 3)$.
4. $3a^2 + 18a + 27 = 3(a^2 + 6a + 9) = 3(a + 3)(a + 3)$.
5. $x^6 - 20x^4 + 100x^2 = x^2(x^4 - 20x^2 + 100) = x^2(x^2 - 10)(x^2 - 10)$.
6. $5cx^6 - 70cx^3 + 245c = 5c(x^6 - 14x^3 + 49) = 5c(x^3 - 7)(x^3 - 7)$.
7. $16x^8 - 24x^6 + 9x^4 = x^4(16x^4 - 24x^2 + 9) = x^4(4x^2 - 3)(4x^2 - 3)$.
8. $28x^2 - 28x^3 + 7x^4 = 7x^2(4 - 4x + x^2) = 7x^2(2 - x)(2 - x)$.
9. $3ax + 3ay + 3bx + 3by = 3[a(x + y) + b(x + y)] = 3(x + y)(a + b)$.

10. $4cx - 4cy + 4dx - 4dy = 4[c(x - y) + d(x - y)] = 4(x - y)(c + d).$
 11. $ax - a + x - 1 = a(x - 1) + (x - 1) = (x - 1)(a + 1).$
 12. $63xy - 84y^2 + 98yz = 7y(9x - 12y + 14z).$
 13. $30ax - 34bx - 15a + 17b = 2x(15a - 17b) - (15a - 17b)$
 $= (15a - 17b)(2x - 1).$
 14. $ax - bx + cx + ay - by + cy = x(a - b + c) + y(a - b + c)$
 $= (a - b + c)(x + y).$
 15. $14anx - 21bny - 7n = 7n(2ax - 3by - 1).$
 16. $3ax - 5by - 5ay + 3bx = 3x(a + b) - 5y(a + b)$
 $= (a + b)(3x - 5y).$
 17. $56a^2 - 40ab + 63ac - 45bc = 8a(7a - 5b) + 9c(7a - 5b)$
 $= (7a - 5b)(8a + 9c).$

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1. $x^2 + 5x + 6 = (x + 2)(x + 3).$ 7. $x^2 - x - 6 = (x + 2)(x - 3).$
 2. $x^2 + 9x + 20 = (x + 4)(x + 5).$ 8. $x^2 - 3x - 40 = (x - 8)(x + 5).$
 3. $x^2 - 7x + 12 = (x - 3)(x - 4).$ 9. $x^2 + x - 30 = (x + 6)(x - 5).$
 4. $x^2 - 9x + 14 = (x - 2)(x - 7).$ 10. $x^2 - 2x - 35 = (x - 7)(x + 5).$
 5. $x^2 + 19x + 18 = (x + 1)(x + 18).$ 11. $x^2 + 11x - 42 = (x + 14)(x - 3).$
 6. $x^2 - 18x + 17 = (x - 1)(x - 17).$ 12. $6 - 5x - x^2 = (6 + x)(1 - x).$
 13. $14 - 5x - x^2 = -1(x^2 + 5x - 14) = -(x + 7)(x - 2) = (x + 7)(2 - x).$
 14. $21 + 4x - x^2 = -1(x^2 - 4x - 21) = -1(x - 7)(x + 3) = (7 - x)(x + 3).$
 15. $24 + 10a - a^2 = -1(a^2 - 10a - 24) = -1(a - 12)(a + 2) = (12 - a)(a + 2).$
 16. $10 + 3x - x^2 = -1(x^2 - 3x - 10) = -1(x - 5)(x + 2) = (5 - x)(x + 2).$
 17. $72 + c - c^2 = -1(c^2 - c - 72) = -1(c - 9)(c + 8) = (9 - c)(c + 8).$
 18. $x^2 + x - 90 = (x + 10)(x - 9).$
 19. $28 + x^2 - 11x = (x - 4)(x - 7).$
 20. $27 - 12x + x^2 = (-9 + x)(-3 + x) = (x - 9)(x - 3).$
 21. $8x + x^2 - 48 = (x + 12)(x - 4).$
 22. $x^2 - x - 90 = (x - 10)(x + 9).$
 23. $x^2 - 5ax + 6a^2 = (x - 2a)(x - 3a).$
 24. $x^2 + 7ax + 12a^2 = (x + 3a)(x + 4a).$
 25. $a^2x^2 - 2ax - 15 = (ax - 5)(ax + 3).$
 26. $h^2k^2 - 9hk + 20 = (hk - 4)(hk - 5).$
 27. $x^4 - 5x^2 - 14 = (x^2 - 7)(x^2 + 2).$
 28. $x^2 + x^4 - 110 = (x^2 + 11)(x^2 - 10).$
 29. $x^6 - 11x^3 + 18 = (x^3 - 9)(x^3 - 2).$
 30. $b^2x^2 - 3bxy - 28y^2 = (bx - 7y)(bx + 4y).$
 31. $h^2k^2 - 5h kx - 36x^2 = (hk - 9x)(hk + 4x).$
 32. $a^6b^2 - 5a^3bx - 24x^2 = (a^3b - 8x)(a^3b + 3x).$
 33. $h^4d^2 + 10h^2dg^3 - 24g^6 = (h^2d + 12g^3)(h^2d - 2g^3).$
 34. $9k^2x^4y^6 + k^4x^8 - 22y^{12} = (k^2x^4 + 11y^6)(k^2x^4 - 2y^6).$

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1. $x^4 + 6x^3 + 9x^2 = x^2(x^2 + 6x + 9) = x^2(x + 3)(x + 3).$
2. $x^3 - 5x^2 + 6x = x(x^2 - 5x + 6) = x(x - 3)(x - 2).$
3. $3b^2 - 9b - 30 = 3(b^2 - 3b - 10) = 3(b - 5)(b + 2).$
4. $3ax^3 - 3ax^2 - 6ax = 3ax(x^2 - x - 2) = 3ax(x - 2)(x + 1).$
5. $4x^5 - 4x^3 - 360x = 4x(x^4 - x^2 - 90) = 4x(x^2 - 10)(x^2 + 9).$
6. $3x^4 - 21x^3 - 54x^2 = 3x^2(x^2 - 7x - 18) = 3x^2(x - 9)(x + 2).$
7. $4a^6y^4 - 64a^4y^2 + 256a^2 = 4a^2(a^4y^4 - 16a^2y^2 + 64)$
 $= 4a^2(a^2y^2 - 8)(a^2y^2 - 8).$
8. $3x - 3ax - 18a^2x = -3x(6a^2 + a - 1)$
 $= -3x(2a + 1)(3a - 1)$
 $= 3x(1 + 2a)(1 - 3a).$
9. $56x + x^2 - x^3 = -x(x^2 - x - 56)$
 $= -x(x - 8)(x + 7)$
 $= x(8 - x)(x + 7).$
10. $140h^2 + 23h^3 - h^4 = -h^2(h^2 - 23h - 140)$
 $= -h^2(h - 28)(h + 5)$
 $= h^2(28 - h)(h + 5).$
11. $3x^3 - 3x^4 + 60x^2 = -3x^2(x^2 - x - 20)$
 $= -3x^2(x - 5)(x + 4)$
 $= 3x^2(5 - x)(x + 4).$
12. $32a^2x^2 + 2a^2 - 16a^2x = 2a^2(16x^2 - 8x + 1) = 2a^2(4x - 1)^2.$
13. $4ay^2x^2 - 40ay^2x + 100ay^2 = 4ay^2(x^2 - 10x + 25)$
 $= 4ay^2(x - 5)(x - 5).$
14. $3x^4 - 3x^3 + 3x^2 - 3x = 3x(x^3 - x^2 + x - 1)$
 $= 3x[x^2(x - 1) + (x - 1)]$
 $= 3x(x - 1)(x^2 + 1).$
15. $2ax^3 - 4ax^2 - 12ax + 24a = 2a(x^3 - 2x^2 - 6x + 12)$
 $= 2a[x^2(x - 2) - 6(x - 2)]$
 $= 2a(x - 2)(x^2 - 6).$
16. $5xyz - 15yz + 10xz - 30z = 5z(xy - 3y + 2x - 6)$
 $= 5z[y(x - 3) + 2(x - 3)]$
 $= 5z(x - 3)(y + 2).$

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1. $2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$
 $= 2x(x + 1) + 3(x + 1)$
 $= (2x + 3)(x + 1).$
2. $3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2$
 $= 3x(x + 1) + 2(x + 1)$
 $= (3x + 2)(x + 1).$

3. $2x^2 - 7x + 3 = 2x^2 - 6x - x + 3$
 $= 2x(x - 3) - (x - 3)$
 $= (2x - 1)(x - 3).$
4. $x^2 + 6x + 9 = x^2 + 3x + 3x + 9$
 $= x(x + 3) + 3(x + 3)$
 $= (x + 3)(x + 3).$
5. $3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$
 $= 3x(x + 2) + 2(x + 2)$
 $= (3x + 2)(x + 2).$
6. $5x^2 - 8x + 3 = 5x^2 - 5x - 3x + 3$
 $= 5x(x - 1) - 3(x - 1)$
 $= (5x - 3)(x - 1).$
7. $9x^2 + 18x + 8 = 9x^2 + 12x + 6x + 8$
 $= 3x(3x + 4) + 2(3x + 4)$
 $= (3x + 2)(3x + 4).$
8. $7x^2 + 9ax + 2a^2 = 7x^2 + 7ax + 2ax + 2a^2$
 $= 7x(x + a) + 2a(x + a)$
 $= (7x + 2a)(x + a).$
9. $4x^2 + 16x + 15 = 4x^2 + 6x + 10x + 15$
 $= 2x(2x + 3) + 5(2x + 3)$
 $= (2x + 5)(2x + 3).$
10. $2x^2 - 3x - 2 = 2x^2 + x - 4x - 2$
 $= x(2x + 1) - 2(2x + 1)$
 $= (x - 2)(2x + 1).$
11. $2x^2 - x - 3 = 2x^2 + 2x - 3x - 3$
 $= 2x(x + 1) - 3(x + 1)$
 $= (2x - 3)(x + 1).$
12. $4x^2 - 7x - 15 = 4x^2 - 12x + 5x - 15$
 $= 4x(x - 3) + 5(x - 3)$
 $= (x - 3)(4x + 5).$
13. $3x^4 + 7x^2 - 6 = 3x^4 - 2x^2 + 9x^2 - 6$
 $= x^2(3x^2 - 2) + 3(3x^2 - 2)$
 $= (x^2 + 3)(3x^2 - 2).$
14. $3x^2 + 5xy - 2y^2 = 3x^2 + 6xy - xy - 2y^2$
 $= 3x(x + 2y) - y(x + 2y)$
 $= (3x - y)(x + 2y).$
15. $3x^2 - x - 2 = 3x^2 - 3x + 2x - 2$
 $= 3x(x - 1) + 2(x - 1)$
 $= (3x + 2)(x - 1).$
16. $2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$
 $= 2x(x - 3) + (x - 3)$
 $= (2x + 1)(x - 3).$

17. $2x^2 + xz - 3z^2 = 2x^2 + 3xz - 2xz - 3z^2$
 $= x(2x + 3z) - z(2x + 3z)$
 $= (x - z)(2x + 3z).$
18. $6x^2 - x - 2 = 6x^2 - 4x + 3x - 2$
 $= 2x(3x - 2) + (3x - 2).$
 $= (2x + 1)(3x - 2).$
19. $25x^2 - 20x + 3 = 25x^2 - 15x - 5x + 3$
 $= 5x(5x - 3) - (5x - 3)$
 $= (5x - 3)(5x - 1).$
20. $49x^2 - 21x + 2 = 49x^2 - 7x - 14x + 2$
 $= 7x(7x - 1) - 2(7x - 1)$
 $= (7x - 2)(7x - 1).$
21. $36x^2 - 36x + 5 = 36x^2 - 30x - 6x + 5$
 $= 6x(6x - 5) - (6x - 5)$
 $= (6x - 5)(6x - 1).$
22. $6x^4 + 13x^2 + 6 = 6x^4 + 9x^2 + 4x^2 + 6$
 $= 3x^2(2x^2 + 3) + 2(2x^2 + 3)$
 $= (3x^2 + 2)(2x^2 + 3).$
23. $9x^6 - 6x^3 - 8 = 9x^6 - 12x^3 + 6x^3 - 8$
 $= 3x^3(3x^3 - 4) + 2(3x^3 - 4)$
 $= (3x^3 + 2)(3x^3 - 4).$
24. $5x^2 + 2x - 16 = 5x^2 + 10x - 8x - 16$
 $= 5x(x + 2) - 8(x + 2)$
 $= (5x - 8)(x + 2).$
25. $3x^2 - 11x - 20 = 3x^2 - 15x + 4x - 20$
 $= 3x(x - 5) + 4(x - 5)$
 $= (3x + 4)(x - 5).$
26. $5x^4 + 18x^2y + 16y^2 = 5x^4 + 10x^2y + 8x^2y + 16y^2$
 $= 5x^2(x^2 + 2y) + 8y(x^2 + 2y)$
 $= (5x^2 + 8y)(x^2 + 2y).$
27. $6x^2 + 23x - 55 = 6x^2 - 10x + 33x - 55$
 $= 2x(3x - 5) + 11(3x - 5)$
 $= (2x + 11)(3x - 5).$
28. $6x^6 + 23x^3 + 21 = 6x^6 + 9x^3 + 14x^3 + 21$
 $= 3x^3(2x^3 + 3) + 7(2x^3 + 3)$
 $= (3x^3 + 7)(2x^3 + 3).$
29. $10x^2 - 7x - 12 = 10x^2 - 15x + 8x - 12$
 $= 5x(2x - 3) + 4(2x - 3)$
 $= (5x + 4)(2x - 3).$
30. $4c^2 + 11cg - 3g^2 = 4c^2 + 12cg - cg - 3g^2$
 $= 4c(c + 3g) - g(c + 3g)$
 $= (4c - g)(c + 3g).$

31. $8x^2 + 26xy + 15y^2 = 8x^2 + 6xy + 20xy + 15y^2$
 $= 2x(4x + 3y) + 5y(4x + 3y)$
 $= (2x + 5y)(4x + 3y).$
32. $35x^2 + 22x + 3 = 35x^2 + 15x + 7x + 3$
 $= 5x(7x + 3) + (7x + 3)$
 $= (7x + 3)(5x + 1).$
33. $14x^2 - x - 3 = 14x^2 - 7x + 6x - 3$
 $= 7x(2x - 1) + 3(2x - 1)$
 $= (7x + 3)(2x - 1).$
34. $21x^4 - x^2 - 2 = 21x^4 - 7x^2 + 6x^2 - 2$
 $= 7x^2(3x^2 - 1) + 2(3x^2 - 1)$
 $= (7x^2 + 2)(3x^2 - 1).$
35. $22x^2 - 3x - 7 = 22x^2 + 11x - 14x - 7$
 $= 11x(2x + 1) - 7(2x + 1)$
 $= (11x - 7)(2x + 1).$
36. $18x^2 + 65x + 7 = 18x^2 + 2x + 63x + 7$
 $= 2x(9x + 1) + 7(9x + 1)$
 $= (2x + 7)(9x + 1).$
37. $26x^2 + 9x - 2 = 26x^2 + 13x - 4x - 2$
 $= 13x(2x + 1) - 2(2x + 1)$
 $= (13x - 2)(2x + 1).$
38. $14x^2 - 39x + 10 = 14x^2 - 35x - 4x + 10$
 $= 7x(2x - 5) - 2(2x - 5)$
 $= (7x - 2)(2x - 5).$
39. $35x^2 - 39x - 36 = 35x^2 - 60x + 21x - 36$
 $= 5x(7x - 12) + 3(7x - 12)$
 $= (5x + 3)(7x - 12).$
40. $42x^2 - 9x - 6 = 3(14x^2 - 3x - 2)$
 $= 3(14x^2 - 7x + 4x - 2)$
 $= 3[7x(2x - 1) + 2(2x - 1)]$
 $= 3(2x - 1)(7x + 2).$

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1. $4ax^2 + 8ax + 3a = a(4x^2 + 8x + 3)$
 $= a(4x^2 + 6x + 2x + 3)$
 $= a[2x(2x + 3) + (2x + 3)]$
 $= a(2x + 1)(2x + 3).$
2. $2c^2x^2 + c^2x - 3c^2 = c^2(2x^2 + x - 3)$
 $= c^2(2x^2 + 3x - 2x - 3)$
 $= c^2[x(2x + 3) - (2x + 3)]$
 $= c^2(x - 1)(2x + 3).$

3. $48ax^4 - 120ax^2 + 75a = 3a(16x^4 - 40x^2 + 25)$
 $= 3a(4x^2 - 5)(4x^2 - 5).$
4. $18c^2d - 30cd - 28d = 2d(9c^2 - 15c - 14)$
 $= 2d(9c^2 - 21c + 6c - 14)$
 $= 2d[3c(3c - 7) + 2(3c - 7)]$
 $= 2d(3c + 2)(3c - 7).$
5. $x^4 + x^3 - 110x^2 = x^2(x^2 + x - 110) = x^2(x + 11)(x - 10).$
6. $75x^3 + 60x^2 + 12x = 3x(25x^2 + 20x + 4) = 3x(5x + 2)(5x + 2).$
7. $4x^2y^2 - 4xy^2 - 120y^2 = 4y^2(x^2 - x - 30) = 4y^2(x - 6)(x + 5).$
8. $70x^3 - 85x^2y - 30xy^2 = 5x(14x^2 - 17xy - 6y^2)$
 $= 5x(14x^2 - 21xy + 4xy - 6y^2)$
 $= 5x[7x(2x - 3y) + 2y(2x - 3y)]$
 $= 5x(7x + 2y)(2x - 3y).$
9. $18x^3 - 39x^2 + 18x = 3x(6x^2 - 13x + 6)$
 $= 3x(6x^2 - 9x - 4x + 6)$
 $= 3x[3x(2x - 3) - 2(2x - 3)]$
 $= 3x(3x - 2)(2x - 3).$
10. $36x^4 - 6x^3 - 12x^2 = 6x^2(6x^2 - x - 2)$
 $= 6x^2(6x^2 - 4x + 3x - 2)$
 $= 6x^2[2x(3x - 2) + (3x - 2)]$
 $= 6x^2(3x - 2)(2x + 1).$
11. $4x^4 - 20x^3 - 24x^2 = 4x^2(x^2 - 5x - 6) = 4x^2(x - 6)(x + 1).$
12. $5ax^2 + 5ax + 5a = 5a(x^2 + x + 1).$

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1. $k^2 - h^2 = (k + h)(k - h).$
2. $h^2 - 1 = (h + 1)(h - 1).$
3. $k^2 - 4 = (k + 2)(k - 2).$
4. $a^4 - x^2 = (a^2 + x)(a^2 - x).$
5. $1 - a^2 = (1 + a)(1 - a).$
6. $1 - 4a^2 = (1 + 2a)(1 - 2a).$
7. $4a^2 - 9 = (2a + 3)(2a - 3).$
8. $25 - x^2 = (5 - x)(5 + x).$
9. $h^4 - 81 = (h^2 + 9)(h^2 - 9)$
 $= (h^2 + 9)(h + 3)(h - 3).$
10. $16x^4 - 25 = (4x^2 + 5)(4x^2 - 5).$
11. $36c^4 - 49d^4 = (6c^4 + 7d^4)(6c^4 - 7d^4).$
12. $a^2b^4 - 121 = (ab^2 + 11)(ab^2 - 11).$
13. $100 - z^8 = (10 + z^4)(10 - z^4).$
14. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$
15. $64h^6 - 169 = (8h^3 + 13)(8h^3 - 13).$
16. $a^8 - b^8 = (a^4 + b^4)(a^4 - b^4)$
 $= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$
 $= (a^4 + b^4)(a^2 + b^2)(a + b)(a - b).$
17. $1 - 16x^4 = (1 + 4x^2)(1 - 4x^2) = (1 + 4x^2)(1 + 2x)(1 - 2x).$
18. $16x^4 - y^8 = (4x^2 + y^4)(4x^2 - y^4) = (4x^2 + y^4)(2x + y^2)(2x - y^2).$

19. $x^{16} - y^{16} = (x^8 + y^8)(x^8 - y^8)$
 $= (x^8 + y^8)(x^4 + y^4)(x^4 - y^4)$
 $= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$
 $= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y).$
20. $x^{16} - 4y^{16} = (x^8 + 2y^8)(x^8 - 2y^8).$
21. $144a^2b^4 - 9 = 9(4ab^2 + 1)(4ab^2 - 1).$
22. $196 - x^4y^6 = (14 + x^2y^3)(14 - x^2y^3).$
23. $225a^4 - 16b^8c^{12} = (15a^2 + 4b^4c^6)(15a^2 - 4b^4c^6).$
24. $x^{2n} - y^2 = (x^n + y)(x^n - y).$
25. $x^2 - y^{4n} = (x + y^{2n})(x - y^{2n}).$
26. $a^{4n} - b^2 = (a^{2n} + b)(a^{2n} - b).$
27. $a^{6n} - b^{2n} = (a^{3n} + b^n)(a^{3n} - b^n).$
28. $a^{4n} - b^{6n} = (a^{2n} + b^{3n})(a^{2n} - b^{3n}).$
29. $4x^{2n} - y^{4n} = (2x^n + y^{2n})(2x^n - y^{2n}).$
30. $9x^{2n} - 4y^{6n} = (3x^n + 2y^{3n})(3x^n - 2y^{3n}).$
31. $(a + b)^2 - 4 = (a + b + 2)(a + b - 2).$
32. $(x - y)^2 - 4c^2 = (x - y + 2c)(x - y - 2c).$
33. $(a - 2b)^2 - 9x^2 = (a - 2b + 3x)(a - 2b - 3x).$
34. $4(x + y)^2 - 1 = (2x + 2y + 1)(2x + 2y - 1).$
35. $25(3b - c)^2 - 64 = (15b - 5c + 8)(15b - 5c - 8).$
37. $81 - (x - y)^2 = [9 + (x - y)][9 - (x - y)]$
 $= (9 + x - y)(9 - x + y).$
38. $4a^2 - (2x + y)^2 = [2a + (2x + y)][2a - (2x + y)]$
 $= (2a + 2x + y)(2a - 2x - y).$

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1. $(a - b)^2 - 9x^2 = (a - b + 3x)(a - b - 3x).$
2. $16y^2 - (h + 2k)^2 = [4y + (h + 2k)][4y - (h + 2k)]$
 $= (4y + h + 2k)(4y - h - 2k).$
3. $25a^4 - (2b - 3c)^2 = [5a^2 + (2b - 3c)][5a^2 - (2b - 3c)]$
 $= (5a^2 + 2b - 3c)(5a^2 - 2b + 3c).$
4. $49x^6 - (7c - 2d)^2 = [7x^3 + (7c - 2d)][7x^3 - (7c - 2d)]$
 $= (7x^3 + 7c - 2d)(7x^3 - 7c + 2d).$
5. $1 - (5h - 3k)^2 = [1 + (5h - 3k)][1 - (5h - 3k)]$
 $= (1 + 5h - 3k)(1 - 5h + 3k).$
7. $(a - b)^2 - (a + b)^2 = [(a - b) + (a + b)][(a - b) - (a + b)]$
 $= (a - b + a + b)(a - b - a - b)$
 $= 2a(-2b)$
 $= -4ab.$
8. $(a - b)^2 - (x + y)^2 = [(a - b) + (x + y)][(a - b) - (x + y)]$
 $= (a - b + x + y)(a - b - x - y).$

9. $(a + 2b)^2 - (a - 3b)^2 = [(a + 2b) + (a - 3b)][(a + 2b) - (a - 3b)]$
 $= (a + 2b + a - 3b)(a + 2b - a + 3b)$
 $= (2a - b)5b.$
10. $(2a - 3b)^2 - (3a - 2b)^2$
 $= [(2a - 3b) + (3a - 2b)][(2a - 3b) - (3a - 2b)]$
 $= (2a - 3b + 3a - 2b)(2a - 3b - 3a + 2b)$
 $= (5a - 5b)(-a - b)$
 $= 5(b - a)(a + b).$
11. $(5h + 3k)^2 - (2c - 9d)^2$
 $= [(5h + 3k) + (2c - 9d)][(5h + 3k) - (2c - 9d)]$
 $= (5h + 3k + 2c - 9d)(5h + 3k - 2c + 9d).$
12. $(4b - a)^2 - (7a - 6b)^2$
 $= [(4b - a) + (7a - 6b)][(4b - a) - (7a - 6b)]$
 $= (4b - a + 7a - 6b)(4b - a - 7a + 6b)$
 $= (6a - 2b)(10b - 8a)$
 $= 4(3a - b)(5b - 4a).$
13. $4(a - 2b)^2 - (2x - y)^2$
 $= [2(a - 2b) + (2x - y)][2(a - 2b) - (2x - y)]$
 $= (2a - 4b + 2x - y)(2a - 4b - 2x + y).$
14. $9(a + b)^2 - (2a - 5b)^2$
 $= [3(a + b) + (2a - 5b)][3(a + b) - (2a - 5b)]$
 $= (3a + 3b + 2a - 5b)(3a + 3b - 2a + 5b)$
 $= (5a - 2b)(a + 8b).$
15. $(a - 2b)^2 - 4(a + b)^2$
 $= [(a - 2b) + 2(a + b)][(a - 2b) - 2(a + b)]$
 $= (a - 2b + 2a + 2b)(a - 2b - 2a - 2b)$
 $= 3a(-a - 4b)$
 $= -3a(a + 4b).$
17. $x^2 + 2ax + a^2 - y^2 = (x + a)^2 - y^2 = (x + a - y)(x + a + y).$
18. $x^2 + 2x + 1 - 4z^2 = (x + 1)^2 - 4z^2 = (x + 1 + 2z)(x + 1 - 2z).$
19. $c^2 - 2cd + d^2 - 16a^2 = (c - d)^2 - 16a^2 = (c - d + 4a)(c - d - 4a).$
20. $4x^2 - 12cx + 9c^2 - 25y^2 = (2x - 3c)^2 - 25y^2$
 $= (2x - 3c + 5y)(2x - 3c - 5y).$
21. $x^2 - y^2 - 2ax + a^2 = x^2 - 2ax + a^2 - y^2$
 $= (x - a)^2 - y^2$
 $= (x - a + y)(x - a - y).$
22. $k^2 - g^4 - 4kh + 4h^2 = k^2 - 4kh + 4h^2 - g^4$
 $= (k - 2h)^2 - g^4$
 $= (k - 2h + g^2)(k - 2h - g^2).$
23. $6xy - c^2 + 9x^2 + y^2 = 9x^2 + 6xy + y^2 - c^2$
 $= (3x + y)^2 - c^2$
 $= (3x + y - c)(3x + y + c).$

24. $12ab - 4h^2 + 4b^2 + 9a^2 = 9a^2 + 12ab + 4b^2 - 4h^2$
 $= (3a + 2b)^2 - 4h^2$
 $= (3a + 2b - 2h)(3a + 2b + 2h).$
25. $1 - 4ax + 4a^2x^2 - x^2 = (1 - 2ax)^2 - x^2$
 $= (1 - 2ax + x)(1 - 2ax - x).$
26. $4c^2 - 20cd + 25d^2 - 9d^4 = (2c - 5d)^2 - (3d^2)^2$
 $= (2c - 5d + 3d^2)(2c - 5d - 3d^2).$
27. $9d^2 - 25a^2 - 6cd + c^2 = 9d^2 - 6cd + c^2 - 25a^2$
 $= (3d - c)^2 - 25a^2$
 $= (3d - c + 5a)(3d - c - 5a).$
29. $a^2 - b^2 - 2bc - c^2 = a^2 - (b^2 + 2bc + c^2)$
 $= a^2 - (b + c)^2$
 $= (a + b + c)(a - b - c).$
30. $x^2 - a^2 - 2ac - c^2 = x^2 - (a^2 + 2ac + c^2)$
 $= x^2 - (a + c)^2$
 $= (x + a + c)(x - a - c).$
31. $y^2 - b^2 + 4bc - 4c^2 = y^2 - (b^2 - 4bc + 4c^2)$
 $= y^2 - (b - 2c)^2$
 $= (y - b + 2c)(y + b - 2c).$
32. $2bc - c^2 - b^2 + a^4 = a^4 - (b^2 - 2bc + c^2)$
 $= a^4 - (b - c)^2$
 $= (a^2 - b + c)(a^2 + b - c).$
33. $9x^2 - 4y^2 - a^2 - 4ay = 9x^2 - (4y^2 + 4ay + a^2)$
 $= 9x^2 - (2y + a)^2$
 $= (3x - 2y - a)(3x + 2y + a).$
34. $6x + 9y^2 - 9 - x^2 = 9y^2 - (x^2 - 6x + 9)$
 $= 9y^2 - (x - 3)^2$
 $= (3y - x + 3)(3y + x - 3).$
35. $4bc + 1 - 4c^2 - b^2 = 1 - (b^2 - 4bc + 4c^2)$
 $= 1 - (b - 2c)^2$
 $= (1 - b + 2c)(1 + b - 2c).$
36. $4bc - 4b^2 + 4x^6 - c^2 = 4x^6 - (4b^2 - 4bc + c^2)$
 $= 4x^6 - (2b - c)^2$
 $= (2x^3 - 2b + c)(2x^3 + 2b - c).$
38. $a^2 + 2ab + b^2 - c^2 - 2cd - d^2$
 $= (a^2 + 2ab + b^2) - (c^2 + 2cd + d^2)$
 $= (a + b)^2 - (c + d)^2$
 $= (a + b + c + d)(a + b - c - d).$
39. $9k^2 - 6hk + h^2 - 4c^2 - 4cd - d^2$
 $= (9k^2 - 6hk + h^2) - (4c^2 + 4cd + d^2)$
 $= (3k - h)^2 - (2c + d)^2$
 $= (3k - h - 2c - d)(3k - h + 2c + d).$

40. $x^2 - 1 + y^2 - a^2 + 2xy + 2a = (x^2 + 2xy + y^2) - (a^2 - 2a + 1)$
 $= (x + y)^2 - (a - 1)^2$
 $= (x + y - a + 1)(x + y + a - 1).$
41. $1 + 2bc + 2a - c^2 - b^2 + a^2 = (a^2 + 2a + 1) - (b^2 - 2bc + c^2)$
 $= (a + 1)^2 - (b - c)^2$
 $= (a + 1 + b - c)(a + 1 - b + c).$

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1. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1).$
2. $x^4 - 2x^2 + 1 = (x^2 - 1)^2 = (x + 1)(x + 1)(x - 1)(x - 1).$
3. $x^8 - 2x^4 + 1 = (x^4 - 1)^2$
 $= (x^2 + 1)(x^2 - 1)(x^2 + 1)(x^2 - 1)$
 $= (x^2 + 1)(x^2 + 1)(x + 1)(x + 1)(x - 1)(x - 1).$
4. $x^5 - 8x^3 + 16x = x(x^4 - 8x^2 + 16)$
 $= x(x^2 - 4)(x^2 - 4)$
 $= x(x + 2)(x - 2)(x + 2)(x - 2).$
5. $x^5 - x = x(x^4 - 1)$
 $= x(x^2 + 1)(x^2 - 1)$
 $= x(x^2 + 1)(x + 1)(x - 1).$
6. $x^{10} - x^2 = x^2(x^8 - 1)$
 $= x^2(x^4 + 1)(x^4 - 1)$
 $= x^2(x^4 + 1)(x^2 + 1)(x^2 - 1)$
 $= x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1).$
7. $x^4 - 10x^2 + 9 = (x^2 - 1)(x^2 - 9) = (x + 1)(x - 1)(x + 3)(x - 3).$
8. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x + 3)(x - 3)(x + 2)(x - 2).$
9. $3a^2x^4 - 12a^4x^2 + 12a^6 = 3a^2(x^4 - 4a^2x^2 + 4a^4)$
 $= 3a^2(x^2 - 2a^2)(x^2 - 2a^2).$
10. $18a^2x^2 - 24a^2x - 10a^2 = 2a^2(9x^2 - 12x - 5)$
 $= 2a^2(9x^2 - 15x + 3x - 5)$
 $= 2a^2[3x(3x - 5) + (3x - 5)]$
 $= 2a^2(3x + 1)(3x - 5).$
11. $3x^4 - 15x^2 + 12 = 3(x^4 - 5x^2 + 4)$
 $= 3(x^2 - 1)(x^2 - 4)$
 $= 3(x + 1)(x - 1)(x + 2)(x - 2).$
12. $12a - 39ay - 51ay^2 = 3a(4 - 13y - 17y^2)$
 $= 3a(4 - 17y + 4y - 17y^2)$
 $= 3a[(4 - 17y) + y(4 - 17y)]$
 $= 3a(4 - 17y)(1 + y).$
13. $4n^6 + 48n^2 - 28n^4 = 4n^2(n^4 - 7n^2 + 12)$
 $= 4n^2(n^2 - 3)(n^2 - 4)$
 $= 4n^2(n^2 - 3)(n + 2)(n - 2).$

14. $16x^4 + 8x^2 - 3 = 16x^4 + 12x^2 - 4x^2 - 3$
 $= 4x^2(4x^2 + 3) - (4x^2 + 3)$
 $= (4x^2 - 1)(4x^2 + 3)$
 $= (2x + 1)(2x - 1)(4x^2 + 3).$
15. $a^3 - a + a^2b - b = a(a^2 - 1) + b(a^2 - 1)$
 $= (a + b)(a^2 - 1)$
 $= (a + b)(a + 1)(a - 1).$
16. $x^3 - x^2 - 4x + 4 = x^2(x - 1) - 4(x - 1)$
 $= (x - 1)(x^2 - 4)$
 $= (x - 1)(x + 2)(x - 2).$
17. $3a^3 + 3a^2 - 27a - 27 = 3(a^3 + a^2 - 9a - 9)$
 $= 3[a^2(a + 1) - 9(a + 1)]$
 $= 3(a + 1)(a^2 - 9)$
 $= 3(a + 1)(a + 3)(a - 3).$
18. $2a^3b + 3a^2b - 8ab - 12b = b(2a^3 + 3a^2 - 8a - 12)$
 $= b[a^2(2a + 3) - 4(2a + 3)]$
 $= b(2a + 3)(a^2 - 4)$
 $= b(2a + 3)(a + 2)(a - 2).$
19. $4a^2 - a^4 + 81 + 10a^2x - 36a - 25x^2$
 $= (4a^2 - 36a + 81) - (a^4 - 10a^2x + 25x^2)$
 $= (2a - 9)^2 - (a^2 - 5x)^2$
 $= (2a - 9 - a^2 + 5x)(2a - 9 + a^2 - 5x).$
20. $12cd^3 - 6a^3x - a^6 + 4c^2 + 9d^6 - 9x^2$
 $= (4c^2 + 12cd^3 + 9d^6) - (a^6 + 6a^3x + 9x^2)$
 $= (2c + 3d^3)^2 - (a^3 + 3x)^2$
 $= (2c + 3d^3 - a^3 - 3x)(2c + 3d^3 + a^3 + 3x).$

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1. $x^3 + 1 = (x + 1)(x^2 - x + 1).$ 4. $27 + b^3 = (3 + b)(9 - 3b + b^2).$
2. $x^3 - 1 = (x - 1)(x^2 + x + 1).$ 5. $64x^3 - 1 = (4x - 1)(16x^2 + 4x + 1).$
3. $a^3 - 8 = (a - 2)(a^2 + 2a + 4).$ 6. $x^3 - y^6 = (x - y^2)(x^2 + xy^2 + y^4).$
7. $x^3 + 8y^3 = (x + 2y)(x^2 - 2xy + 4y^2).$
8. $x^3 + y^3 = (x + y^3)(x^2 - xy^3 + y^6).$
9. $8x^3 - 27y^3 = (2x - 3y)[(2x)^2 + (2x)(3y) + (3y)^2]$
 $= (2x - 3y)(4x^2 + 6xy + 9y^2).$
10. $27a^3 + 64b^3 = (3a + 4b)(9a^2 - 12ab + 16b^2).$
11. $125y^3 + 8x^3 = (5y + 2x)(25y^2 - 10xy + 4x^2).$
12. $216 - 27x^3 = 27(8 - x^3) = 27(2 - x)(4 + 2x + x^2).$
13. $x^6 + y^9 = (x^2 + y^3)(x^4 - x^2y^3 + y^6).$
14. $a^6 + b^6 = (a^2 + b^2)(a^4 - a^2b^2 + b^4).$
15. $a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$
 $= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2).$

16. $x^6 - 64 = (x^3 + 8)(x^3 - 8)$
 $= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4).$
17. $64x^6 + 1 = (4x^2 + 1)(16x^4 - 4x^2 + 1).$
18. $1 - 64x^6 = (1 - 8x^3)(1 + 8x^3)$
 $= (1 - 2x)(1 + 2x + 4x^2)(1 + 2x)(1 - 2x + 4x^2).$
19. $a^6b^6 - 64 = (a^3b^3 - 8)(a^3b^3 + 8)$
 $= (ab - 2)(a^2b^2 + 2ab + 4)(ab + 2)(a^2b^2 - 2ab + 4).$
20. $x^{12} + y^{12} = (x^4)^3 + (y^4)^3 = (x^4 + y^4)(x^8 - x^4y^4 + y^8).$
21. $27x^{27} - 1 = (3x^9)^3 - 1^3 = (3x^9 - 1)(9x^{18} + 3x^9 + 1).$
22. $k^{12} - y^6 = (k^6 + y^3)(k^6 - y^3)$
 $= [(k^2)^3 + y^3][(k^2)^3 - y^3]$
 $= (k^2 + y)(k^4 - k^2y + y^2)(k^2 - y)(k^4 + k^2y + y^2).$
23. $x^{3a} + 1 = (x^{2a} - x^a + 1)(x^a + 1).$
24. $x^{3a} - y^{3a} = (x^a - y^a)(x^{2a} + x^ay^a + y^{2a}).$
25. $x^{3a} + y^{6a} = (x^a + y^{2a})(x^{2a} - x^ay^{2a} + y^{4a}).$
26. $8x^{6a} - y^{9a} = (2x^{2a})^3 - (y^3)^3 = (2x^{2a} - y^3)(4x^{4a} + 2x^{2a}y^3 + y^{6a}).$
27. $27x^{3m} - 64 = (3x^m - 4)(9x^{2m} + 12x^m + 16).$

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1. $x^6 + 2x^3 + 1 = (x^3 + 1)(x^3 + 1)$
 $= (x + 1)(x^2 - x + 1)(x + 1)(x^2 - x + 1).$
2. $x^6 - 2x^3y^3 + y^6 = (x^3 - y^3)(x^3 - y^3)$
 $= (x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2).$
3. $x^4 + x = x(x^3 + 1) = x(x + 1)(x^2 - x + 1).$
4. $x^5 - x^2 = x^2(x^3 - 1) = x^2(x - 1)(x^2 + x + 1).$
5. $8x^3 - 64 = 8(x^3 - 8) = 8(x - 2)(x^2 + 2x + 4).$
6. $8x^6 + 216x^3 = 8x^3(x^3 + 27) = 8x^3(x + 3)(x^2 - 3x + 9).$
7. $x^5 - x^3y^2 - x^2y^3 + y^5 = x^3(x^2 - y^2) - y^3(x^2 - y^2)$
 $= (x^3 - y^3)(x^2 - y^2)$
 $= (x - y)(x^2 + xy + y^2)(x + y)(x - y).$
8. $x^5 + x^3 + 8x^2 + 8 = x^3(x^2 + 1) + 8(x^2 + 1)$
 $= (x^3 + 8)(x^2 + 1)$
 $= (x + 2)(x^2 - 2x + 4)(x^2 + 1).$
9. $x^7 - x^4 - 16x^3 + 16 = x^4(x^3 - 1) - 16(x^3 - 1)$
 $= (x^4 - 16)(x^3 - 1)$
 $= (x^2 + 4)(x^2 - 4)(x^3 - 1)$
 $= (x^2 + 4)(x + 2)(x - 2)(x - 1)(x^2 + x + 1).$
10. $x^6 - 7x^3 - 8 = (x^3 + 1)(x^3 - 8)$
 $= (x + 1)(x^2 - x + 1)(x - 2)(x^2 + 2x + 4).$
11. $8x^6 - 3 - 23x^3 = 8x^3 - 24x^3 + x^3 - 3$
 $= (x^3 - 3)(2x + 1)(4x^2 - 2x + 1).$

12. $x^4 + x^3 - x - 1 = x^3(x + 1) - (x + 1) = (x + 1)(x - 1)(x^2 + x + 1)$.
13. $x^6 - x^2y^2 = x^2(x^2 + y)(x^2 - y)$.
14. $x^8 + 64x^2 = x^2(x^6 + 64)$
 $= x^2[(x^2)^3 + 4^3]$
 $= x^2(x^2 + 4)(x^4 - 4x^2 + 16)$.
16. $1 - 3x + 3x^2 - x^3 = (1 - x^3) - (3x - 3x^2)$
 $= (1 - x)(1 + x + x^2) - 3x(1 - x)$
 $= (1 - x)(1 + x + x^2 - 3x)$
 $= (1 - x)(1 - 2x + x^2)$
 $= (1 - x)(1 - x)(1 - x)$.
17. $x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + y^3) + (3x^2y + 3xy^2)$
 $= (x + y)(x^2 - xy + y^2) + 3xy(x + y)$
 $= (x + y)(x^2 - xy + y^2 + 3xy)$
 $= (x + y)(x^2 + 2xy + y^2)$
 $= (x + y)(x + y)(x + y)$.
18. $x^3 - 3x^2y + 3xy^2 - y^3 = (x^3 - y^3) - (3x^2y - 3xy^2)$
 $= (x - y)(x^2 + xy + y^2) - 3xy(x - y)$
 $= (x - y)(x^2 + xy + y^2 - 3xy)$
 $= (x - y)(x^2 - 2xy + y^2)$
 $= (x - y)(x - y)(x - y)$.
19. $x^3 + 2x^2 + 4x + 8 = x^2(x + 2) + 4(x + 2)$
 $= (x + 2)(x^2 + 4)$.
20. $8 - 4x + 2x^2 - x^3 = 4(2 - x) + x^2(2 - x)$
 $= (2 - x)(4 + x^2)$.
21. $x^3 + 9x^2 + 27x + 27 = x^3 + 27 + 9x^2 + 27x$
 $= (x + 3)(x^2 - 3x + 9) + 9x(x + 3)$
 $= (x + 3)(x^2 - 3x + 9 + 9x)$
 $= (x + 3)(x + 3)(x + 3)$.
22. $x^3 - 9x^2 + 27x - 27 = x^3 - 27 - 9x^2 + 27x$
 $= (x - 3)(x^2 + 3x + 9) - 9x(x - 3)$
 $= (x - 3)(x^2 + 3x + 9 - 9x)$
 $= (x - 3)(x - 3)(x - 3)$.
23. $x^3 + 12x^2 + 48x + 64 = (x^3 + 64) + (12x^2 + 48x)$
 $= (x + 4)(x^2 - 4x + 16) + 12x(x + 4)$
 $= (x + 4)(x^2 - 4x + 16 + 12x)$
 $= (x + 4)(x^2 + 8x + 16)$
 $= (x + 4)(x + 4)(x + 4)$.
24. $64 - 48x + 12x^2 - x^3 = (64 - x^3) - (48x - 12x^2)$
 $= (4 - x)(16 + 4x + x^2) - 12x(4 - x)$
 $= (4 - x)(16 + 4x + x^2 - 12x)$
 $= (4 - x)(16 - 8x + x^2)$
 $= (4 - x)(4 - x)(4 - x)$.

25. $8x^3 - 12x^2 + 6x - 1 = (8x^3 - 1) - (12x^2 - 6x)$
 $= (2x - 1)(4x^2 + 2x + 1) - 6x(2x - 1)$
 $= (2x - 1)^3.$
26. $x^4 + x^3 + x^2 - 1 = x^3(x + 1) + (x - 1)(x + 1)$
 $= (x + 1)(x^3 + x - 1).$
27. $x^3 + a^3 + x^2 - a^2 = (x + a)(x^2 - ax + a^2) + (x + a)(x - a)$
 $= (x + a)(x^2 - ax + a^2 + x - a).$

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1. $x^4 + 16x^2y^2 + 64y^4 = (x^2 + 8y^2)(x^2 + 8y^2):$
2. $4a^2x^4 + 4a^2x^2 - 120a^2 = 4a^2(x^4 + x^2 - 30) = 4a^2(x^2 + 6)(x^2 - 5).$
3. $9x^2 + 27x + 14 = 9x^2 + 6x + 21x + 14$
 $= 3x(3x + 2) + 7(3x + 2)$
 $= (3x + 7)(3x + 2).$
4. $3x^4 + x^2 - 2 = 3x^4 + 3x^2 - 2x^2 - 2$
 $= 3x^2(x^2 + 1) - 2(x^2 + 1)$
 $= (3x^2 - 2)(x^2 + 1).$
5. $3x^3 - 75x = 3x(x^2 - 25) = 3x(x + 5)(x - 5).$
6. $x^2 + 2a^2x^2 + x^2a^4 = x^2(1 + 2a^2 + a^4) = x^2(1 + a^2)(1 + a^2).$
7. $x^3 - 27 = (x - 3)(x^2 + 3x + 9).$
8. $x^2 - xz + 2xy - 2yz = x(x - z) + 2y(x - z)$
 $= (x + 2y)(x - z).$
9. $2ac - bc + 6ad - 3bd = c(2a - b) + 3d(2a - b)$
 $= (c + 3d)(2a - b).$
10. $25x^2y^2 + 16x^4 - 40x^3y = x^2(25y^2 - 40xy + 16x^2)$
 $= x^2(5y - 4x)(5y - 4x).$
11. $x^4 - 15x^3 + 56x^2 = x^2(x^2 - 15x + 56) = x^2(x - 8)(x - 7).$
12. $9x^6 - 6x^3 - 35 = 9x^6 - 21x^3 + 15x^3 - 35$
 $= 3x^3(3x^3 - 7) + 5(3x^3 - 7)$
 $= (3x^3 + 5)(3x^3 - 7).$
13. $a^6 - 4096 = (a^3 + 64)(a^3 - 64)$
 $= (a + 4)(a^2 - 4a + 16)(a - 4)(a^2 + 4a + 16)$
14. $a^{2x+2} - 1 = (a^{x+1} + 1)(a^{x+1} - 1).$
15. $4x^4 - 13x^2y^2 + 9y^4 = 4x^4 - 4x^2y^2 - 9x^2y^2 + 9y^4$
 $= 4x^2(x^2 - y^2) - 9y^2(x^2 - y^2)$
 $= (4x^2 - 9y^2)(x^2 - y^2)$
 $= (2x + 3y)(2x - 3y)(x + y)(x - y).$
16. $216 + x^3 = 6^3 + x^3 = (6 + x)(36 - 6x + x^2).$
17. $x^9 - 512 = (x^3)^3 - 8^3$
 $= (x^3 - 8)[(x^3)^2 + (x^3)8 + 8^2]$
 $= (x - 2)(x^2 + 2x + 4)(x^6 + 8x^3 + 64).$

18. $x^2 - 4y^2 + 9 - 6x = x^2 - 6x + 9 - 4y^2$
 $= (x - 3)^2 - 4y^2$
 $= (x - 3 + 2y)(x - 3 - 2y).$
19. $ah - 2ak + 2bh - 4bk = a(h - 2k) + 2b(h - 2k)$
 $= (h - 2k)(a + 2b).$
20. $4x^4 - 25y^8 = (2x^2 + 5y^4)(2x^2 - 5y^4).$
21. $a^2 - 25c^2 + 10ab + 25b^2 = (a^2 + 10ab + 25b^2) - 25c^2$
 $= (a + 5b)^2 - 25c^2$
 $= (a + 5b + 5c)(a + 5b - 5c).$
22. $y^6 - 343 = (y^2)^3 - 7^3 = (y^2 - 7)(y^4 + 7y^2 + 49).$
23. $9x^4 - 9x^2 - 28 = 9x^4 - 21x^2 + 12x^2 - 28$
 $= 3x^2(3x^2 - 7) + 4(3x^2 - 7)$
 $= (3x^2 + 4)(3x^2 - 7).$
24. $9x^2y^2 + 39xy^2 - 30y^2 = 3y^2(3x^2 + 13x - 10)$
 $= 3y^2(3x^2 + 15x - 2x - 10)$
 $= 3y^2[3x(x + 5) - 2(x + 5)]$
 $= 3y^2(3x - 2)(x + 5).$
25. $3x^6y^2 + 63x^3y^2 - 300y^2 = 3y^2(x^6 + 21x^3 - 100)$
 $= 3y^2(x^3 + 25)(x^3 - 4).$
26. $12 - 15a + 16x - 20ax = 3(4 - 5a) + 4x(4 - 5a)$
 $= (4 - 5a)(3 + 4x).$
27. $2cx + 3dx - 2cy - 3dy = x(2c + 3d) - y(2c + 3d)$
 $= (x - y)(2c + 3d).$
28. $y^4 - 2y^2z^4 + z^8 = (y^2 - z^4)(y^2 - z^4) = (y + z^2)(y - z^2)(y + z^2)(y - z^2).$
29. $81c^{10} - 64d^{10} = (9c^5 - 8d^5)(9c^5 + 8d^5).$
30. $100x^8 - 220x^4y^2 + 121y^4 = (10x^4 - 11y^2)(10x^4 - 11y^2).$
31. $27y^3 - 512 = (3y)^3 - 8^3 = (3y - 8)(9y^2 + 24y + 64).$
32. $81 - 16x^8 = (9 + 4x^4)(9 - 4x^4) = (9 + 4x^4)(3 + 2x^2)(3 - 2x^2).$
33. $25x^4 - 3025x^8 = 25x^4(1 - 121x^4) = 25x^4(1 + 11x^2)(1 - 11x^2).$
34. $16c^7 - 4c^5 - 72c^3 = 4c^3(4c^4 - c^2 - 18)$
 $= 4c^3(4c^4 + 8c^2 - 9c^2 - 18)$
 $= 4c^3[4c^2(c^2 + 2) - 9(c^2 + 2)]$
 $= 4c^3(c^2 + 2)(4c^2 - 9)$
 $= 4c^3(c^2 + 2)(2c + 3)(2c - 3).$
35. $c^3 + c^2d + 3cd^2 + 3d^3 = c^2(c + d) + 3d^2(c + d) = (c^2 + 3d^2)(c + d).$
36. $1 - 18x^4 + 81x^8 = (1 - 9x^4)(1 - 9x^4)$
 $= (1 + 3x^2)(1 - 3x^2)(1 + 3x^2)(1 - 3x^2).$
37. $x^4 + 12x^2 - 64 = (x^2 + 16)(x^2 - 4) = (x^2 + 16)(x + 2)(x - 2).$
38. $c^9 - 512c^{12} = c^9(1 - 512c^3)$
 $= c^9[1 - (8c)^3]$
 $= c^9(1 - 8c)(1 + 8c + 64c^2).$
39. $h^6a - k^8a = (h^3a + k^4a)(h^3a - k^4a).$

40. $a^2 - b^4 - 12ab + 36b^2 = a^2 - 12ab + 36b^2 - b^4$
 $= (a - 6b)^2 - b^4$
 $= (a - 6b - b^2)(a - 6b + b^2).$
41. $5h^2 + 21hk - 20k^2 = 5h^2 - 4hk + 25hk - 20k^2$
 $= h(5h - 4k) + 5k(5h - 4k)$
 $= (5h - 4k)(h + 5k).$
42. $x^6 - x^3 - 2 = (x^3 - 2)(x^3 + 1) = (x^3 - 2)(x + 1)(x^2 - x + 1).$
43. $2x^2y^2 - y^4 - x^4 = -1(x^4 - 2x^2y^2 + y^4)$
 $= -1(x^2 - y^2)(x^2 - y^2)$
 $= (y^2 - x^2)(x^2 - y^2)$
 $= (y + x)(y - x)(x + y)(x - y).$
44. $5h^3 - 9h^2k - 9k^3 + 5h^2k = (5h^3 + 5h^2k) - (9h^2k + 9k^3)$
 $= 5h(h^2 + k^2) - 9k(h^2 + k^2)$
 $= (h^2 + k^2)(5h - 9k).$
45. $3x^2 - 17x + 10 = 3x^2 - 15x - 2x + 10$
 $= 3x(x - 5) - 2(x - 5)$
 $= (x - 5)(3x - 2).$
46. $9x^{5n} - x^{3n} = x^{3n}(9x^{2n} - 1) = x^{3n}(3x^n + 1)(3x^n - 1).$
47. $27a - 18ab^2 - 3a^9 + 3ab^4 = 3a(9 - 6b^2 - a^8 + b^4)$
 $= 3a[(9 - 6b^2 + b^4) - a^8]$
 $= 3a[(3 - b^2)^2 - a^8]$
 $= 3a(3 - b^2 - a^4)(3 - b^2 + a^4).$
48. $1 + 2x^2 - 3x^4 = -1(3x^4 - 2x^2 - 1)$
 $= -1(3x^4 - 3x^2 + x^2 - 1)$
 $= -1[3x^2(x^2 - 1) + (x^2 - 1)]$
 $= -1(x^2 - 1)(3x^2 + 1)$
 $= -1(x + 1)(x - 1)(3x^2 + 1)$
 $= (x + 1)(1 - x)(3x^2 + 1).$
49. $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3) = (x + 2)(x - 2)(x^2 + 3).$
50. $16x^4 + 8x^2 - 3 = 16x^4 - 4x^2 + 12x^2 - 3$
 $= 4x^2(4x^2 - 1) + 3(4x^2 - 1)$
 $= (4x^2 - 1)(4x^2 + 3)$
 $= (2x + 1)(2x - 1)(4x^2 + 3).$
51. $1024 - 64h^3 + h^6 = (32 - h^3)(32 - h^3).$
52. $h^2k^2 - h^2 - k^2 + 1 = h^2(k^2 - 1) - (k^2 - 1)$
 $= (k^2 - 1)(h^2 - 1)$
 $= (k + 1)(k - 1)(h + 1)(h - 1).$
53. $12 - 2h - 4h^2 = -2(2h^2 + h - 6)$
 $= -2(2h^2 - 3h + 4h - 6)$
 $= -2[h(2h - 3) + 2(2h - 3)]$
 $= -2(2h - 3)(h + 2)$
 $= 2(3 - 2h)(h + 2).$

$$\begin{aligned}
 54. \quad 3 - y^2 + 3y^3 - y^5 &= (3 - y^2) + y^3(3 - y^2) \\
 &= (3 - y^2)(1 + y^3) \\
 &= (3 - y^2)(1 + y)(1 - y + y^2).
 \end{aligned}$$

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- | | |
|---|--|
| <p>1. $x^2 - 9 = 0.$
Factoring the left member,
$(x + 3)(x - 3) = 0.$
Whence $x = -3$ or $+3.$</p> <p>2. $x^2 = 16.$
Then
$(x + 4)(x - 4) = 0.$
Whence $x = -4$ or $+4.$</p> <p>3. $x^2 - 3x = 0.$
Then $x(x - 3) = 0.$
Whence $x = 0$ or $3.$</p> <p>4. $x^2 = 7x.$
Then $x(x - 7) = 0.$
Whence $x = 0$ or $7.$</p> <p>5. $x^2 - 7x = -12.$
Then
$(x - 3)(x - 4) = 0.$
Whence $x = 3$ or $4.$</p> <p>6. $x^2 - x = 20.$
Then
$(x - 5)(x + 4) = 0.$
Whence $x = 5$ or $-4.$</p> <p>7. $3x^2 - 15x = 0.$
Then $3x(x - 5) = 0.$
Whence $x = 0$ or $5.$</p> <p>8. $5x^2 + 35x = 0.$
Then $5x(x + 7) = 0.$
Whence $x = 0$ or $-7.$</p> <p>9. $8 - 9x = -x^2.$
Then
$(x - 8)(x - 1) = 0.$
Whence $x = 8$ or $1.$</p> | <p>10. $12x - 28 = -x^2.$
Then
$(x + 14)(x - 2) = 0.$
Whence $x = -14$ or $2.$</p> <p>11. $4x^2 = 16x.$
Then $4x(x - 4) = 0.$
Whence $x = 0$ or $4.$</p> <p>12. $x^2 - a^2 = 0.$
Then
$(x + a)(x - a) = 0.$
Whence $x = -a$ or $a.$</p> <p>13. $x^2 = 9b^2.$
Then
$(x + 3b)(x - 3b) = 0.$
Whence $x = -3b$ or $3b.$</p> <p>14. $x^2 - 2ax + a^2 = 0.$
Then
$(x - a)(x - a) = 0.$
Whence $x = a.$</p> <p>15. $x^2 + 4b^2 = 4bx.$
Then
$(x - 2b)(x - 2b) = 0.$
Whence $x = 2b.$</p> <p>16. $x^2 + ax + 3x + 3a = 0.$
Then
$(x + a)(x + 3) = 0.$
Whence $x = -a$ or $-3.$</p> <p>17. $x^2 + bx = 4x + 4b.$
Then
$(x - 4)(x + b) = 0.$
Whence $x = 4$ or $-b.$</p> |
|---|--|

18. $x^2 + ax + bx + ab = 0.$

Then

$$(x + a)(x + b) = 0.$$

Whence $x = -a$ or $-b.$

19. $4x^2 + 8x + 3 = 0.$

Then

$$(2x + 1)(2x + 3) = 0.$$

Whence $x = -\frac{1}{2}$ or $-\frac{3}{2}.$

20. $9x^2 = 3x + 2.$

Then

$$(3x - 2)(3x + 1) = 0.$$

Whence $x = \frac{2}{3}$ or $-\frac{1}{3}.$

21. $16x^2 - 12x = 10.$

Then

$$(4x - 5)(2x + 1) = 0.$$

Whence $x = \frac{5}{4}$ or $-\frac{1}{2}.$

22. $50x + 24 = 25x^2.$

Then

$$(5x - 12)(5x + 2) = 0.$$

Whence $x = \frac{12}{5}$ or $-\frac{2}{5}.$

28. $x^3 + x^2 = 4x + 4.$

Then $(x + 2)(x - 2)(x + 1) = 0.$

Whence $x = -2, 2,$ or $-1.$

29. $x^3 - 5x^2 + 6x = 0.$

Then $x(x - 3)(x - 2) = 0.$

Whence $x = 0, 3,$ or $2.$

30. $2x^3 - x^2 = 32x - 16.$

Then $(x - 4)(x + 4)(2x - 1) = 0.$

Whence $x = 4, -4,$ or $\frac{1}{2}.$

31. $5x^2 + x^3 = 45 + 9x.$

Then $(x + 3)(x - 3)(x + 5) = 0.$

Whence $x = -3, 3,$ or $-5.$

32. $x^4 - 5x^2 + 4 = 0.$

Then $(x + 2)(x - 2)(x + 1)(x - 1) = 0.$

Whence $x = -2, 2, -1,$ or $1.$

33. $9 + x^4 = 10x^2.$

Then $(x + 3)(x - 3)(x + 1)(x - 1) = 0.$

Whence $x = -3, 3, -1,$ or $1.$

23. $x^2 + bx = 0.$

Then $x(x + b) = 0.$

Whence $x = 0$ or $-b.$

24. $x^2 - ax - bx = 0.$

Then

$$x(x - a - b) = 0.$$

Whence $x = 0$ or $a + b.$

25. $x^2 + 3x = ax.$

Then

$$x(x + 3 - a) = 0.$$

Whence $x = 0$ or $a - 3.$

26. $4x = x^2 + 4.$

Then

$$(x - 2)(x - 2) = 0.$$

Whence $x = 2.$

27. $x^3 - 9x = 0.$

Then

$$x(x + 3)(x - 3) = 0.$$

Whence $x = 0, -3,$ or $3.$

34. $x^4 - 36x^2 = 0.$

Then

$x^2(x + 6)(x - 6) = 0.$

Whence $x = 0, -6, \text{ or } 6.$

35. $x^4 + 15x^2 = 8x^3.$

Then

$x^2(x - 5)(x - 3) = 0.$

Whence $x = 0, 5, \text{ or } 3.$

36. $x^4 - 2x^2 + 1 = 0.$

Then $(x + 1)^2(x - 1)^2 = 0.$

Whence $x = -1 \text{ or } 1.$

37. It does not follow from $(x + 1)(x - 1) = x - 1$ that $x + 1 = 1$, for by the condition of the problem $x - 1 = 0$, and it is not permissible to divide each member of an equation by zero.

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1. Let $n = \text{the number.}$

Then $n^2 + n = 90,$

or $(n - 9)(n + 10) = 0.$

Whence $n = 9 \text{ or } -10.$

2. Let $n = \text{the number.}$

Then $n^2 - 2n = 35,$

or $(n - 7)(n + 5) = 0.$

Whence $n = 7 \text{ or } -5.$

3. Let $n = \text{the number.}$

Then $n^2 + 2n + 5 = 148,$

or $(n + 13)(n - 11) = 0.$

Whence $n = -13 \text{ or } 11.$

4. Let $n = \text{the number.}$

Then $4n^2 = 7n,$

or $n(n - \frac{7}{4}) = 0.$

Whence $n = 0 \text{ or } \frac{7}{4}.$

5. Let $n = \text{the number.}$

Then

$(n + 20)(n + 21) = 930,$

or $(n - 10)(n + 51) = 0.$

Whence $n = 10 \text{ or } -51.$

6. Let $n = \text{the number}$

Then

$(17 - n)(23 - n) = 216,$

or $(n - 35)(n - 5) = 0.$

Whence $n = 35 \text{ or } 5.$

7. Let $n = \text{the number.}$

Then

$(27 - n)(n + 21) = 540,$

or $(n - 9)(n + 3) = 0.$

Whence $n = 9 \text{ or } -3.$

8. Let $n = \text{the number.}$

Then $(n + 15)(22 - n) = 70 + 23n,$

or $(n - 10)(n + 26) = 0.$

Whence $n = 10 \text{ or } -26.$

9. Let $n = \text{the one number.}$

Then $n + 6 = \text{the other.}$

Therefore $(n + 6)^2 - n^2 = 120.$

Whence $n = 7,$

and $n + 6 = 13.$

Also $n^2 - (n + 6)^2 = 120.$

Whence $n = -13,$

and $n + 6 = -7.$

The negative numbers are rejected.

10. Let $n =$ the number.

Then $9n^2 - 5n = 8n^2,$

or $n(n - 5) = 0.$

Whence $n = 0$ or $5.$

11. Let $f =$ the frontage in feet.

Then $4f =$ the depth in feet.

Therefore $4f^2 = 2500,$

or $(f - 25)(f + 25) = 0.$

Whence $f = 25$ or $-25,$

and $4f = 100$ or $-100.$

The negative lengths, being meaningless, are rejected.

12. Let $b =$ the breadth in feet.

Then $b + 6 =$ the length in feet.

Therefore $b(b + 6) = 216,$

or $(b + 18)(b - 12) = 0.$

Whence $b = -18$ or $12,$

and $b + 6 = -12$ or $18.$

The negative values are rejected.

13. Let $w =$ the width in rods.

Then $w + 6 =$ the length in rods.

Therefore $w(w + 6) = 24 \cdot 9,$

or $(w + 18)(w - 12) = 0.$

Whence $w = -18$ or $12,$

and $w + 6 = -12$ or $18.$

The negative values are rejected.

14. Let $n =$ the first number.

Then $n + 1 =$ the second.

Hence $n^2 + (n + 1)^2 = 145,$

or $(n + 9)(n - 8) = 0.$

Whence $n = -9$ or $8,$

and $n + 1 = -8$ or $9.$

Therefore the numbers are -9 and $-8,$ or 8 and $9.$

15. Let $n =$ the first odd number.

Then $n + 2 =$ the second.

Hence $n^2 + (n + 2)^2 = 290,$

or $(n + 13)(n - 11) = 0.$

Whence $n = -13$ or $11,$

and $n + 2 = -11$ or $13.$

Therefore the numbers are -13 and $-11,$ or 11 and $13.$

16. Let n , $n + 2$, and $n + 4$ represent the odd numbers.

Then $n^2 + (n + 2)^2 + (n + 4)^2 = 251,$

or $(n + 11)(n - 7) = 0.$

Whence $n = -11$ or $7.$

Therefore the numbers are -11 , -9 , and -7 ; or 7 , 9 , and $11.$

17. Let $n =$ the side of the square in inches.

Then $n^2 + 4 \cdot 8 \cdot n = 185,$

or $(n + 37)(n - 5) = 0.$

Whence $n = -37$ or $5.$

Therefore n is $5.$

18. Let $x =$ the edge of the cube in inches.

Then $6x^2 = 294,$

or $(x - 7)(x + 7) = 0.$

Whence $x = 7$ or $-7.$

Therefore the edge of the cube is 7 inches.

19. Let $d =$ the depth in feet,

$4d =$ the length in feet,

and $3d =$ the width in feet.

Then $2 \cdot d \cdot 4d + 2 \cdot d \cdot 3d + 2 \cdot 4d \cdot 3d = 608,$

or $(d - 4)(d + 4) = 0.$

Whence $d = 4$ or $-4.$

Therefore the depth is 4 feet, the length is 16 feet, and the width is 12 feet.

20. Let $x =$ the depth in inches.

Then $x + 3 =$ the length in inches,

and $x + 1 =$ the width in inches.

$2x(x + 3) + 2x(x + 1) + 2(x + 3)(x + 1) = 62,$

or $(3x + 14)(x - 2) = 0.$

Whence $x = -4\frac{2}{3}$ or $2.$

Therefore the depth is 2 inches, the length is 5 inches, and the width is 3 inches.

21. Let $b =$ the base of the triangle in feet.

Then $\frac{6 \cdot b}{2} = 30.$

Whence $b = 10.$

22. Let $b =$ the base of the triangle in feet.

Then $3b =$ the altitude in feet.

Therefore $\frac{3b \cdot b}{2} = 54,$

or $(b - 6)(b + 6) = 0.$

Whence $b = 6$ or $-6.$

Hence the base is 6 feet and the altitude is 18 feet.

23. Let $a =$ the altitude in feet.

Then $5a =$ the base in feet.

Therefore
$$\frac{5a \cdot a}{2} = 40,$$

or $(a - 4)(a + 4) = 0.$

Whence $a = 4$ or $-4.$

Hence the altitude is 4 feet and the base is 20 feet.

24. Let $a =$ the altitude in meters.

Then $6a =$ the base in meters.

Therefore
$$\frac{6a \cdot a}{2} = 75,$$

or $(a - 5)(a + 5) = 0.$

Whence $a = 5$ or $-5.$

Hence the altitude is 5 meters and the base is 30 meters.

25. If $x =$ the base in feet,

then $x + 2 =$ the altitude in feet.

Therefore
$$\frac{x(x + 2)}{2} = 24,$$

or $(x - 6)(x + 8) = 0.$

Whence $x = 6$ or $-8.$

Therefore the base is 6 feet and the altitude is 8 feet.

26. Let $b =$ the base in feet.

Then $b + 3 =$ the altitude in feet.

Therefore
$$\frac{b(b + 3)}{2} = 54,$$

or $(b - 9)(b + 12) = 0.$

Whence $b = 9$ or $-12.$

Therefore the base is 9 feet and the altitude is 12 feet.

27. If x and $x + 2$ represent the legs in feet,

then
$$\frac{x(x + 2)}{2} = 24,$$

or $(x - 6)(x + 8) = 0.$

Whence $x = 6$ or $-8.$

Therefore the legs are 6 feet and 8 feet.

28. If x and $x + 7$ represent the legs in feet,

then
$$\frac{x(x + 7)}{2} = 30,$$

or
$$(x - 5)(x + 12) = 0.$$

Whence
$$x = 5 \text{ or } -12.$$

Therefore the legs are 5 feet and 12 feet.

29. Let a = the altitude in inches.

Then $6 + 2a$ = the base in inches.

Therefore
$$\frac{a(6 + 2a)}{2} = 2\frac{5}{8} \cdot 144,$$

or
$$(a - 18)(a + 21) = 0.$$

Whence
$$a = 18 \text{ or } -21.$$

Therefore the altitude is 18 inches and the base is 42 inches.

30. Let b = the base in feet.

Then $6 + 3b$ = the altitude in feet.

Therefore
$$\frac{b(6 + 3b)}{2} = 4 \cdot 9,$$

or
$$(b - 4)(b + 6) = 0.$$

Whence
$$b = 4 \text{ or } -6.$$

Therefore the base is 4 feet and the altitude is 18 feet.

31. Let b = the base in centimeters.

Then $b - 5$ = the altitude in centimeters.

Therefore
$$\frac{b(b - 5)}{2} = 150,$$

or
$$(b + 15)(b - 20) = 0.$$

Whence
$$b = -15 \text{ or } 20.$$

Therefore the base is 20 centimeters and the altitude is 15 centimeters.

32.
$$\frac{12(10 + 18)}{2} = 168, \text{ area.}$$

33. If x and $x + 4$ represent the bases in inches,

then
$$8x + 16 = 96.$$

Whence
$$x = 10,$$

and
$$x + 4 = 14.$$

34. If 12 = one base in feet,

and a = the altitude in feet,

then $2a$ = the other base in feet.

Therefore
$$\frac{a(2a + 12)}{2} = 112,$$

or $(a - 8)(a + 14) = 0.$

Whence $a = 8$ or $-14.$

Therefore the altitude is 8 feet and the other base is 16 feet.

35. If $b =$ the longer base in feet,

then $\frac{2b}{3} =$ the other base in feet,

and $\frac{b}{3} =$ the altitude in feet.

Hence $\frac{b}{3} \cdot \frac{1}{2} \left(b + \frac{2b}{3} \right) = 360,$

or $(b - 36)(b + 36) = 0.$

Whence $b = 36$ or $-36.$

Therefore the longer base is 36 feet, the other base is 24 feet, and the altitude is 12 feet.

36. If $a =$ the altitude in feet,

then $a + 4 =$ the shorter base in feet,

and $a + 6 =$ the longer base in feet.

Therefore $\frac{a(a + 4 + a + 6)}{2} = 66,$

or $(a - 6)(a + 11) = 0.$

Whence $a = 6$ or $-11.$

Therefore the altitude is 6 feet, the shorter base is 10 feet, and the longer base is 12 feet.

37. If $a =$ the altitude in feet,

then $a + 8 =$ the shorter base in feet,

and $a + 12 =$ the longer base in feet.

Therefore $\frac{a(a + 8 + a + 12)}{2} = 16 \cdot 9,$

or $(a - 8)(a + 18) = 0.$

Whence $a = 8$ or $-18.$

Therefore the altitude is 8 feet, the shorter base is 16 feet, and the longer base is 20 feet.

38. If $b =$ the shorter base in feet,

then $b + 4 =$ the longer base in feet,

and $b + 2 =$ the altitude in feet.

Therefore $\frac{(b + 2)(b + b + 4)}{2} = 4 \cdot 9,$

or $(b - 4)(b + 8) = 0.$

Whence $b = 4$ or -8 .

Therefore the shorter base is 4 feet, the longer base is 8 feet, and the altitude is 6 feet.

39. If $b =$ one base in feet,
then $b =$ the altitude in feet,
and $b + 2 =$ the other base in feet.

$$\text{Therefore } \frac{b(b + b + 2)}{2} = 10 \cdot 9,$$

$$\text{or } (b - 9)(b + 10) = 0.$$

Whence $b = 9$ or -10 .

Therefore one base is 9 feet, the altitude is 9 feet, and the other base is 11 feet.

40. If $b =$ the shorter base in feet,
then $b + 10 =$ the longer base in feet,
and $5b + 2 =$ the altitude in feet.

$$\text{Therefore } \frac{(5b + 2)(b + b + 10)}{2} = 22 \cdot 9,$$

$$\text{or } (b - 4)(5b + 47) = 0.$$

Whence $b = 4$ or $-9\frac{2}{5}$.

Therefore the shorter base is 4 feet, the longer base is 14 feet, and the altitude is 22 feet.

41. If $s =$ the shorter base in centimeters,
then $2s =$ the longer base in centimeters,
and $2s - 10 =$ the altitude in centimeters.

$$\text{Therefore } (2s - 10)\left(\frac{s + 2s}{2}\right) = 900,$$

$$\text{or } (s - 20)(s + 15) = 0.$$

Whence $s = 20$ or -15 .

Therefore the shorter base is 20 centimeters, the longer base is 40 centimeters, and the altitude is 30 centimeters.

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$$1. 12 = 2^2 \cdot 3 \text{ and } 18 = 2 \cdot 3^2. \therefore \text{H.C.F.} = 2 \cdot 3 = 6.$$

$$2. 24 = 2^3 \cdot 3 \text{ and } 56 = 2^3 \cdot 7. \therefore \text{H.C.F.} = 2^3 = 8.$$

$$3. 96 = 2^5 \cdot 3 \text{ and } 144 = 2^4 \cdot 3^2. \therefore \text{H.C.F.} = 2^4 \cdot 3 = 48.$$

$$4. 84 = 2^2 \cdot 3 \cdot 7 \text{ and } 196 = 2^2 \cdot 7^2. \therefore \text{H.C.F.} = 2^2 \cdot 7 = 28.$$

$$5. 125 = 5^3 \text{ and } 225 = 5^2 \cdot 3^2. \therefore \text{H.C.F.} = 5^2 = 25.$$

$$6. 64 = 2^6, 96 = 2^5 \cdot 3, \text{ and } 256 = 2^8. \therefore \text{H.C.F.} = 2^5 = 32.$$

$$7. 90 = 3^2 \cdot 2 \cdot 5, 108 = 3^3 \cdot 2^2, \text{ and } 324 = 3^4 \cdot 2^2. \therefore \text{H.C.F.} = 3^2 \cdot 2 = 18.$$

$$8. 12x^4 = 2^2 \cdot 3 \cdot x^4 \text{ and } 18x^3 = 2 \cdot 3^2 \cdot x^3. \therefore \text{H.C.F.} = 2 \cdot 3 \cdot x^3 = 6x^3.$$

9. $16x^2y^4 = 2^4 \cdot x^2 \cdot y^4$ and $24x^3y^5 = 2^3 \cdot 3 \cdot x^3 \cdot y^5$. $\therefore$ H.C.F. $= 2^3x^2y^4 = 8x^2y^4$.
10. $27c^2d = 3^3 \cdot c^2 \cdot d$ and $21cd^4 = 3 \cdot 7 \cdot c \cdot d^4$. $\therefore$ H.C.F. $= 3cd$.
11. $32a^3bc^4 = 2^5 \cdot a^3 \cdot b \cdot c^4$ and $48a^2b^2d = 2^4 \cdot 3 \cdot a^2 \cdot b^2 \cdot d$.
 $\therefore$ H.C.F. $= 2^4a^2b = 16a^2b$.
12. $125m^7n^5p = 5^3 \cdot m^7 \cdot n^5 \cdot p$ and $100ng^4 = 5^2 \cdot 2^2 \cdot n \cdot g^4$.
 $\therefore$ H.C.F. $= 5^2 \cdot n = 25n$.
13. $18h^2k^2 = 2 \cdot 3^2 \cdot h^2 \cdot k^2$, $36h^4k = 2^2 \cdot 3^2 \cdot h^4 \cdot k$,
 and $24h^3k^3 = 2^3 \cdot 3 \cdot h^3 \cdot k^3$. $\therefore$ H.C.F. $= 2 \cdot 3 \cdot h^2 \cdot k = 6h^2k$.
14. $9xy^4 = 3^2 \cdot x \cdot y^4$, $54x^5y = 2 \cdot 3^3 \cdot x^5 \cdot y$, and $15x^4y^5 = 3 \cdot 5 \cdot x^4 \cdot y^5$.
 $\therefore$ H.C.F. $= 3xy$.
15. $27a^4b^5c^2 = 3^3 \cdot a^4 \cdot b^5 \cdot c^2$, $54a^3b^2d = 2 \cdot 3^3 \cdot a^3 \cdot b^2 \cdot d$,
 and $81a^2b^2e^3 = 3^4 \cdot a^2 \cdot b^2 \cdot e^3$. $\therefore$ H.C.F. $= 3^3 \cdot a^2 \cdot b^2 = 27a^2b^2$.
16. $x^2 - 9 = (x + 3)(x - 3)$ and $x^2 - 5x + 6 = (x - 2)(x - 3)$.
 $\therefore$ H.C.F. $= x - 3$.
17. $x^2 + 3x - 10 = (x + 5)(x - 2)$ and $x^2 + 6x + 5 = (x + 5)(x + 1)$.
 $\therefore$ H.C.F. $= x + 5$.
18. $x^3 - 4x = x(x + 2)(x - 2)$ and $x^3 - 8x^2 + 12x = x(x - 2)(x - 6)$.
 $\therefore$ H.C.F. $= x^2 - 2x$.
19. $2c^3 + 12c^2 + 18c = 2c(c + 3)^2$ and $c^3 - 2c^2 - 15c = c(c + 3)(c - 5)$.
 $\therefore$ H.C.F. $= c^2 + 3c$.
20. $8 + y^3 = (2 + y)(4 - 2y + y^2)$ and $y^2 + 4y + 4 = (2 + y)^2$.
 $\therefore$ H.C.F. $= 2 + y$.
21. $x^4 - 2x^2 + 1 = (x + 1)^2(x - 1)^2$ and $x^2 - 2x + 1 = (x - 1)^2$.
 $\therefore$ H.C.F. $= x^2 - 2x + 1$.
22. $ab + 3b + ac + 3c = (a + 3)(b + c)$
 and $2ab + 6b - 2ac - 6c = 2(a + 3)(b - c)$. $\therefore$ H.C.F. $= a + 3$.
23. $c^2 + 3cd + 2d^2 = (c + d)(c + 2d)$, $c^2 + 5cd + 6d^2 = (c + 2d)(c + 3d)$,
 and $c^2 + cd - 2d^2 = (c - d)(c + 2d)$. $\therefore$ H.C.F. $= c + 2d$.

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1. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. $\therefore$ L.C.M. $= 2^2 \cdot 3^2 = 36$.
2. $32 = 2^5$ and $48 = 2^4 \cdot 3$. $\therefore$ L.C.M. $= 2^5 \cdot 3 = 96$.
3. $20 = 2^2 \cdot 5$ and $28 = 2^2 \cdot 7$. $\therefore$ L.C.M. $= 2^2 \cdot 5 \cdot 7 = 140$.
4. $96 = 2^5 \cdot 3$ and $144 = 2^4 \cdot 3^2$. $\therefore$ L.C.M. $= 2^5 \cdot 3^2 = 288$.
5. $64 = 2^6$, $120 = 2^3 \cdot 3 \cdot 5$, and $216 = 2^3 \cdot 3^3$. $\therefore$ L.C.M. $= 2^6 \cdot 3^3 \cdot 5 = 8640$.
6. $128 = 2^7$, $160 = 2^5 \cdot 5$, and $200 = 2^3 \cdot 5^2$. $\therefore$ L.C.M. $= 2^7 \cdot 5^2 = 3200$.
7. $x^2y = x^2 \cdot y$, $xy^2 = x \cdot y^2$, and $xy^3 = x \cdot y^3$. $\therefore$ L.C.M. $= x^2y^3$.
8. $6cd^3 = 2 \cdot 3 \cdot c \cdot d^3$, $4c^5de = 2^2 \cdot c^5 \cdot d \cdot e$, and $10c^2d^2e^3 = 2 \cdot 5 \cdot c^2 \cdot d^2 \cdot e^3$.
 $\therefore$ L.C.M. $= 60c^5d^3e^3$.
9. $8abc = 2^3 \cdot a \cdot b \cdot c$, $3b^2c = 3 \cdot b^2 \cdot c$, and $12c^2 = 2^2 \cdot 3 \cdot c^2$.
 $\therefore$ L.C.M. $= 24ab^2c^2$.

10. $18m^2 = 2 \cdot 3^2 \cdot m^2$, $15mn^2p = 3 \cdot 5 \cdot m \cdot n^2 \cdot p$, and $20m^3p^2 = 2^2 \cdot 5m^3 \cdot p^2$.
 $\therefore$ L.C.M. = $180m^3n^2p^2$.
11. $20 = 2^2 \cdot 5$, $18xy^4 = 2 \cdot 3^2 \cdot x \cdot y^4$, and $27ax^3y = 3^3 \cdot a \cdot x^3 \cdot y$.
 $\therefore$ L.C.M. = $540ax^3y^4$.
12. $36ab^2 = 2^2 \cdot 3^2 \cdot a \cdot b^2$, $42abc = 2 \cdot 3 \cdot 7 \cdot a \cdot b \cdot c$, and $63b^2c = 3^2 \cdot 7 \cdot b^2 \cdot c$.
 $\therefore$ L.C.M. = $252ab^2c$.
13. $4a = 2^2 \cdot a$, $a^2 - ab = a(a - b)$. $\therefore$ L.C.M. = $4a^2 - 4ab$.
14. $12ax = 2^2 \cdot 3 \cdot ax$, $3a^3x^2 - 3ax^3 = 3ax^2(a^2 - x)$.
 $\therefore$ L.C.M. = $12a^3x^2 - 12ax^3$.
15. $cx + cy = c(x + y)$, $dy + dx = d(x + y)$. $\therefore$ L.C.M. = $cdx + cdy$.
16. $3x + 3z = 3(x + z)$, $6a^2x + 6a^2z = 2 \cdot 3a^2(x + z)$.
 $\therefore$ L.C.M. = $6a^2x + 6a^2z$.
17. $x^2 - xy = x(x - y)$, $ax + ay = a(x + y)$. $\therefore$ L.C.M. = $ax^3 - axy^2$.
18. $x^2 - 9 = (x - 3)(x + 3)$, $x^2 - 5x + 6 = (x - 2)(x - 3)$.
 $\therefore$ L.C.M. = $x^3 - 2x^2 - 9x + 18$.
19. $c^2 - 4 = (c + 2)(c - 2)$, $c^2 - 8c - 20 = (c - 10)(c + 2)$.
 $\therefore$ L.C.M. = $c^3 - 10c^2 - 4c + 40$.
20. $4ax = 2^2 \cdot a \cdot x$, $4x^2 - 1 = (2x + 1)(2x - 1)$,
 $4x^2 + 4x + 1 = (2x + 1)^2$.
 $\therefore$ L.C.M. = $32ax^4 - 8ax^2 + 16ax^3 - 4ax$.
21. $x^2 + 1 = x^2 + 1$, $x^4 - 1 = (x^2 + 1)(x + 1)(x - 1)$,
 $x^4 - 2x^2 + 1 = (x + 1)^2(x - 1)^2$. $\therefore$ L.C.M. = $x^6 - x^4 - x^2 + 1$.
22. $4 - c^2 = (2 + c)(2 - c)$, $c^3 + 8 = (c + 2)(c^2 - 2c + 4)$,
 $c^2 + 6c + 8 = (c + 2)(c + 4)$.
 $\therefore$ L.C.M. = $64 - 16c - 8c^2 + 8c^3 - 2c^4 - c^5$.
23. $ac - 2bd + 2ad - bc = (a - b)(c + 2d)$,
 $a^2 + ab - 2b^2 = (a + 2b)(a - b)$.
 $\therefore$ L.C.M. = $a^2c + abc - 2b^2c + 2a^2d + 2abd - 4b^2d$.
24. $x^2 - y^2 = (x + y)(x - y)$, $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$,
 $x^2 + 2xy + y^2 = (x + y)^2$.
 $\therefore$ L.C.M. = $x^5 + 2x^4y + x^3y^2 - x^2y^3 - 2xy^4 - y^5$.
25. $ax^2 + bxy = x(ax + by)$, $2ax + 2by = 2(ax + by)$,
 $a^2x^2 - b^2y^2 = (ax + by)(ax - by)$. $\therefore$ L.C.M. = $2a^2x^3 - 2b^2xy^2$.
26. $2x^3 - 2x = 2x(x + 1)(x - 1)$, $3x^4 + 15x^3 - 18x^2 = 3x^2(x + 6)(x - 1)$,
 $x^2 - 36 = (x + 6)(x - 6)$.
 $\therefore$ L.C.M. = $6x^6 - 222x^4 + 216x^2$.
27. $8 - y^3 = (2 - y)(4 + 2y + y^2)$, $y^2 - 4 = (y + 2)(y - 2)$,
 $4y^2 + 2y^3 + y^4 = y^2(4 + 2y + y^2)$.
 $\therefore$ L.C.M. = $-y^6 - 2y^5 + 8y^3 + 16y^2$.
28. $2x^3 - 6x^2 + 4x^2y = 2x^2(x - 3 + 2y)$,
 $x^2 - 4y^2 - 6x + 9 = (x - 3 + 2y)(x - 3 - 2y)$.
 $\therefore$ L.C.M. = $2x^4 - 8x^2y^2 - 12x^3 + 18x^2$.

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1. $\frac{a^3b}{a^2b^4} = \frac{a}{b^3}.$
2. $\frac{12x^2y^6}{18xy^7} = \frac{2x}{3y}.$
3. $\frac{32a^4b^4c}{48a^3b^6c^2} = \frac{2a}{3b^2c}.$
4. $\frac{45cd^5e^2}{20c^5d^3} = \frac{9d^2e^2}{4c^4}.$
5. $\frac{36a^2b^2}{54a^5bc^3} = \frac{2b}{3a^3c^3}.$
6. $\frac{2a}{2a+2} = \frac{a}{a+1}.$
7. $\frac{3xy+3y^2}{3y^4} = \frac{x+y}{y^3}.$
8. $\frac{21d^2+14c}{14c} = \frac{3d^2+2c}{2c}.$
9. $\frac{a^2-1}{a^2-2a+1} = \frac{a+1}{a-1}.$
10. $\frac{4x^2+4x+1}{4x^2-1} = \frac{2x+1}{2x-1}.$
11. $\frac{2a^2-2b^2}{4a^2-8ab+4b^2} = \frac{a+b}{2a-2b}.$
12. $\frac{18ax^3+9ay}{12bx^3+6by} = \frac{3a}{2b}.$
13. $\frac{x^2-25}{x^2-x-30} = \frac{x-5}{x-6}.$
14. $\frac{c^2-5cd+4d^2}{c^2-16d^2} = \frac{c-d}{c+4d}.$
15. $\frac{2x^2-2x-180}{2x^2-162} = \frac{x-10}{x-9}.$
16. $\frac{21+10x+x^2}{x^2-9} = \frac{x+7}{x-3}.$
17. $\frac{4c^3-5c^2-4c+5}{8c^4-10c^3+12c-15} = \frac{(4c-5)(c^2-1)}{(4c-5)(2c^3+3)} = \frac{c^2-1}{2c^3+3}.$
18. $\frac{x^2-y^2}{(x-y)^2} = \frac{x+y}{x-y}.$
19. $\frac{a^2-b^2}{(a-b)^3} = \frac{a+b}{a^2-2ab+b^2}.$
20. $\frac{c^2-d^2}{c^3-d^3} = \frac{(c+d)(c-d)}{(c^2+cd+d^2)(c-d)} = \frac{c+d}{c^2+cd+d^2}.$
21. $\frac{y^3-z^3}{(y-z)^3} = \frac{(y-z)(y^2+yz+z^2)}{(y-z)(y^2-2yz+z^2)} = \frac{y^2+yz+z^2}{y^2-2yz+z^2}.$
22. $\frac{x^3-8}{x^2-4} = \frac{(x^2+2x+4)(x-2)}{(x+2)(x-2)} = \frac{x^2+2x+4}{x+2}.$
23. $\frac{x^6-1}{x^2-1} = \frac{(x+1)(x-1)(x^2+x+1)(x^2-x+1)}{(x+1)(x-1)} = x^4+x^2+1.$
24. $\frac{x^4-y^4}{x^6-y^6} = \frac{(x+y)(x-y)(x^2+y^2)}{(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)} = \frac{x^2+y^2}{x^4+x^2y^2+y^4}.$
25. $\frac{64x^3+1}{1+8x+16x^2} = \frac{(16x^2-4x+1)(4x+1)}{(1+4x)(4x+1)} = \frac{16x^2-4x+1}{1+4x}.$

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1. $\frac{-x}{y} = -\frac{x}{y} = \frac{x}{-y} = -\frac{-x}{-y}.$
2. $\frac{x}{-y} = -\frac{x}{y} = \frac{-x}{y} = -\frac{-x}{-y}.$
3. $\frac{a}{a-b} = \frac{-a}{-a+b} = -\frac{-a}{a-b} = -\frac{a}{-a+b}.$

4. $\frac{x-3}{2x+4} = \frac{-x+3}{-2x-4} = -\frac{-x+3}{2x+4} = -\frac{x-3}{-2x-4}.$
5. $-\frac{c^2-d}{-2c+d} = -\frac{-c^2+d}{2c-d} = +\frac{-c^2+d}{-2c+d} = +\frac{c^2-d}{2c-d}.$
6. $-\frac{x-y}{x+y-3} = -\frac{-x+y}{-x-y+3} = +\frac{-x+y}{x+y-3} = +\frac{x-y}{-x-y+3}.$
7. $\frac{x^2-5x+6}{x^2-7x+12} = \frac{-x^2+5x-6}{-x^2+7x-12} = -\frac{-x^2+5x-6}{x^2-7x+12} = -\frac{x^2-5x+6}{-x^2+7x-12}.$
8. $-\frac{-x+x^2-2}{x^3-x^2y-1} = -\frac{x-x^2+2}{-x^3+x^2y+1} = +\frac{x-x^2+2}{x^3-x^2y-1} = +\frac{-x+x^2-2}{-x^3+x^2y+1}.$

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1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ equal $\frac{3}{6}, \frac{2}{6},$ and $\frac{1}{6}$ respectively.
2. $\frac{5}{24}, \frac{7}{32}$ equal $\frac{20}{96}$ and $\frac{21}{96}$ respectively.
3. $\frac{4}{3d}, \frac{2c}{5d}$ equal $\frac{20}{15d}$ and $\frac{6c}{15d}$ respectively.
4. $\frac{3a+b}{12}, \frac{3a}{16}$ equal $\frac{12a+4b}{48}$ and $\frac{9a}{48}$ respectively.
5. $\frac{5x+y}{21}, \frac{4x+3y}{35}$ equal $\frac{25x+5y}{105}$ and $\frac{12x+9y}{105}$ respectively.
6. $\frac{4}{5a^2b}, \frac{3}{abc}$ equal $\frac{4c}{5a^2bc}$ and $\frac{15a}{5a^2bc}$ respectively.
7. $\frac{5c}{3mn^3}, \frac{12d}{11m^2n^2}$ equal $\frac{55cm}{33m^2n^3}$ and $\frac{36nd}{33m^2n^3}$ respectively.
8. $\frac{3a}{2b^2c}, \frac{2b}{3a^2c}, \frac{c}{6ab}$ equal $\frac{9a^3}{6a^2b^2c}, \frac{4b^3}{6a^2b^2c}$ and $\frac{abc^2}{6a^2b^2c}$ respectively.
9. $\frac{3b}{2cd^3}, \frac{9a}{4cde}, \frac{7ab}{5cde^2}$ equal $\frac{30be^2}{20cd^3d^2}, \frac{45ad^2e}{20cd^3e^2}$ and $\frac{28abd^2}{20cd^3e^2}$ respectively.
10. $\frac{x+y}{3xy^2}, \frac{x-2y}{2x^2y}$ equal $\frac{2x^2+2xy}{6x^2y^2}$ and $\frac{3xy-6y^2}{6x^2y^2}$ respectively.
11. $\frac{2+m^2}{2m}, \frac{4+m^2}{4n}$ equal $\frac{4n+2m^2n}{4mn}$ and $\frac{4m+m^3}{4mn}$ respectively.
12. $\frac{2}{x+2}, \frac{4}{3x+6}$ equal $\frac{6}{3x+6}$ and $\frac{4}{3x+6}$ respectively.
13. $\frac{4}{x+1}, \frac{5}{x-1}$ equal $\frac{4x-4}{x^2-1}$ and $\frac{5x+5}{x^2-1}$ respectively.
14. $\frac{3c}{c^2-d^2}, \frac{4}{c+d}$ equal $\frac{3c}{c^2-d^2}$ and $\frac{4c-4d}{c^2-d^2}$ respectively.
15. $\frac{3x}{x+3}, \frac{5}{x^2+5x+6}$ equal $\frac{3x^2+6x}{x^2+5x+6}$ and $\frac{5}{x^2+5x+6}$ respectively.

$$16. \frac{2x}{x^2 - xy}, \frac{3y}{x^2 - 2xy + y^2} \text{ equal } \frac{2x^2 - 2xy}{x^3 - 2x^2y + xy^2} \text{ and } \frac{3xy}{x^3 - 2x^2y + xy^2} \text{ respectively.}$$

$$17. \frac{3c + 2}{c^2 - d^2}, \frac{4 - c}{c^2 - 7cd + 6d^2} \text{ equal } \frac{3c^2 + 2c - 18cd - 12d}{c^3 - cd^2 - 6c^2d + 6d^3} \text{ and } \frac{4c - c^2 + 4d - cd}{c^3 - cd^2 - 6c^2d + 6d^3} \text{ respectively.}$$

$$18. \frac{2x + 5}{x^2 - 1}, \frac{2x - 4}{x^3 - 3x^2 + 2x} \text{ equal } \frac{2x^3 + x^2 - 10x}{x^4 - 2x^3 - x^2 + 2x} \text{ and } \frac{2x^2 - 2x - 4}{x^4 - 2x^3 - x^2 + 2x} \text{ respectively.}$$

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$$1. \frac{2x}{3} + \frac{3x}{5} = \frac{10x}{15} + \frac{9x}{15} = \frac{19x}{15}.$$

$$2. \frac{2a}{3} - \frac{5a}{6} + \frac{a}{4} = \frac{8a}{12} - \frac{10a}{12} + \frac{3a}{12} = \frac{a}{12}.$$

$$3. \frac{3c}{7} + \frac{c}{21} - \frac{1}{3} = \frac{9c}{21} + \frac{c}{21} - \frac{7}{21} = \frac{10c - 7}{21}.$$

$$4. \frac{9a}{16} - \frac{12m}{8} - \frac{5m}{24} = \frac{27a}{48} - \frac{72m}{48} - \frac{10m}{48} = \frac{27a - 82m}{48}.$$

$$5. \frac{x - 3}{4} + \frac{2x - 5}{6} = \frac{3x - 9}{12} + \frac{4x - 10}{12} = \frac{7x - 19}{12}.$$

$$6. \frac{5a + 7}{14} - \frac{9a + x}{21} = \frac{15a + 21}{42} - \frac{18a + 2x}{42} = \frac{21 - 3a - 2x}{42}.$$

$$7. \frac{5c}{3} - \frac{3c - x}{5} + \frac{x - 3c}{10} = \frac{50c}{30} - \frac{18c - 6x}{30} + \frac{3x - 9c}{30} = \frac{23c + 9x}{30}.$$

$$8. \frac{4m - 3}{6} - \frac{7 - 9m}{9} - \frac{3a + 5m - 4}{27} \\ = \frac{36m - 27}{54} - \frac{42 - 54m}{54} - \frac{6a + 10m - 8}{54} \\ = \frac{80m - 6a - 61}{54}.$$

$$9. \frac{2}{a} + \frac{3}{4a^2} = \frac{8a}{4a^2} + \frac{3}{4a^2} = \frac{8a + 3}{4a^2}.$$

$$10. \frac{3}{x^2} + \frac{4}{x} - \frac{8}{3x^3} = \frac{9x}{3x^3} + \frac{12x^2}{3x^3} - \frac{8}{3x^3} = \frac{12x^2 + 9x - 8}{3x^3}.$$

$$11. \frac{x}{y} - \frac{5}{2y^3} + \frac{a}{4} = \frac{4xy^2}{4y^3} - \frac{10}{4y^3} + \frac{ay^3}{4y^3} = \frac{4xy^2 + ay^3 - 10}{4y^3}.$$

$$12. \frac{a}{b} + \frac{b}{c} - \frac{c}{a} = \frac{a^2c}{abc} + \frac{ab^2}{abc} - \frac{bc^2}{abc} = \frac{a^2c + ab^2 - bc^2}{abc}.$$

$$13. \frac{7}{m^2} - \frac{5}{mn} + \frac{6}{n^2} = \frac{7n^2}{m^2n^2} - \frac{5mn}{m^2n^2} + \frac{6m^2}{m^2n^2} = \frac{7n^2 - 5mn + 6m^2}{m^2n^2}.$$

$$14. \frac{3}{xy} - \frac{5}{2x^2} - \frac{6}{5xy^3} = \frac{30xy^2}{10x^2y^3} - \frac{25y^3}{10x^2y^3} - \frac{12x}{10x^2y^3} = \frac{30xy^2 - 25y^3 - 12x}{10x^2y^3}.$$

$$15. \frac{3x-1}{6x^2} - \frac{7-2x^2}{4x^3} + \frac{5x^3-8}{9x^4} = \frac{18x^3-6x^2}{36x^4} - \frac{63x-18x^3}{36x^4} + \frac{20x^3-32}{36x^4} \\ = \frac{56x^3 - 6x^2 - 63x - 32}{36x^4}.$$

$$16. \frac{4c^2-9}{2cd} - \frac{6-c}{5c^2} - \frac{c^2-4}{cd^2} = \frac{20c^3d-45cd}{10c^2d^2} - \frac{12d^2-2cd^2}{10c^2d^2} - \frac{10c^3-40c}{10c^2d^2} \\ = \frac{20c^3d - 45cd - 12d^2 + 2cd^2 - 10c^3 + 40c}{10c^2d^2}.$$

$$17. \frac{3+x}{x-3} + \frac{3}{4} = \frac{(3+x)4}{(x-3)4} + \frac{3(x-3)}{4(x-3)} = \frac{7x+3}{4x-12}.$$

$$18. \frac{3}{x-5} - \frac{2}{x+5} = \frac{3(x+5)}{(x-5)(x+5)} - \frac{2(x-5)}{(x+5)(x-5)} = \frac{x+25}{x^2-25}.$$

$$19. \frac{5x}{x^2+xy} - \frac{7}{x} = \frac{5x}{x(x+y)} - \frac{7(x+y)}{x(x+y)} = \frac{-2x-7y}{x^2+xy}.$$

$$20. \frac{3a-b}{a^2-b^2} + \frac{2}{a-b} = \frac{3a-b}{a^2-b^2} + \frac{2(a+b)}{(a-b)(a+b)} = \frac{5a+b}{a^2-b^2}.$$

$$21. \frac{10}{25-m^2} - \frac{1}{m^2+16m+55} \\ = \frac{10(m+11)}{(5+m)(5-m)(m+11)} - \frac{1(5-m)}{(m+5)(m+11)(5-m)} \\ = \frac{105+11m}{275+25m-11m^2-m^3}.$$

$$22. \frac{3}{a^2-16} + \frac{5}{a^2-6a+8} = \frac{3(a-2)}{(a+4)(a-4)(a-2)} + \frac{5(a+4)}{(a-2)(a-4)(a+4)} \\ = \frac{8a+14}{a^3-2a^2-16a+32}.$$

$$23. \frac{2x+1}{x^2-1} + \frac{4}{x^2-3x+2} = \frac{(2x+1)(x-2)}{(x+1)(x-1)(x-2)} + \frac{4(x+1)}{(x-1)(x-2)(x+1)} \\ = \frac{2x^2+x+2}{x^3-2x^2-x+2}.$$

$$24. \frac{c-5}{c^2-6c} + \frac{2c-3}{c^2-8c+12} = \frac{(c-5)(c-2)}{c(c-6)(c-2)} + \frac{(2c-3)c}{(c-6)(c-2)c} \\ = \frac{3c^2-10c+10}{c^3-8c^2+12c}.$$

25. $\frac{x^2 - 3xy + y^2}{9 - 6x + x^2} - \frac{5x - 3y}{9 - 3x} = \frac{(x^2 - 3xy + y^2)3}{(3-x)^2 \cdot 3} - \frac{(5x - 3y)(3-x)}{3(3-x)(3-x)}$
 $= \frac{8x^2 - 12xy - 15x + 9y + 3y^2}{3x^2 - 18x + 27}.$
26. $\frac{x+2}{x^2+x} + \frac{1}{x} + \frac{3-x}{x^2+2x+1} = \frac{(x+2)(x+1)}{x(x+1)(x+1)} + \frac{1(x+1)^2}{x(x+1)^2} + \frac{(3-x)x}{(x+1)^2 \cdot x}$
 $= \frac{x^2 + 8x + 3}{x^3 + 2x^2 + x}.$
27. $\frac{a+b}{a^2-ab} + \frac{a-2b}{a^2-2ab+b^2} - \frac{2a-b}{a^2-b^2}$
 $= \frac{(a+b)(a+b)(a-b)}{(a^2-ab)(a-b)(a+b)} + \frac{(a-2b)(a+b)a}{(a^2-2ab+b^2)(a+b)a} - \frac{(2a-b)(a-b)a}{(a^2-b^2)(a-b)a}$
 $= \frac{-4ab^2 + 3a^2b - b^3}{a^4 - a^3b - a^2b^2 + ab^3}.$
28. $\frac{m-2n}{m^2+mn+n^2} - \frac{m^2-3n^2}{m^3-n^3} + \frac{3m-n}{m-n}$
 $= \frac{(m-2n)(m-n)}{(m^2+mn+n^2)(m-n)} - \frac{m^2-3n^2}{m^3-n^3} + \frac{(3m-n)(m^2+mn+n^2)}{(m-n)(m^2+mn+n^2)}$
 $= \frac{3m^3 + 2m^2n + 2mn^2 - 3mn + 5n^2 - n^3}{m^3 - n^3}.$
29. $\frac{c^2+cd+d^2}{c^2-cd+d^2} - \frac{c-d}{2c+2d} - \frac{2c^2+5cd}{c^3+d^3}$
 $= \frac{(c^2+cd+d^2)(c+d)2}{(c^2-cd+d^2)(c+d)2} - \frac{(c-d)(c^2-cd+d^2)}{(2c+2d)(c^2-cd+d^2)} - \frac{(2c^2+5cd)2}{(c^3+d^3)2}$
 $= \frac{c^3 + 6c^2d + 2cd^2 + 3d^3 - 4c^2 - 10cd}{2c^3 + 2d^3}.$
30. $\frac{x+y}{x^3-y^3} - \frac{x-2y}{(x-y)^3} = \frac{(x+y)(x-y)^2}{(x^3-y^3)(x-y)^2} - \frac{(x-2y)(x^2+xy+y^2)}{(x-y)^3(x^2+xy+y^2)}$
 $= \frac{x^5 - 2x^4y + x^3y^2 - x^2y^3 + 2xy^4 - y^5}{3y^3}.$
31. $R^2 + \frac{R^2}{4} = \frac{5R^2}{4}.$
32. $x-3 - \frac{x+3}{5} = \frac{(x-3)5}{5} - \frac{x+3}{5} = \frac{4x-18}{5}.$
33. $\frac{m^2+n^2}{m-n} + n+m = \frac{m^2+n^2}{m-n} + \frac{(n+m)(m-n)}{1(m-n)} = \frac{2m^2}{m-n}.$
34. $3a+b - \frac{a^2-3b^2}{3a-b} = \frac{(3a+b)(3a-b)}{1(3a-b)} - \frac{a^2-3b^2}{3a-b} = \frac{8a^2+2b^2}{3a-b}.$
35. $x^2+y^2 - \frac{3x^3-y^3}{x+y} - xy = \frac{(x^2-xy+y^2)(x+y)}{1(x+y)} - \frac{3x^3-y^3}{x+y} = \frac{2y^3-2x^3}{x+y}.$

$$36. m^2 - \frac{m^4 - 2m^2}{m^2 + m + 1} + 1 - m = \frac{(m^2 - m + 1)(m^2 + m + 1)}{1(m^2 + m + 1)} - \frac{m^4 - 2m^2}{m^2 + m + 1}$$

$$= \frac{3m^2 + 1}{m^2 + m + 1}.$$

$$37. \frac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + m - 2n$$

$$= \frac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + \frac{(m - 2n)(m^2 + 2mn + 4n^2)}{m^2 + 2mn + 4n^2}$$

$$= \frac{2m^3}{m^2 + 2mn + 4n^2}.$$

$$38. a^2 - \frac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2} - ab + b^2$$

$$= \frac{(a^2 - ab + b^2)(a^2 + ab + b^2)}{a^2 + ab + b^2} - \frac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2}$$

$$= \frac{2b^4 - a^2b^2}{a^2 + ab + b^2}.$$

$$39. x^3 + x^2y - \frac{x^4 + 3y^4}{x - y} + xy^2 + y^3$$

$$= \frac{(x^3 + x^2y + xy^2 + y^3)(x - y)}{x - y} - \frac{x^4 + 3y^4}{x - y}$$

$$= \frac{4y^4}{y - x}.$$

$$40. \frac{2c^2 + d^2}{c^2 - d^2} - \frac{c}{c + d} + 5 = \frac{2c^2 + d^2}{c^2 - d^2} - \frac{c(c - d)}{(c + d)(c - d)} + \frac{5(c^2 - d^2)}{c^2 - d^2}$$

$$= \frac{6c^2 + cd - 4d^2}{c^2 - d^2}.$$

$$41. 6 + \frac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \frac{3r - 5s}{r - 7s} + 5$$

$$= \frac{11(r^2 - 9rs + 14s^2)}{r^2 - 9rs + 14s^2} + \frac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \frac{(3r - 5s)(r - 2s)}{(r - 7s)(r - 2s)}$$

$$= \frac{14r^2 - 88rs + 143s^2}{r^2 - 9rs + 14s^2}.$$

$$42. \left(x - 3 - \frac{3}{x + 3}\right) - \left(2x - 7 - \frac{4}{x + 4}\right)$$

$$= -x + 4 - \frac{3}{x + 3} + \frac{4}{x + 4}$$

$$= \frac{(-x + 4)(x + 3)(x + 4)}{(x + 3)(x + 4)} - \frac{3(x + 4)}{(x + 3)(x + 4)} + \frac{4(x + 3)}{(x + 4)(x + 3)}$$

$$= \frac{48 + 17x - 3x^2 - x^3}{x^2 + 7x + 12}.$$

$$43. \left(5m + \frac{6m}{2n}\right) - \left(m - \frac{m-3n}{n}\right) = 4m + \frac{3m}{n} + \frac{m-3n}{n}$$

$$= \frac{4mn + 4m - 3n}{n}.$$

$$44. \left(7c - \frac{2d}{3de}\right) - \left(5c + \frac{c}{3de}\right) = 2c - \frac{2d+c}{3de} = \frac{6cde - 2d - c}{3de}.$$

$$45. \left(\frac{2a}{a-b} - 5a\right) - \left(2a - \frac{4a}{a+b}\right) = -7a + \frac{2a}{a-b} + \frac{4a}{a+b}$$

$$= -\frac{7a(a-b)(a+b)}{(a-b)(a+b)} + \frac{2a(a+b)}{(a-b)(a+b)} + \frac{4a(a-b)}{(a+b)(a-b)}$$

$$= \frac{6a^2 - 2ab + 7ab^2 - 7a^3}{a^2 - b^2}.$$

$$46. \left(2a - 3b + \frac{3a}{2a+b}\right) - \left(3a + 2b - \frac{5a}{2a-b}\right)$$

$$= -a - 5b + \frac{3a}{2a+b} + \frac{5a}{2a-b}$$

$$= \frac{(-a-5b)(2a-b)(2a+b)}{(2a-b)(2a+b)} + \frac{3a(2a-b)}{(2a+b)(2a-b)} + \frac{5a(2a+b)}{(2a-b)(2a+b)}$$

$$= \frac{-4a^3 + 16a^2 - 20a^2b + ab^2 + 2ab + 5b^3}{4a^2 - b^2}.$$

$$47. \left(-\frac{4r^2}{2r+s} + 2r - s\right) - \left(-\frac{4s^2}{2s-3r} + 2s + 3r\right)$$

$$= -r - 3s - \frac{4r^2}{2r+s} + \frac{4s^2}{2s-3r}$$

$$= \frac{(-r-3s)(2r+s)(2s-3r)}{(2r+s)(2s-3r)} - \frac{4r^2(2s-3r)}{(2r+s)(2s-3r)} + \frac{4s^2(2r+s)}{(2s-3r)(2r+s)}$$

$$= \frac{18r^3 + 9r^2s + 3rs^2 - 2s^3}{2s^2 + rs - 6r^2}.$$

$$48. \frac{x-1}{2-x} + \frac{x^2-3}{x^2-4} = \frac{(1-x)(x+2)}{(x-2)(x+2)} + \frac{x^2-3}{(x+2)(x-2)} = \frac{x+1}{4-x^2}.$$

$$49. \frac{5}{x-3} + \frac{3}{3-x} = \frac{5}{x-3} - \frac{3}{x-3} = \frac{2}{x-3}.$$

$$50. \frac{6}{x^2-25} - \frac{3}{5-x} = \frac{6}{(x+5)(x-5)} + \frac{3(x+5)}{(x-5)(x+5)} = \frac{3x+21}{x^2-25}.$$

$$51. \frac{3a}{a^2-4} + \frac{2a-1}{2-a} = \frac{3a}{(a+2)(a-2)} - \frac{(2a-1)(a+2)}{(a-2)(a+2)} = \frac{2-2a^2}{a^2-4}.$$

$$52. \frac{3c}{9-c^2} - \frac{4c-2}{c-3} = \frac{3c}{(3+c)(3-c)} + \frac{(4c-2)(3+c)}{(3-c)(3+c)} = \frac{4c^2+13c-6}{9-c^2}.$$

$$53. \frac{x}{2x-1} - \frac{x}{1+2x} - \frac{x}{1-4x^2}$$

$$= \frac{x(2x+1)}{(2x-1)(2x+1)} - \frac{x(2x-1)}{(2x+1)(2x-1)} + \frac{x}{(2x+1)(2x-1)}$$

$$= \frac{3x}{4x^2-1}.$$

$$54. \frac{7}{x^2-13x+42} + \frac{4x-1}{7-x} - \frac{2x+3}{6-x}$$

$$= \frac{7}{(x-6)(x-7)} - \frac{(4x-1)(x-6)}{(x-7)(x-6)} + \frac{(2x+3)(x-7)}{(x-6)(x-7)}$$

$$= \frac{-2x^2+14x-20}{x^2-13x+42}.$$

$$55. \frac{3x-1}{x^2+7x-8} - \frac{4x-1}{1-x} + \frac{x+2}{8+x}$$

$$= \frac{3x-1}{(x+8)(x-1)} + \frac{(4x-1)(x+8)}{(x-1)(x+8)} + \frac{(x+2)(x-1)}{(x+8)(x-1)}$$

$$= \frac{5x^2+35x-11}{x^2+7x-8}.$$

$$56. \frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} - \frac{1}{(c-a)(c-b)}$$

$$= \frac{1}{(a-b)(a-c)} - \frac{1}{(b-c)(a-b)} - \frac{1}{(a-c)(b-c)}$$

$$= \frac{b-c}{(a-b)(a-c)(b-c)} - \frac{a-c}{(b-c)(a-b)(a-c)} - \frac{a-b}{(a-c)(b-c)(a-b)}$$

$$= \frac{-2}{(a-c)(b-c)}.$$

$$57. \frac{2}{(x-y)(x-z)} - \frac{1}{(x-y)(z-x)} = \frac{2}{(x-y)(x-z)} + \frac{1}{(x-y)(x-z)}$$

$$= \frac{3}{(x-y)(x-z)}.$$

$$58. \frac{3a}{(a-4)(a-3)} - \frac{2a}{(3-a)(4-a)} = \frac{3a}{(a-4)(a-3)} - \frac{2a}{(a-3)(a-4)}$$

$$= \frac{a}{(a-3)(a-4)}.$$

$$59. \frac{2}{(m-n)(m+n)} + \frac{3}{(n-m)(m-7)} - \frac{4}{(n-m)(7-m)}$$

$$= \frac{2}{(m-n)(m+n)} - \frac{3}{(m-n)(m-7)} - \frac{4}{(m-n)(m-7)}$$

$$= \frac{2(m-7) - 3(m+n) - 4(m+n)}{(m-n)(m+n)(m-7)}$$

$$= \frac{-5m-7n-14}{(m-n)(m+n)(m-7)}.$$

$$\begin{aligned}
 60. \quad & \frac{c}{c^2 - 10c + 24} - \frac{1}{6c - c^2 - 8} + \frac{3}{(6 - c)(2 - c)} \\
 &= \frac{c}{(c - 4)(c - 6)} + \frac{1}{(c - 4)(c - 2)} + \frac{3}{(c - 6)(c - 2)} \\
 &= \frac{c(c - 2) + 1(c - 6) + 3(c - 4)}{(c - 4)(c - 6)(c - 2)} \\
 &= \frac{c^2 + 2c - 18}{(c - 6)(c - 4)(c - 2)}.
 \end{aligned}$$

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Performing the indicated division:

1. $\frac{15x^2 - 10x + 2}{5x} = 3x - 2 + \frac{2}{5x}.$
2. $\frac{24a^3 - 6a^2 - 14}{6a} = 4a^2 - a - \frac{14}{6a}.$
3. $\frac{c}{c + 1} = 1 - \frac{1}{c + 1}.$
4. $\frac{d^3 + 1}{d - 1} = d^2 + d + 1 + \frac{2}{d - 1}.$
5. $\frac{27x^3 - y^3}{3x + y} = 9x^2 - 3xy + y^2 - \frac{2y^3}{3x + y}.$
6. $\frac{y^4 + y^2 + 1}{y^2 - y - 1} = y^2 + y + 3 + \frac{4y + 4}{y^2 - y - 1}.$
7. $\frac{a^4 + a^2b^2 + b^4}{a^2 + ab - b^2} = a^2 - ab + 3b^2 + \frac{4b^4 - 4ab^3}{a^2 + ab - b^2}.$
8. $\frac{3y^3 - 11}{y + 3} = 3y^2 - 9y + 27 - \frac{92}{y + 3}.$
9. $\frac{(a + b)^3}{a^3 + b^3} = 1 + \frac{3a^2b + 3ab^2}{a^3 + b^3} = 1 + \frac{3ab}{a^2 - ab + b^2}.$
10. $\frac{16a^4 + b^4}{2a - 1} = 8a^3 + 4a^2 + 2a + 1 + \frac{1 + b^4}{2a - 1}.$
11. $\frac{5a^3 + 3a^2 - 6}{5a^2 + 3a + 2} = a + \frac{-2a - 6}{5a^2 + 3a + 2}.$
12. $\frac{x^6 + y^6}{x^2 - y^2} = x^4 + x^2y^2 + y^4 + \frac{2y^6}{x^2 - y^2}.$
13. $\frac{x^4}{x + 1} = x^3 - x^2 + x - 1 + \frac{1}{x + 1}.$
14. $\frac{(a^2 + b)^2}{a^2 + b^2} = a^2 - b^2 + 2b + \frac{b^2 - 2b^3 + b^4}{a^2 + b^2}.$

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$$1. \frac{3x^2y^3}{4a^4} \cdot \frac{8a^3}{12xy} = \frac{xy^2}{2a}.$$

$$2. \frac{10a^3b^2}{15c^3} \cdot \frac{18c}{7a^2} = \frac{12ab^2}{7c^2}.$$

$$3. \frac{3x}{8y} \cdot \frac{6ax}{9b} \cdot \frac{12b^2y}{16x} = \frac{3abx}{16}.$$

$$4. \frac{14m^2n^4}{5n} \cdot \frac{25n^3}{6m^2} \cdot \frac{4x}{7m} = \frac{20n^6x}{3m}.$$

$$5. \left(\frac{2x}{3ay}\right)^2 \cdot \frac{12a^3}{16b^2xy^3} \cdot 8b^4y^2 = \frac{8ab^2x}{3y^3}.$$

$$6. \frac{a}{4c^2} \cdot \left(\frac{c}{2a}\right)^3 \cdot 16 = \frac{c}{2a^2}.$$

$$7. \left(\frac{2a}{c}\right)^2 \cdot \left(\frac{2c}{a}\right)^3 \cdot \left(\frac{1}{4}\right)^2 = \frac{2c}{a}.$$

$$8. \left(\frac{-3a}{3}\right)^2 \cdot \frac{c^5}{(2a)^5} \cdot \left(\frac{2}{3}\right)^2 = \frac{c^5}{72a^3}.$$

$$9. \frac{6c}{a} \cdot \left(\frac{9c^2}{4a}\right)^2 \cdot \left(\frac{-2a}{3c}\right)^3 = -9c^2.$$

$$10. \frac{(4ax)^2}{225c^4} \cdot \frac{(5ac^2)^3}{(2x^2)^3} \cdot \left(\frac{-x}{c}\right)^5 = \frac{-10a^5x}{9c^3}.$$

$$11. \frac{2a+4x}{5y^2} \cdot \frac{15y}{a+2x} = \frac{6}{y}.$$

$$12. \frac{3c}{9-c^2} \cdot \frac{c^2+5c+6}{18cd} = \frac{3c}{(3+c)(3-c)} \cdot \frac{(c+2)(c+3)}{18cd} = \frac{c+2}{18d-6cd}.$$

$$13. \frac{3a-6y}{4a+2y} \cdot \frac{8(2a+y)^2}{4a^2-24ay+24y^2} = \frac{3(a-2y)}{2(2a+y)} \cdot \frac{8(2a+y)^2}{4(a^2-6ay+6y^2)} \\ = \frac{6a^2-9ay-6y^2}{a^2-6ay+6y^2}.$$

$$14. \frac{c^2+6ce+9e^2}{6(c+d)^2} \cdot \frac{3c^2e-12d^2e}{c^2-2cd+3ce-6de} = \frac{(c+3e)^2 3e(c+2d)(c-2d)}{6(c+d)^2(c-2d)(c+3e)} \\ = \frac{c^2e+2cde+3ce^2+6de^2}{2(c+d)^2}.$$

$$15. \frac{a^2+4ab+4b^2}{9-a^2} \cdot \frac{a^2-5a+6}{a^2-4b^2} = \frac{(a+2b)^2}{(3+a)(3-a)} \cdot \frac{(a-2)(a-3)}{(a-2b)(a+2b)} \\ = \frac{(a+2b)(a-2)}{(a+3)(2b-a)} \\ = \frac{2a-2ab+4b-a^2}{a^2-2ab+3a-6b}.$$

$$16. \frac{5c^2-20d^2}{c^3+8d^3} \cdot \frac{c^2-2cd+4d^2}{25cd^4} \\ = \frac{5(c+2d)(c-2d)}{(c+2d)(c^2-2cd+4d^2)} \cdot \frac{c^2-2cd+4d^2}{25cd^4} \\ = \frac{c-2d}{5cd^4}.$$

$$\begin{aligned}
 17. \quad & \frac{(x+2)^2}{3x^3+6x^2+12x} \cdot (15-9x^2) \cdot \frac{x^3-8}{4-x^2} \\
 &= \frac{(x+2)^2}{3x(x^2+2x+4)} \cdot \frac{3(5-3x^2)}{1} \cdot \frac{(x-2)(x^2+2x+4)}{(2-x)(2+x)} \\
 &= \frac{3x^3+6x^2-5x-10}{x}.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{8c-24d}{c^2-6cd+9d^2} \cdot \left(4c-\frac{d^2}{c}\right) \cdot \frac{4c^2+d^2}{64c^4-4d^4} \\
 &= \frac{8(c-3d)}{(c-3d)^2} \cdot \frac{(2c+d)(2c-d)}{c} \cdot \frac{4c^2+d^2}{4(4c^2+d^2)(4c^2-d^2)} \\
 &= \frac{2}{c^2-3cd}.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \left(\frac{6m-9}{2m-3}+2m\right)\left(2m-9+\frac{36}{2m+3}\right) = \frac{(2m+3)(2m-3)}{2m-3} \cdot \frac{(2m-3)^2}{(2m+3)} \\
 &= 4m^2-12m+9.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \left(\frac{3}{2a}-1\right)^2 \cdot \frac{8a^2x}{9-4a^2} \cdot \frac{3+2a}{2a-3} = \frac{(3-2a)^2}{4a^2} \cdot \frac{8a^2x}{(3-2a)(3+2a)} \cdot \frac{3+2a}{2a-3} \\
 &= -2x.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \left(1+\frac{2}{x}-\frac{3}{x^2}\right) \cdot \frac{9x^3}{3(x^2-18x+17)} = \frac{(x+3)(x-1)}{x^2} \cdot \frac{9x^3}{3(x-1)(x-17)} \\
 &= \frac{3x^2+9x}{x-17}.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{x^3-8}{6xy} \cdot \left(3-\frac{4}{2-x}\right) \cdot \frac{18x^2y^2}{x^2+2x+4} \\
 &= \frac{(x-2)(x^2+2x+4)}{6xy} \cdot \frac{2-3x}{2-x} \cdot \frac{18x^2y^2}{x^2+2x+4} \\
 &= 9x^2y-6xy.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{a^3-b^3}{(a-b)^3} \cdot \frac{6a+6b}{2a^2+2ab+2b^2} \cdot \frac{(a-b)^2}{a^2-b^2} \\
 &= \frac{(a-b)(a^2+ab+b^2)}{(a-b)^3} \cdot \frac{6(a+b)}{2(a^2+ab+b^2)} \cdot \frac{(a-b)^2}{(a+b)(a-b)} \\
 &= \frac{3}{a-b}.
 \end{aligned}$$

$$24. \quad \left(4-\frac{20-9x}{5-x}-x\right)\left(\frac{1}{x}+\frac{1}{x^2}-\frac{30}{x^3}\right) = \frac{x^2}{5-x} \cdot \frac{(x+6)(x-5)}{x^3} = -\frac{x+6}{x}.$$

$$\begin{aligned}
 25. \quad & \left(x+2y-\frac{5x+10y}{x+y}\right)\left(\frac{y^2+x^2+2yx}{x^2-3xy-4y^2}\right) \cdot \frac{1}{x+2y} \\
 &= \frac{(x+2y)(x+y-5)}{x+y} \cdot \frac{(x+y)^2}{(x-4y)(x+y)} \cdot \frac{1}{(x+2y)} = \frac{x+y-5}{x-4y}.
 \end{aligned}$$

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1. $\frac{6a}{3b} \div \frac{4a}{12b^2} = \frac{6a}{3b} \cdot \frac{12b^2}{4a} = 6b.$
2. $\frac{9a^2}{6b^2} \div \frac{14ab}{8x} = \frac{9a^2}{6b^2} \cdot \frac{8x}{14ab} = \frac{6ax}{7b^3}.$
3. $\left(\frac{2ax}{3c}\right)^2 \div \frac{6ax^2}{9c^4} = \left(\frac{2ax}{3c}\right)^2 \cdot \frac{9c^4}{6ax^2} = \frac{2ac^2}{3}.$
4. $\frac{a}{b} \div \frac{c}{d} \div \frac{ad^2}{bc^2} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{bc^2}{ad^2} = \frac{c}{d}.$
5. $2a \cdot \left(\frac{2a^2}{3x}\right)^2 \div \frac{(4a^2)^2}{18x^2} = 2a \cdot \frac{(2a^2)^2}{(3x)^2} \cdot \frac{18x^2}{(4a^2)^2} = a.$
6. $\frac{5x}{5y} \div \frac{10x^2}{12y^3} \div \frac{4ax}{3by^2} = \frac{5x}{5y} \cdot \frac{12y^3}{10x^2} \cdot \frac{3by^2}{4ax} = \frac{9by^4}{10ax^2}.$
7. $\frac{6}{(2x)^2} \div \frac{12}{(4x^2)^3} \cdot \left(\frac{2}{x}\right)^2 = \frac{6}{(2x)^2} \cdot \frac{(4x^2)^3}{12} \cdot \frac{2^2}{x^2} = 32x^2.$
8. $\frac{3m^2}{5n} \div \frac{21m^3}{10mn^2} \div 14m^4n = \frac{3m^2}{5n} \cdot \frac{10mn^2}{21m^3} \cdot \frac{1}{14m^4n} = \frac{1}{49m^4}.$
9. $\left(\frac{4x^2}{3a}\right)^2 \div \left(\frac{2x}{3a}\right)^3 \cdot \left(\frac{a}{x}\right)^2 = \frac{(4x^2)^2}{(3a)^2} \cdot \frac{(3a)^3}{(2x)^3} \cdot \frac{a^2}{x^2} = \frac{6a^3}{x}.$
10. $\frac{6a-3}{5x} \div \frac{2a-1}{15bx^3} = \frac{3(2a-1)}{5x} \cdot \frac{15bx^3}{2a-1} = 9bx^2.$
11. $\frac{4-x^2}{3x^2+x^3} \div \frac{4-4x+x^2}{x^3-x^2-12x} = \frac{(2-x)(2+x)}{x^2(3+x)} \cdot \frac{x(x-4)(x+3)}{(2-x)^2}$
 $= \frac{x^2-2x-8}{2x-x^2}.$
12. $\frac{a^2-7a+12}{a-1} \div \frac{a^2-16}{1-a^2} = \frac{(a-3)(a-4)}{a-1} \cdot \frac{(1-a)(1+a)}{(a+4)(a-4)}$
 $= -\frac{a^2-2a-3}{a+4}.$
13. $(-14-5c+c^2) \div \frac{ac^2-49a}{bc^2+9bc+14b} = \frac{(c-7)(c+2)}{1} \cdot \frac{b(c+2)(c+7)}{a(c+7)(c-7)}$
 $= \frac{bc^2+4bc+4b}{a}.$
14. $\frac{4a^2-4ab-3b^2}{8a^3x} \div \left(a - \frac{9b^2}{4a}\right) = \frac{(2a-3b)(2a+b)}{8a^3x} \cdot \frac{4a}{(2a-3b)(2a+3b)}$
 $= \frac{2a+b}{4a^3x+6a^2bx}.$
15. $\left(\frac{2a}{x}\right)^4 \div \left(\frac{-6a^2c}{5x}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \left(\frac{2a}{x}\right)^4 \cdot \left(\frac{5x}{-6a^2c}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \frac{-5c^3}{6a^4x^3}.$

$$16. \left(\frac{-a^2}{x}\right) \div \left(\frac{-a}{x^2}\right) \cdot \left(\frac{a}{x}\right)^3 \cdot \left(\frac{-1}{x^2}\right)^3 = \frac{-a^2}{x} \cdot \left(\frac{x^2}{-a}\right)^5 \cdot \frac{a^3}{x^3} \cdot \frac{-1}{x^6} = -1.$$

$$17. \left(\frac{m^2}{n^2} - \frac{n^2}{m^2}\right) \div \left(\frac{m^4 + 2m^2n^2 + n^4}{m^2n - 4mn^2}\right) = \frac{m^4 - n^4}{m^2n^2} \cdot \frac{mn(m-4n)}{(m^2 + n^2)^2}$$

$$= \frac{(m^2 - n^2)(m - 4n)}{mn(m^2 + n^2)}$$

$$= \frac{m^3 - 4m^2n - mn^2 + 4n^3}{m^3n + mn^3}.$$

$$18. \frac{9x^2 + 6xy - 8y^2}{2x + y} \div (3x - 2y)^2 \div \left(2 - \frac{x - 2y}{2x + y}\right)$$

$$= \frac{(3x + 4y)(3x - 2y)}{2x + y} \cdot \frac{1}{(3x - 2y)^2} \cdot \frac{2x + y}{3x + 4y}$$

$$= \frac{1}{3x - 2y}.$$

$$19. \left(\frac{a}{c} + \frac{c}{a}\right) \div \left(\frac{a^6 + c^6}{a^3c^3}\right) \left(a^2 - c^2 + \frac{c^4}{a^2}\right) \cdot \frac{1}{ac^2}$$

$$= \frac{a^2 + c^2}{ac} \cdot \frac{a^3c^3}{a^6 + c^6} \cdot \frac{a^4 - a^2c^2 + c^4}{a^2} \cdot \frac{1}{ac^2} = \frac{1}{a}.$$

$$20. \frac{2a - 5}{2a^2 + 2} \div \frac{4a^4 - 25a^2}{4 - 4a^4} \cdot \left(3a^2 - \frac{a^3 - 8a^2}{a - 1}\right)$$

$$= \frac{2a - 5}{2(a^2 + 1)} \cdot \frac{4(1 - a^4)}{a^2(2a + 5)(2a - 5)} \cdot \frac{a^2(2a + 5)}{a - 1}$$

$$= -2 - 2a.$$

$$21. \frac{8m^3 - 125n^3}{m^2 + mn} \div \left(\frac{2m}{5n} - \frac{5n}{2m}\right) \div \left[3mn \left(2m + 5n + \frac{25n^2}{2m}\right)\right]$$

$$= \frac{8m^3 - 125n^3}{m(m + n)} \cdot \frac{10mn}{(2m + 5n)(2m - 5n)} \cdot \frac{2m}{3mn(4m^2 + 10mn + 25n^2)}$$

$$= \frac{20}{6m^2 + 21mn + 15n^2}.$$

$$22. \left(\frac{4y}{x} - \frac{15y^2}{x^2} + 4\right) \div \left(4 - \frac{16y}{x} + \frac{15y^2}{x^2}\right) \left(3 - \frac{4x + 20y}{2x + 5y}\right)$$

$$= \frac{(2x + 5y)(2x - 3y)}{x^2} \cdot \frac{x^2}{(2x - 3y)(2x - 5y)} \cdot \frac{2x - 5y}{2x + 5y}$$

$$= 1.$$

$$23. \left(\frac{9c^2 - 4d^2}{6c^2}\right) \div \left(d + 4c + \frac{15c^2}{4d}\right) \cdot \frac{15c^2d + 6cd^2}{9c^2d + 24cd^2 - 20d^3}$$

$$= \frac{(3c + 2d)(3c - 2d)}{6c^2} \cdot \frac{4d}{(2d + 5c)(2d + 3c)} \cdot \frac{3cd(5c + 2d)}{d(3c + 10d)(3c - 2d)}$$

$$= \frac{2d}{3c^2 + 10cd}.$$

$$\begin{aligned}
 24. \quad & \left(6x - 11 - \frac{7}{x}\right) \div \left(2 + \frac{11}{x} + \frac{5}{x^2}\right) \div \left(\frac{1}{3x^3 - 75x}\right) \\
 &= \frac{(3x - 7)(2x + 1)}{x} \cdot \frac{x^2}{(x + 5)(2x + 1)} \cdot \frac{3x(x + 5)(x - 5)}{1} \\
 &= 9x^4 - 66x^3 + 105x^2.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \left(x^2 - y^2 + \frac{4xy(y + x)}{x - y}\right) \div \frac{x^2 + y(y + 2x)}{2x^2 - 3xy + y^2} \\
 &= \frac{(x + y)^3}{x - y} \cdot \frac{(x - y)(2x - y)}{(x + y)^2} \\
 &= 2x^2 + xy - y^2.
 \end{aligned}$$

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$$1. \quad \frac{4 - \frac{1}{4}}{2 + \frac{1}{2}} = \frac{\frac{15}{4}}{\frac{5}{2}} = \frac{15}{4} \cdot \frac{2}{5} = \frac{3}{2}.$$

$$2. \quad \frac{\frac{4}{9} + 1}{\frac{7}{2} - 2} = \frac{\frac{13}{9}}{\frac{3}{2}} = \frac{13}{9} \cdot \frac{2}{3} = \frac{26}{27}.$$

$$3. \quad \frac{9 - (\frac{1}{5})^2}{4 - \frac{4}{5}} = \frac{\frac{224}{25}}{\frac{16}{5}} = \frac{224}{25} \cdot \frac{5}{16} = \frac{14}{5}.$$

$$4. \quad \frac{\frac{3}{5} + \frac{4}{7}}{2 - \frac{3}{5} \cdot \frac{4}{7}} = \frac{\frac{41}{35}}{\frac{58}{35}} = \frac{41}{35} \cdot \frac{35}{58} = \frac{41}{58}.$$

$$5. \quad \frac{(\frac{8}{5})^2 - 2}{3 \cdot \frac{8}{5}} = \frac{\frac{14}{25}}{\frac{24}{5}} = \frac{14}{25} \cdot \frac{5}{24} = \frac{7}{60}.$$

$$6. \quad \frac{4 - \frac{3}{2} + \frac{2}{3}}{\frac{3}{3} - \frac{2}{3} + \frac{3}{2}} = \frac{\frac{19}{6}}{\frac{23}{6}} = \frac{19}{23}.$$

$$7. \quad \frac{\frac{2^3}{27} + 1}{\frac{9}{2} - 1} = \frac{\frac{35}{27}}{\frac{7}{2}} = \frac{35}{27} \cdot \frac{2}{7} = \frac{10}{27}.$$

$$8. \quad \frac{2\frac{1}{2} - 3\frac{1}{3} + 4\frac{3}{4}}{2\frac{1}{2} \cdot 3\frac{1}{3} - 4\frac{3}{4}} = \frac{\frac{47}{12}}{\frac{43}{12}} = \frac{47}{43}.$$

$$9. \quad \frac{2 - \frac{9}{2}}{(\frac{1}{2})^2 - (\frac{1}{2})^3 - 12(\frac{1}{2})^4} = \frac{-\frac{5}{2}}{-\frac{5}{8}} = 4.$$

$$10. \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

$$11. \quad \frac{\frac{1+x}{x}}{1 - \frac{1}{x^2}} = \frac{1+x}{x} \cdot \frac{x^2}{x^2 - 1} = \frac{x}{x - 1}.$$

$$12. \quad \frac{c + \frac{c}{d}}{1 + \frac{1}{d}} = \frac{\frac{cd + c}{d}}{\frac{d + 1}{d}} = \frac{cd + c}{d} \cdot \frac{d}{d + 1} = c.$$

$$13. \quad \frac{9 - \frac{b^2}{4a^2}}{1 - \frac{b}{6a}} = \frac{\frac{36a^2 - b^2}{4a^2}}{\frac{6a - b}{6a}} = \frac{36a^2 - b^2}{4a^2} \cdot \frac{6a}{6a - b} = \frac{18a + 3b}{2a}.$$

$$14. \quad \frac{\frac{a^2}{b^2} - \frac{b^2}{a^2}}{a + \frac{1}{a}} = \frac{\frac{a^4 - b^4}{a^2b^2}}{\frac{a^2 + 1}{a}} = \frac{a^4 - b^4}{a^2b^2} \cdot \frac{a}{a^2 + 1} = \frac{a^2 - b^2}{ab^2}.$$

$$15. \frac{a - \frac{9}{a}}{\frac{1}{a^2} - \frac{1}{a^3} - \frac{12}{a^4}} = \frac{a^2 - 9}{a} \cdot \frac{a^4}{(a+3)(a-4)} = \frac{a^4 - 3a^3}{a-4}.$$

$$16. \frac{\frac{m^3}{n^3} + 1}{m + \frac{n^2}{m} - n} = \frac{m^3 + n^3}{n^3} \cdot \frac{m}{m^2 - mn + n^2} = \frac{m^2 + mn}{n^3}.$$

$$17. \frac{2 + \frac{1}{a-2} + a}{a + \frac{1}{a+2} - 2} = \frac{a^2 - 3}{a-2} \cdot \frac{a+2}{a^2 - 3} = \frac{a+2}{a-2}.$$

$$18. \frac{\frac{a}{\frac{b}{c}} - \frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \cdot \frac{1}{c} - \frac{a}{1} \cdot \frac{c}{b} = \frac{a - ac^2}{bc}.$$

$$19. \frac{1 + \frac{b}{a} - \frac{20a}{b}}{\frac{b}{a} - \frac{8a}{b} - 2} = \frac{(b+5a)(b-4a)}{ab} \cdot \frac{ab}{(b-4a)(b+2a)} = \frac{b+5a}{b+2a}.$$

$$20. \frac{\frac{16}{x} - x}{\frac{24}{x^4} + \frac{10}{x^3} + \frac{1}{x^2}} = \frac{16 - x^2}{x} \cdot \frac{x^4}{x^2 + 10x + 24} = \frac{x^3(4-x)}{6+x}.$$

$$21. \frac{x + \frac{x^2 + y^2}{y}}{\frac{x}{y} - 1} = \frac{x^2 + xy + y^2}{y} \cdot \frac{y}{x-y} = \frac{x^2 + xy + y^2}{x-y}.$$

$$22. \frac{2 + \frac{a}{b}}{\frac{(a-2b)^2}{4ab} + 2} = \frac{2b+a}{b} \cdot \frac{4ab}{a^2 + 4ab + 4b^2} = \frac{4a}{a+2b}.$$

$$23. \frac{\left(\frac{2x^2}{x-y}\right)^2}{8x^3} = \frac{4x^4}{(x-y)^2} \cdot \frac{(x-y)^3}{8x^3} = \frac{x(x-y)}{2}.$$

$$24. \frac{\frac{a-b}{a} - \frac{a+b}{b}}{\frac{a-b}{b} + \frac{a+b}{a}} = \frac{-(a^2 + b^2)}{ab} \cdot \frac{ab}{a^2 + b^2} = -1.$$

$$\begin{aligned}
 25. \quad 1 - \frac{\left(1 - \frac{c^4}{9}\right) - \left(1 - \frac{c^4}{16}\right)}{1 - \frac{7c^4}{144}} &= 1 - \frac{\frac{-7c^4}{144}}{\frac{144 - 7c^4}{144}} \\
 &= 1 + \frac{7c^4}{144 - 7c^4} \\
 &= \frac{144}{144 - 7c^4}.
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \frac{\frac{a}{1+a} + \frac{1-a}{a}}{1+a - \frac{1-a}{a}} &= \frac{\frac{1}{a(a+1)}}{\frac{a(a+1)}{2a^2-1}} = \frac{1}{2a^2-1}.
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \frac{\frac{9x}{8} - \frac{9y}{8} - \frac{7y^2}{2x}}{\frac{3x}{4} + \frac{9y}{4} + \frac{5y^2}{3x}} &= \frac{\frac{9x^2 - 9xy - 28y^2}{8x}}{\frac{9x^2 + 27xy + 20y^2}{12x}} \\
 &= \frac{(3x - 7y)(3x + 4y)}{8x} \cdot \frac{12x}{(3x + 5y)(3x + 4y)} \\
 &= \frac{9x - 21y}{6x + 10y} \cdot \frac{x}{x}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad \frac{\frac{2ax + 3bx + 6ab + x^2}{1} - \frac{1}{x+3b}}{\frac{1}{x+2a} - \frac{1}{x+3b}} &= \frac{x}{(x+2a)(x+3b)} \cdot \frac{(x+2a)(x+3b)}{3b-2a} \\
 &= \frac{x}{3b-2a}.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \frac{\left(\frac{3a+4b}{3a}\right)^2 - \frac{6b}{a}}{8a - \frac{(3a+2b)^2}{3b}} &= \frac{(3a-8b)(3a-2b)}{9a^2} \cdot \frac{3b}{(2b-3a)(3a-2b)} \\
 &= \frac{b(3a-8b)}{3a^2(2b-3a)}.
 \end{aligned}$$

$$\begin{aligned}
 30. \quad 1 - \frac{\frac{x^2-1}{5x^2-6x+1}}{1 + \frac{3x+2}{5x-1}} &= \frac{\frac{4x^2-6x+2}{5x^2-6x+1}}{\frac{8x+1}{5x-1}} \\
 &= \frac{2(x-1)(2x-1)}{(5x-1)(x-1)} \cdot \frac{5x-1}{8x+1} \\
 &= \frac{4x-2}{8x+1}.
 \end{aligned}$$

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1. $\frac{x}{2} + \frac{x}{3} = 10.$ (1)
 (1) · 6, $3x + 2x = 60.$
 Whence $x = 12.$
2. $\frac{4}{3}x + \frac{2}{5}x = 5\frac{1}{5}.$ (1)
 (1) · 15, $20x + 6x = 78.$
 Whence $x = 3.$
3. $\frac{x+5}{4} - \frac{2x+4}{9} = 1.$ (1)
 (1) · 36, $9(x+5) - 4(2x+4) = 36.$
 Whence $x = 7.$
4. $\frac{2x+3}{5} - \frac{1}{3}(x-3) = 2.$ (1)
 (1) · 15, $3(2x+3) - 5(x-3) = 30.$
 Whence $x = 6.$
5. $\frac{4x+2}{11} - \frac{1}{5}(x+5) = 0.$ (1)
 (1) · 55, $5(4x+2) - 11(x+5) = 0.$
 Whence $x = 5.$
6. $\frac{3}{4}(x+1) - \frac{5x-7}{6} = \frac{7}{3}.$ (1)
 (1) · 12, $9(x+1) - 2(5x-7) = 28.$
 Whence $x = -5.$
7. $\frac{x+6}{10} + \frac{3}{2}(x+4) = -3.$ (1)
 (1) · 10, $x+6 + 15(x+4) = -30.$
 Whence $x = -6.$
8. $\frac{5x-12}{6} - \frac{4}{11}(2x-7) = \frac{1}{3}.$ (1)
 (1) · 66, $11(5x-12) - 24(2x-7) = 22.$
 Whence $x = -2.$
9. $2x - 1 - \frac{12x-7}{6} - \frac{1}{6} = 0.$ (1)
 (1) · 6, $6(2x-1) - (12x-7) - 1 = 0.$
 Whence $0 = 0$, or any number will satisfy the equation.
10. $\frac{10x-7}{6} + \frac{5}{2}\left(\frac{2}{5} - x\right) = \frac{15x-11}{3}.$ (1)
 (1) · 6, $(10x-7) + 15\left(\frac{2}{5} - x\right) = 2(15x-11).$
 Whence $x = \frac{3}{5}.$
11. $\frac{5x}{6} - \frac{1}{2} - \frac{3}{8}\left(x - \frac{5}{3}\right) + \frac{7}{32} = 0.$ (1)
 (1) · 96, $80x - 48 - 12(3x-5) + 21 = 0.$
 Whence $x = -\frac{3}{4}.$
12. $\frac{5}{x} + \frac{4}{3} = \frac{9}{x}.$ (1)
 (1) · 3x, $15 + 4x = 27.$
 Whence $x = 3.$
13. $\frac{9x}{4} - \frac{3}{4} - \frac{3x-7}{3} + 4 = -\frac{17}{24}.$ (1)
 (1) · 24, $54x - 18 - 8(3x-7) + 96 = -17.$
 Whence $x = -\frac{151}{30}.$

$$14. \quad \frac{c}{3x} - \frac{c}{5x} = \frac{1}{15}.$$

$$\text{Whence} \quad x = 2c.$$

$$15. \quad \frac{1}{2x} - \frac{13}{24} = \frac{8}{3x}.$$

$$\text{Whence} \quad x = -4.$$

$$16. \quad \frac{2a - 3x}{6a} + \frac{5a - 2x}{5a} + \frac{41}{30} = 0.$$

$$\text{Whence} \quad x = 3a.$$

$$17. \quad \frac{x}{a} - \frac{1}{3}(a - 3x) + \frac{19a}{3} = -6.$$

$$\text{Whence} \quad x = -6a.$$

$$18. \quad \frac{cx}{3} - \frac{n}{5}(3x - 5cn) = cn\left(\frac{2c}{3} - \frac{n}{5}\right). \quad (1)$$

$$(1) \cdot 15, \quad 5cx - 9nx + 15cn^2 = 10c^2n - 3cn^2.$$

$$\text{Whence} \quad x(5c - 9n) = 2cn(5c - 9n),$$

$$\text{or} \quad x = 2cn.$$

$$19. \quad 2x - b - \frac{b}{a}(3x - 4b) + 2a = \frac{2(2a^2 - b^2)}{a}.$$

$$\text{Whence} \quad 2ax - 3bx = 2a^2 + ab - 6b^2,$$

$$\text{or} \quad x = a + 2b.$$

$$20. \quad (x + 5)(x - 6) = x(x - \frac{5}{2}).$$

$$\text{Whence} \quad x^2 - x - 30 = x^2 - \frac{5x}{2},$$

$$\text{or} \quad x = 20.$$

$$21. \quad \left(x + \frac{3}{4}\right)\left(3 + \frac{x}{2}\right) = \frac{x}{2}(x - 5) + 8\frac{1}{8}. \quad (1)$$

$$(1) \cdot 8, \quad 4\left(x + \frac{3}{4}\right)2\left(3 + \frac{x}{2}\right) = 4x(x - 5) + 65.$$

$$\text{Whence} \quad x = 1.$$

$$22. \quad (x - \frac{1}{2})(x + \frac{3}{7}) = (x - 1)(x + 2) + 1\frac{1}{4}. \quad (1)$$

$$(1) \cdot 14, \quad 2(x - \frac{1}{2})7(x + \frac{3}{7}) = 14(x^2 + x - 2) + 25.$$

$$\text{Whence} \quad x = 0.$$

$$23. \quad (x - \frac{2}{3})(x + \frac{2}{3}) - (x - \frac{1}{3})^2 - 1\frac{1}{9} = 0. \quad (1)$$

$$(1) \cdot 18, \quad 6(x - \frac{2}{3})3(x + \frac{2}{3}) - 18(x - \frac{1}{3})^2 - 20 = 0.$$

$$\text{Whence} \quad x = -\frac{40}{3}.$$

$$24. \quad (x + \frac{2}{5})^2 - (x - \frac{1}{2})(x + \frac{1}{5}) + \frac{3}{50} = 0. \quad (1)$$

$$(1) \cdot 50, \quad 2 \cdot 25(x + \frac{2}{5})^2 - 10(x - \frac{1}{2})(x + \frac{1}{5}) + 31 = 0.$$

$$\text{Whence} \quad x = -\frac{4}{5}.$$

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1. If $n =$ the number,

then $\frac{n}{4} + \frac{n}{12} = 16.$

Whence $n = 48.$

2. If $n =$ the number,

then $\frac{n}{3} - \frac{n}{17} = 70.$

Whence $n = 255.$

3. If $n =$ the greater number,

then $38 - n =$ the less,

and $\frac{n}{10} = \frac{38 - n}{9}.$

Whence $n = 20,$

and $38 - n = 18.$

4. If $l =$ the length in centimeters,

then $\frac{4l}{5} =$ the width,

and $2l + \frac{8l}{5} = 216.$

Whence $l = 60,$

and $\frac{4l}{5} = 48.$

Therefore the area of the rectangle is 2880 square centimeters.

5. Let $n =$ the number.

Then $\frac{4+n}{7} = \frac{n}{5}.$

Whence $n = 10.$

6. Let $n, n + 1,$ and $n + 2$ represent the numbers.

Then $\frac{3n}{4} = \frac{n + 1 + n + 2}{3}.$

Whence $n = 12.$

7. Let $n, n + 2,$ and $n + 4$ represent the numbers.

Then $\frac{n}{11} = \frac{n + 2 + n + 4}{24}.$

Whence $n = 33,$

$n + 2 = 35,$

and $n + 4 = 37.$

8. If $n =$ the number,

then $\frac{10 + n}{17 + n} = \frac{5}{7}.$

Whence $n = \frac{15}{2}.$

9. Let $g =$ the greater part.

Then $42 - g =$ the less,

and $\frac{g - (42 - g)}{6} = \frac{1}{3}.$

Whence $g = 22,$

and $42 - g = 20.$

10. Let $n =$ the number.

Then $\frac{3n - 4}{4} = \frac{5n - 4}{7}.$

Whence $n = 12.$

11. Let $g =$ the greater part.

Then $112 - g =$ the less,

and $\frac{112 - g}{g} = \frac{2}{9}.$

Whence $g = 91\frac{7}{11},$

and $112 - g = 20\frac{4}{11}.$

12. Let $n =$ one number.

Then $24 - n =$ the other,

and

$\frac{n - (24 - n)}{n + 24 - n} = n - (24 - n) - 3\frac{5}{8}.$

Whence $n = 14,$

and $24 - n = 10.$

13. Let $n =$ the number.

Then $\frac{27 + 7n}{2n} = \frac{90 + 5n}{3n}.$

Whence $n = 9.$

14. Let $x = \text{B's age in years.}$ 15. Let $x = \text{B's age in years.}$
 Then $\frac{5x}{2} = \text{A's age in years,}$ Then $\frac{2x}{3} = \text{A's age in years,}$
 and $\frac{5x}{2} + 10 = 2(x + 10).$ and $\frac{2x}{3} - 14 = \frac{1}{2}(x - 14).$
 Whence $x = 20,$ Whence $x = 42,$
 and $\frac{5x}{2} = 50.$ and $\frac{2x}{3} = 28.$

16. Let $x = \text{B's age in years.}$
 Then $x + 16 = \text{A's age in years,}$
 and $x - 8 = \frac{3}{5}(x + 16 - 8).$
 Whence $x = 32,$
 and $x + 16 = 48.$

17. Let $x = \text{the number of moons of Uranus.}$
 Then $x + 4 = \text{the number of moons of Jupiter,}$
 $2x + 2 = \text{the number of moons of Saturn,}$
 $x - 2 = \text{the number of moons of Mars,}$
 and $\frac{x - 2}{2} = \text{the number of moons of Neptune.}$
 Then $x + x + 4 + 2x + 2 + x - 2 + \frac{x - 2}{2} = 25.$
 Whence $x = 4,$
 $x + 4 = 8, \quad 2x + 2 = 10, \quad x - 2 = 2,$
 and $\frac{x - 2}{2} = 1.$

18. Let $x = \text{the other base of the trapezoid in meters.}$
 Then $\frac{8 \cdot 30}{2} = \frac{(8 + x) 10}{2}.$
 Whence $x = 16.$

19. Let $d = \text{the distance to the target in feet.}$
 Then $\frac{d}{1100} + \frac{d}{1925} = 3.$
 Whence $d = 2100,$
 and $\frac{2100}{1925} = \frac{12}{11}.$
 Therefore $\frac{12}{11}$ seconds is the time the bullet was in the air.

20. Let $v = \text{the velocity of the bullet in feet per second.}$
 Then $\frac{3 \cdot 2640}{v} + \frac{3 \cdot 2640}{1100} = 9\frac{2}{5}.$
 Whence $v = 3600.$

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1. $\frac{32}{x} = 5.$

Then $32 = 5x,$

or $x = \frac{32}{5}.$

2. $\frac{25}{3x} = 5.$

Then $5 = 3x,$

or $x = \frac{5}{3}.$

3. $5x + \frac{x-2}{4} = 4x + 7.$

Then $20x + x - 2 = 16x + 28,$

or $x = 6.$

4. $3x - \frac{x-1}{4} - \frac{8x}{5} = 6.$

Then $60x - 5x + 5 - 32x = 120,$

or $x = 5.$

5. $\frac{4}{x} - \frac{12+x}{3x} = \frac{4}{3}.$

Then $x = 0,$

or the equation is "impossible," for division by zero is excluded.

6. $\frac{5}{6} - \frac{3x-5}{4x} + \frac{x-2}{3x} = 0.$

Then

$10x - 9x + 15 + 4x - 8 = 0,$

or $x = -\frac{7}{5}.$

13.

$\frac{5}{7x+5} - \frac{1}{8} = 0,$

or

$40 - 7x - 5 = 0.$

Then

$x = 5.$

14.

$\frac{3x^2 - 7x - 4}{4x^2 - 10x - 8} = \frac{3}{4}.$

Then

$6x^2 - 14x - 8 = 6x^2 - 15x - 12,$

or

$x = -4.$

15.

$\frac{1}{x-2} = \frac{3}{x-3},$

or

$x - 3 = 3(x - 2).$

Then

$x = \frac{3}{2}.$

7. $\frac{x+5}{5x} - \frac{3(x+1)}{x} = 3\frac{1}{5}.$

Then

$x + 5 - 15x - 15 = 16x,$

or

$x = -\frac{1}{3}.$

8. $5x - 4x\left(3 - \frac{2}{x}\right) + \frac{1}{4} = 3.$

Then

$20x - 48x + 32 + 1 = 12,$

or

$x = \frac{3}{4}.$

9. $x - \frac{3x}{5}\left(\frac{10}{x} - 4\right) + 3\frac{3}{5} = 18,$

or

$5x - 30 + 12x + 18 = 90.$

Then

$x = 6.$

10. $\frac{8x-7}{4x} + 3\frac{1}{2} = \frac{\frac{3}{2}-x}{x}.$

Then $8x - 7 + 14x = 6 - 4x,$

or

$x = \frac{1}{2}.$

11. $\frac{2x-3}{3+2x} = 4,$

or

$2x - 3 = 12 + 8x.$

Then

$x = -\frac{5}{2}.$

12. $\frac{x-2}{x-3} = \frac{15}{16}.$

Then

$16(x-2) = 15(x-3),$

or

$x = -13.$

16.
$$\frac{1}{x-3} = \frac{x-6}{x+3} + \frac{x}{x+3}.$$

Then
$$x+3 = x^2 - 9x + 18 + x^2 - 3x.$$

Whence
$$x = \frac{3}{2} \text{ or } 5.$$

17.
$$\frac{x-3}{x+4} = \frac{x-9}{x+5},$$

or
$$(x-3)(x+5) = (x+4)(x-9).$$

Then
$$x = -3.$$

18.
$$\frac{x-2}{x+2} + \frac{4}{x+2} + 2 = 0,$$

or
$$x-2+4+2x+4=0.$$

Then
$$x = -2.$$

Since $x = -2$ the equation is "impossible," for division by zero is excluded.

19.
$$\frac{3x}{4} - \frac{2}{x-2} = \frac{3x-2}{4},$$

or
$$3x(x-2) - 8 = (3x-2)(x-2).$$

Then
$$x = 6.$$

20.
$$\frac{4}{x-3} + \frac{3x+4}{6} = \frac{x}{2},$$

or
$$24 + (3x+4)(x-3) = 3x(x-3).$$

Then
$$x = -3.$$

21.
$$\frac{2x+3}{x-5} + 4 = \frac{x+8}{x-5},$$

or
$$2x+3+4(x-5) = x+8.$$

Then
$$x = 5.$$

Since $x = 5$, the equation is "impossible," for division by zero is excluded.

22.
$$\frac{x}{4} - \frac{5}{4x-12} = \frac{2x+\frac{5}{4}}{3},$$

or
$$3x(x-3) - 15 = 4(x-3)(2x+\frac{5}{4}).$$

Then
$$5x^2 - 10x = 0,$$

and
$$x = 2 \text{ or } 0.$$

23.
$$\frac{x-4}{x+5} + \frac{7}{5} = \frac{3}{x+5},$$

or
$$5(x-4) + 7(x+5) = 15.$$

Then
$$x = 0.$$

$$24. \quad \frac{x+4}{15x-5} - \frac{x}{5} = \frac{2x+\frac{20}{3}}{10}.$$

Then $36x^2 + 42x - 44 = 0,$

or $(3x-2)(6x+11) = 0.$

Therefore $x = \frac{2}{3}$ or $-\frac{11}{6}.$

$$25. \quad \frac{4}{3x+6} + \frac{5}{7x+14} + 43 = 0,$$

or $28 + 15 + 43 \cdot 21 \cdot (x+2) = 0.$

Then $x = -\frac{43}{21}.$

$$26. \quad \frac{7}{4x-12} + \frac{47}{220} = \frac{-3}{5x-15}, \quad 27. \quad \frac{4x}{x+3} - \frac{6}{2x+6} = \frac{10x+11}{3x+9},$$

or $7(55) + 47(x-3) = -132.$ or $24x - 18 = (10x+11)2.$

Then $x = -8.$ Then $x = 10.$

$$28. \quad \frac{3}{x-2} = \frac{5}{x^2-25} + \frac{3x}{x^2-25},$$

or $3(x^2-25) = 5(x-2) + 3x(x-2).$

Then $x = 65.$

$$29. \quad \frac{1}{x-3} + \frac{2}{x+3} = \frac{-3}{x^2-9},$$

or $x+3+2(x-3) = -3.$

Then $x = 0.$

$$30. \quad \frac{x+2}{x-2} = \frac{10-x^2}{4-x^2} - \frac{10}{x^2-4},$$

or $\frac{x+2}{x-2} = \frac{x^2-20}{x^2-4},$

or $(x+2)(x+2) = x^2-20.$

Then $x = -6.$

$$31. \quad \frac{x-4}{x-5} + \frac{x-15}{x+4} = \frac{2x^2-10x-1}{x^2-x-20},$$

or $(x-4)(x+4) + (x-15)(x-5)$
 $= 2x^2 - 10x - 1.$

Then $x = 6.$

$$32. \quad \frac{x+2}{x+3} + \frac{x+3}{x+2} = \frac{4x+9}{x^2+5x+6},$$

or $(x+2)^2 + (x+3)^2 = 4x+9,$

or $(x+2)(x+1) = 0.$

Then $x = -1,$

for -2 does not satisfy, since division by zero is excluded.

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$$\begin{array}{ll} 1. & .3x + 4 = .25. \\ [(1) \cdot 100] \div 30, & x = -\frac{25}{2}. \end{array} \quad (1)$$

$$\begin{array}{ll} 2. & .15x - .4x = 235x - 2352.5. \\ [(1) \cdot 100, \text{ collected}] \div -23,525, & x = 10. \end{array} \quad (1)$$

$$\begin{array}{ll} 3. & 1.3x + 8.24 = -5.26 - 3.2x. \\ [(1) \cdot 100, \text{ collected}] \div 450, & x = -3. \end{array} \quad (1)$$

$$\begin{array}{ll} 4. & 3x - 1.245x + .6x = 1.5 + .355x. \\ [(1) \cdot 1000, \text{ collected}] \div 2000, & x = .75. \end{array} \quad (1)$$

$$\begin{array}{ll} 5. & 3.5x + .0564 - .1x = 4.9128 - .02x. \\ [(1) \cdot 10,000, \text{ collected}] \div 34,200, & x = 1.42. \end{array} \quad (1)$$

$$\begin{array}{ll} 6. & .12(2x + .05) - .15(1.5x - 2) = 0.246. \\ & .24x + .006 - .225x + .30 = .246. \end{array} \quad \begin{array}{l} (1) \\ (2) \end{array}$$

$$[(2) \cdot 1000, \text{ collected}] \div 15, \quad x = -4.$$

$$\begin{array}{ll} 7. & \frac{.01x + .003}{6} + \frac{.02x + .0008}{7} = .0017. \end{array} \quad (1)$$

$$(1) \cdot 10,000, \quad \frac{100x + 30}{6} + \frac{200x + 8}{7} = 17. \quad (2)$$

$$[(2) \cdot 42] \div 1900, \quad x = .24.$$

$$\begin{array}{ll} 8. & \frac{.3(x + 5)}{8} - \frac{4(.25x - .35)}{7} = \frac{14.325}{56}. \end{array} \quad (1)$$

$$(1) \cdot 1000, \quad \frac{300(x + 5)}{8} - \frac{4(250x - 350)}{7} = \frac{14325}{56}. \quad (2)$$

$$[(2) \div 56] \div -5900, \quad x = \frac{5}{4}.$$

$$\begin{array}{ll} 9. & \frac{0.5(6 - .2x)}{.80} - \frac{.3(.4x - 3)}{.16} = 5. \end{array} \quad (1)$$

100 (numerator and denominator of each fraction),

$$\frac{5(60 - 2x)}{80} - \frac{3(4x - 30)}{16} = 5. \quad (2)$$

$$(2) \cdot 80 \text{ and solved gives } x = 5.$$

$$\begin{array}{ll} 10. & \frac{.32x}{.05} + \frac{.045x}{.125} = 13.52. \end{array} \quad (1)$$

(First numerator and denominator) $\cdot 100$, and (second numerator and denominator) $\cdot 1000$,

$$\frac{32x}{5} + \frac{45x}{125} = 13.52. \quad (2)$$

$$[(2) \cdot (125)] \div 845, \quad x = 2.$$

$$11. \quad \frac{33}{x+5} + \frac{3.75}{.5(x-8.5)} = 0. \quad (1)$$

(Numerator and denominator of second fraction) $\cdot 100$,

$$\frac{33}{x+5} + \frac{375}{5(10x-85)} = 0. \quad (2)$$

$$[(2) \cdot (x+5)(10x-85) \cdot 5] \div 2025, \quad x = 6.$$

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$$1. \quad 5cx - 8c^2 = 4c^2 - cx.$$

$$6cx = 12c^2.$$

$$\text{Therefore} \quad x = 2c.$$

$$2. \quad 2(x+1) - 4k = 2.$$

$$2x = 4k.$$

$$\text{Therefore} \quad x = 2k.$$

$$3. \quad 3(2x - a) = 2(x - 2a).$$

$$6x - 3a = 2x - 4a.$$

$$\text{Therefore} \quad x = -\frac{a}{4}.$$

$$4. \quad ax + bx = a^2 + ab.$$

$$x(a+b) = a(a+b).$$

$$\text{Therefore} \quad x = a.$$

$$5. \quad cx + b^2 = bx + bc.$$

$$x(c-b) = b(c-b).$$

$$\text{Therefore} \quad x = b.$$

$$6. \quad mx + n^2 = m^2 - nx.$$

$$x(m+n) = (m^2 - n^2).$$

$$\text{Therefore} \quad x = m - n.$$

$$7. \quad 6ac + cx + 4a^2 = 2ax + 3c^2 + 2ca.$$

$$x(c-2a) = 3c(c-2a) + 2a(c-2a).$$

$$\text{Therefore} \quad x = 3c + 2a.$$

$$8. \quad 5ax - 5a^2 + 6b^2 = 7ab + 3bx.$$

$$x(5a-3b) = (5a-3b)(a+2b).$$

$$\text{Therefore} \quad x = a + 2b.$$

$$9. \quad \frac{x}{2a} = b.$$

$$\text{Therefore} \quad x = 2ab.$$

$$13. \quad \frac{x}{a} + \frac{x}{b} = a + b.$$

$$x(a+b) = ab(a+b).$$

$$\text{Therefore} \quad x = ab.$$

$$10. \quad \frac{3ab}{x} = a.$$

$$\text{Therefore} \quad x = 3b.$$

$$11. \quad \frac{a}{x} + \frac{3a}{2x} = \frac{5}{4}.$$

$$4a + 6a = 5x.$$

$$\text{Therefore} \quad x = 2a.$$

$$12. \quad \frac{4a}{3x} + \frac{4a}{x} = \frac{3}{2} + \frac{5a}{6x}.$$

$$8a + 24a = 9x + 5a.$$

$$\text{Therefore} \quad x = 3a.$$

$$14. \quad \frac{c^2}{x} - c = \frac{d^2}{x} + d.$$

$$-x(c+d) = d^2 - c^2.$$

$$\text{Therefore} \quad x = c - d.$$

$$15. \quad \frac{ax}{2b} - 4b^2 = \frac{2bx}{a} - a^2.$$

$$x(a^2 - 4b^2) = -2ab(a^2 - 4b^2).$$

$$\text{Therefore} \quad x = -2ab.$$

$$16. \quad \frac{x}{c} + \frac{c - 2x}{3} - 3c = -4.$$

$$x(3 - 2c) = -4c(3 - 2c).$$

Therefore

$$x = -4c.$$

$$17. \quad \frac{x}{a} + \frac{x}{c} + ac = bc + ab + \frac{x}{b}.$$

$$x(bc + ab - ac) = abc(bc + ab - ac).$$

Therefore

$$x = abc.$$

$$18. \quad \frac{3b + 4x}{5b} + \frac{3b + 2x}{4b} = \frac{1}{20}.$$

$$26x = b - 27b.$$

Therefore

$$x = -b.$$

$$19. \quad \frac{ax}{2} - \frac{3b}{5} \left(x - \frac{2ab}{3} \right) = ab \left(\frac{a}{2} - \frac{b}{5} \right).$$

$$5ax - 6bx = 5a^2b - 6ab^2.$$

Therefore

$$x = ab.$$

$$20. \quad \frac{2x - 3b}{a} + \frac{2}{b} \left(\frac{3x}{2} - a \right) + 5 + \frac{9b}{a} = \frac{4a}{b}.$$

$$x(3a + 2b) = 6a^2 - 5ab - 6b^2.$$

Therefore

$$x = 2a - 3b.$$

$$21. \quad \frac{x - m^2}{x - n^2} = \frac{n}{m}.$$

$$x(m - n) = m^3 - n^3.$$

Therefore

$$x = m^2 + mn + n^2.$$

$$22. \quad \frac{x}{c} - \frac{d}{c} = \frac{x + 2c}{d} - 3.$$

$$x(d - c) = d^2 - 3cd + 2c^2.$$

Therefore

$$x = d - 2c.$$

$$23. \quad \frac{b}{b(b - x)} + \frac{3}{a(b - x)} + \frac{3a + 9}{2ab} = 0.$$

$$2ab + 6b + 3ab + 9b - 3ax - 9x = 0.$$

Therefore

$$x = \frac{5b}{3}.$$

$$24. \quad \frac{c}{a(x + c)} + \frac{a}{c(x - a)} = \frac{c^2 - ac + 2a^2}{2ac(x - a)}.$$

$$2c^2x - 2ac^2 + 2a^2x + 2a^2c = c^2x - acx + 2a^2x + c^3 - ac^2 + 2a^2c.$$

$$x(c + a) = c^2 + ac.$$

Therefore

$$x = c.$$

$$25. \quad \frac{1}{ab} + 1 - \frac{ab}{x} + \frac{1}{abx} = 0.$$

$$x + abx - a^2b^2 + 1 = 0.$$

Therefore

$$x = ab - 1.$$

$$26. \quad \frac{a^2}{bx} + \frac{b^2}{ax} = \frac{1}{a} + \frac{1}{b}.$$

$$a^3 + b^3 = x(a + b).$$

Therefore

$$x = a^2 - ab + b^2.$$

$$27. \quad \frac{c^2}{dx} - \frac{d^2}{cx} - \frac{3c - 3d}{x} = \frac{c - d}{cd}.$$

$$x(c - d) = c^3 - 3c^2d + 3cd^2 + d^3.$$

Therefore

$$x = c^2 - 2cd + d^2.$$

$$28. \quad \frac{a + x}{b + x} - \frac{a - x}{b - x} = \frac{2}{x^2 - b^2}.$$

$$(a + x)(b - x) - (a - x)(b + x) = -2.$$

Whence

$$x = \frac{1}{a - b}.$$

$$29. \quad \frac{\frac{x}{a - b} - a}{\frac{x}{a - b} + a} + 1 = \frac{2}{a^2 + ab + 1}.$$

$$\frac{x - a^2 + ab}{x + a^2 - ab} + 1 = \frac{2}{a^2 + ab + 1}.$$

$$2a^2 + 2abx + 2x = 2x + 2a^2 - 2ab,$$

or

$$ax + bx = a - b.$$

Therefore

$$x = \frac{a - b}{a + b}.$$

$$30. \quad \frac{\frac{a}{2} + \frac{x}{3}}{\frac{a}{3} - \frac{x}{2}} + \frac{7a}{4a - 6x} = -7.$$

$$\frac{3a + 2x}{2a - 3x} + \frac{7a}{2(2a - 3x)} = -7,$$

or

$$(3a + 2x)2 + 7a = -14(2a - 3x).$$

Therefore

$$x = \frac{41a}{38}.$$

$$31. \quad \frac{a^2 + ac}{x + 3c} + \frac{2cx(a + c)}{x^2 + 5cx + 6c^2} = \frac{a^2 + 2ac + c^2}{x + 2c}.$$

$$(a^2 + ac)(x + 2c) + 2acx + 2c^2x = (a^2 + 2ac + c^2)(x + 3c).$$

Therefore

$$x = a + 3c.$$

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1. Solve for R , $K = 2\pi RH$.

$$R = \frac{K}{2\pi H}.$$

2. Solve for a , $A = \frac{ab}{2}$.

$$a = \frac{2A}{b}.$$

3. Solve for R , $C = 2\pi R$.

$$R = \frac{C}{2\pi}.$$

4. Solve for r and t , $d = rt$.

$$r = \frac{d}{t}, \quad t = \frac{d}{r}.$$

5. Solve for a and A , $\frac{a}{A} = \frac{D}{360}$.

$$a = \frac{AD}{360}, \quad A = \frac{360a}{D}.$$

6. Solve for C , $\frac{D}{360} = \frac{l}{C}$.

$$C = \frac{360l}{D}.$$

7. Solve for r , $C = \frac{E}{R+r}$.

$$r = \frac{E - CR}{C}.$$

8. Solve for r and n , $C = \frac{E}{R+nr}$.

$$r = \frac{E - CR}{Cn}, \quad n = \frac{E - CR}{Cr}.$$

9. Solve for r and n , $C = \frac{n \cdot e}{R+nr}$.

$$r = \frac{n \cdot e - CR}{Cn},$$

$$n = \frac{CR}{e - Cr}.$$

10. Solve for F , $C = \frac{5}{9}(F - 32)$.

$$F = \frac{9C}{5} + 32.$$

11. Solve for W_2 , $\frac{W_1}{W_2} = \frac{L_1}{L_2}$.

$$W_2 = \frac{W_1 L_2}{L_1}.$$

12. Solve for r and t , $A = P(1 + rt)$.

$$r = \frac{A - P}{Pt}, \quad t = \frac{A - P}{Pr}.$$

13. Solve for P_2 , $\frac{V_1}{V_2} = \frac{P_2}{P_1}$.

$$P_2 = \frac{V_1 P_1}{V_2}.$$

14. Solve for n and l , $s = \frac{n(a+l)}{2}$.

$$n = \frac{2s}{a+l}, \quad l = \frac{2s - an}{n}.$$

15. Solve for a , l , and r , $s = \frac{rl - a}{r - 1}$.

$$a = rl + s - rs,$$

$$l = \frac{a + rs - s}{r},$$

$$r = \frac{s - a}{s - l}.$$

16. Solve for θ , $\frac{D}{180} = \frac{\theta}{\pi}$.

$$\theta = \frac{\pi D}{180}.$$

17. Solve for t_1 ,

$$V_1 = V_0(1 + .00365 t_1).$$

$$t_1 = \frac{V_1 - V_0}{.00365 V_0}.$$

18. Solve for b_2 , $A = \frac{(b_1 + b_2)a}{2}$.

$$b_2 = \frac{2A - ab_1}{a}.$$

19. Solve for x , $\frac{a}{b} = \frac{x}{c - x}$.

$$x = \frac{ac}{a + b}.$$

20. Solve for F , D_1 , and D_0 , $\frac{1}{F} = \frac{1}{D_1} + \frac{1}{D_0}$.

$$F = \frac{D_1 D_0}{D_1 + D_0}, \quad D_1 = \frac{F D_0}{D_0 - F}, \quad D_0 = \frac{F D_1}{D_1 - F}.$$

21. $C(t_m - t_1)H_c + W_c(t_m - t_1) = W_h(t_2 - t_m)$.

Find the value of t_m in the preceding equation, when $C = 80$, $t_1 = 20$, $t_2 = 99$, $H_c = .09$, $W_c = 1000$, and $W_h = 800$.

$$t_m = 54.9+.$$

22. $C(t_m - t_w)H_c + W(t_m - t_w) = I(t - t_m)H_I$.

Solve the preceding equation for t , when $C = 80$, $t_m = 54$, $t_w = 18$, $H_c = .09$, $W = 100$, $I = 440$, and $H_I = .11$.

$$t = 133.7+.$$

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1. Let b = the weight of B in pounds.
Then $4 \cdot 96 = 6 \cdot b$.
Whence $b = 64$.
2. Let a = the distance in feet of A from the fulcrum.
Then $100 \cdot a = 6\frac{2}{3} \cdot 120$.
Whence $a = 8$.
3. Let a = the distance in feet of A from the fulcrum.
Then $9 - a$ = the distance in feet of B from the fulcrum,
and $125 \cdot a = 100 \cdot (9 - a)$.
Whence $a = 4$,
and $9 - a = 5$.
4. Let a = the weight in pounds of A.
Then $210 - a$ = the weight in pounds of B,
and $3\frac{3}{4} \cdot a = 5(210 - a)$.
Whence $a = 120$,
and $b = 90$.
5. Let x = C's distance in feet from the fulcrum.
Then $108 \cdot x = 90 \cdot 4 + 60 \cdot 3$,
or $x = 5$.

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1. Let x = the greater part.
Then $300 - x$ = the less part,
and $\frac{x}{300 - x} = 5$.

Whence
and

$$x = 250,$$

$$300 - x = 50.$$

2. Let
Then
and

$$x = \text{the greater part.}$$

$$60 - x = \text{the less part,}$$

$$\frac{2x}{3} = \frac{3}{4}(60 - x).$$

Whence
and

$$x = 31\frac{3}{7},$$

$$60 - x = 28\frac{4}{7}.$$

3. Let
Then
and
Whence
and

$$x = \text{the greater part.}$$

$$45 - x = \text{the less part,}$$

$$\frac{4}{9}x + \frac{2}{3}(45 - x) = 24.$$

$$x = 27,$$

$$45 - x = 18.$$

4. Let
Then
and
Whence
and

$$x = \text{the less part.}$$

$$\frac{3}{5} - x = \text{the greater part,}$$

$$\frac{3}{4}x = \frac{1}{3}(\frac{3}{5} - x).$$

$$x = \frac{1}{6}\frac{2}{5},$$

$$\frac{3}{5} - x = \frac{2}{6}\frac{7}{5}.$$

6. Let
Then
and
Whence
and

$$g = \text{the greater part.}$$

$$126 - g = \text{the less part,}$$

$$\frac{g}{126 - g} = 6 + \frac{7}{126 - g}.$$

$$g = 109,$$

$$126 - g = 17.$$

7. Let
Then
and
Whence
and

$$l = \text{the less number.}$$

$$1906 - l = \text{the greater number,}$$

$$\frac{1906 - l}{l} = 41 + \frac{16}{l}.$$

$$l = 45,$$

$$1906 - l = 1861.$$

8. Let
Then
and
Whence
and

$$g = \text{the greater part.}$$

$$\frac{8}{5} - g = \text{the less part,}$$

$$g(\frac{8}{5} - g) = g^2 - \frac{2}{5}.$$

$$g = 1 \text{ or } -\frac{1}{5},$$

$$\frac{8}{5} - g = \frac{3}{5} \text{ or } \frac{9}{5} \text{ respectively.}$$

9. Let
Then

$$x = \text{one part.}$$

$$71 - x = \text{the other part,}$$

and $40 - \frac{2x}{3} = 71 - x - 16.$

Whence $x = 45,$
and $71 - x = 26.$

10. Let $x =$ the boy's age in years.

Then $x = \frac{2}{5}(x + 12).$

Therefore $x = 8.$

11. Let $m =$ the man's age in years.

Then $\frac{2m}{3} = \frac{6}{5}(m - 30).$

Therefore $m = 67\frac{1}{2}.$

12. Let $m =$ the man's age in years.

Then $\frac{m - 8}{6} = \frac{m + 12}{8}.$

Therefore $m = 68.$

13. Let $x =$ the number of dollars invested at 6%.

Then $3100 - x =$ the number of dollars invested at 5%,

and $.05(3100 - x) - .06x = 18.60.$

Whence $x = 1240,$

and $3100 - x = 1860.$

14. Let $n =$ the number of dollars invested at 5%.

Then $5360 - n =$ the number of dollars invested at 6%,

and $.05n - .06(5360 - n) = 63.40.$

Whence $n = 3500,$

and $5360 - n = 1860.$

15. Let $n =$ the number of dollars invested at 4%.

Then $3880 - n =$ the number of dollars invested at 6%,

and $.04n + .06(3880 - n) = 171.20.$

Whence $n = 3080,$

and $3880 - n = 800.$

16. Let $n =$ the number of quarters.

Then $80 - n =$ the number of nickels,

and $25n + 5(80 - n) = 1600.$

Whence $n = 60,$

and $80 - n = 20.$

17. Let $n =$ the number of quarters.

Then $28 - n =$ the number of dimes,

and $25n + 10(28 - n) = 505.$

Whence $n = 15,$

and $28 - n = 13.$

18. Let n , $n + 2$, and $n + 4$ represent the required numbers.

Then $\frac{n^2}{4} = \frac{1}{4}(n + 2)(n + 4) - 11.$

Whence $n = 6,$

$n + 2 = 8,$

and $n + 4 = 10.$

19. Let n , $n + 1$, and $n + 3$ represent the required numbers.

Then $\frac{16n^2}{9} = \frac{16}{9}(n + 1)(n + 3) - 2864.$

Whence $n = 402,$

$n + 1 = 403,$

and $n + 3 = 405.$

20. Let $m =$ the number of meters in width.

Then $4m =$ the number of meters in length,

and $4m^2 = (4m - 4)\left(m + \frac{3}{2}\right) - 11.$

Whence $m = 8\frac{1}{2},$

and $4m = 34.$

21. Let $m =$ the number of meters in width.

Then $\frac{5m}{2} =$ the number of meters in length,

and $\frac{5m^2}{2} = \left(\frac{5m}{2} + 5\right)(m - 4) + 50.$

Whence $m = 6,$

and $\frac{5m}{2} = 15.$

22. Let $s =$ the number of meters in the side of the square.

Then $20 \cdot s^2 = 80 \cdot 4s.$

Whence $s = 16.$

23. Let $w =$ the width in yards.

Then $2w =$ the length in yards,

and $50 \cdot 6w = 15 \cdot 2w^2.$

Whence $w = 10,$

and $2w = 20.$

24. Let

 w = the width in inches.

Then

 $\frac{5w}{2}$ = the length in inches,

and $(w + 4) \left(\frac{5w}{2} + 4 \right) - \frac{5w^2}{2} = 128.$

Whence

$$w = 8,$$

and

$$\frac{5w}{2} = 20.$$

25. Let

 s = the side of the square in yards.

Then $s + 2\frac{7}{9}$ and $s - 2\frac{1}{2}$ represent the length and width of the rectangle respectively.

Hence

$$s^2 = (s + 2\frac{7}{9})(s - \frac{5}{2}).$$

Therefore

$$s = 25,$$

$$s + 2\frac{7}{9} = 27\frac{7}{9},$$

and

$$s - 2\frac{1}{2} = 22\frac{1}{2}.$$

26. Let

 n = the number of apples bought.

Then

$$\frac{n}{5} \cdot \frac{4}{3} + \frac{4n}{5} \cdot \frac{3}{4} = n \cdot \frac{3}{2} - 76.$$

Whence

$$n = 120.$$

28. Let

 n = the number of days required for both.

Then

$$\frac{2}{5} + \frac{4}{15} = \frac{1}{n}.$$

Whence

$$n = \frac{3}{2}.$$

29. Let

 n = the number of days required for all three.

Then

$$\frac{1}{2} + \frac{5}{14} + \frac{2}{7} = \frac{1}{n}.$$

Whence

$$n = \frac{7}{8}.$$

30. Let

 x = the number of days required by B alone.

Then

$$\frac{5}{24} - \frac{1}{8} = \frac{1}{x}.$$

Whence

$$x = 12.$$

31. Let

 x = the number of days required by C alone.

Then

$$\frac{6}{7} - \frac{5}{14} - \frac{2}{7} = \frac{1}{x}.$$

Whence

$$x = 4\frac{2}{3}.$$

32. Let $n =$ the number of days they work together.

Then $\frac{3}{12} + \frac{n}{12} + \frac{n}{15} = 1.$

Whence $n = 5.$

35. Let $r =$ the rate of automobile in miles per hour.

Then $10 \cdot 5\frac{2}{3} = r \cdot 4.$

Whence $r = 14\frac{1}{6}.$

36. Let $r =$ the rate of A in miles per hour.

Then $r + 3 =$ the rate of B in miles per hour,

and $\frac{54}{r} = \frac{72}{r + 3}.$

Whence $r = 9,$

and $r + 3 = 12.$

37. Let $t =$ the required time in hours.

Then $4\frac{1}{2} \cdot t + 10\frac{1}{2}(t - 2\frac{1}{2}) = 40.$

Whence $t = 4\frac{5}{12}.$

38. Let $r =$ the rate of return in miles per hour.

Then $\frac{80}{8} + \frac{80}{r} = 18.$

Whence $r = 10.$

39. Let $t =$ A's time of actual traveling in hours.

Then $t - \frac{3}{2} =$ B's time of actual traveling in hours,

and $9t + 12(t - \frac{3}{2}) = 150.$

Whence $t = 8,$

and $t - \frac{3}{2} = 6\frac{1}{2}.$

Therefore A's distance is 72 miles and B's 78 miles.

40. Let $r =$ B's rate in miles per hour.

Then $\frac{7r}{2} =$ A's rate in miles per hour,

and $8r + 8 \cdot \frac{7r}{2} = 126.$

Whence $r = 3\frac{1}{2},$

and $\frac{7r}{2} = 12\frac{1}{4}.$

41. From Hint. $5(4\frac{1}{4} - x) = 3(4\frac{1}{4} + x).$

Whence $x = 1\frac{1}{16},$

or the rate of the current is $1\frac{1}{16}$ miles per hour.

42. Let d = the distance in miles upstream.

Then
$$\frac{d}{4-2} + \frac{d}{4+2} = 12.$$

Whence
$$d = 18.$$

43. Let t = the time, in hours, for the return.

Then $13 - t$ = the time, in hours, going,

and
$$(4\frac{1}{3} + 2\frac{1}{4})(13 - t) = (4\frac{1}{3} - 2\frac{1}{4})t.$$

Whence
$$t = 9\frac{7}{8},$$

and
$$13 - t = 3\frac{1}{8}.$$

44.
$$5x^2 - 7x - 6 = 0$$

becomes
$$(5x + 3)(x - 2) = 0.$$

Whence
$$x = 2, \text{ or } -\frac{3}{5},$$

and
$$x + 1 = 3, \text{ or } \frac{2}{5}.$$

The second set of values is rejected.

45. Let x = the number of days required by B alone.

Then $x + 2$ = the number of days required by A alone,

and
$$\frac{1}{x} + \frac{1}{x+2} = \frac{5}{12},$$

or
$$(x - 4)(5x + 6) = 0.$$

Whence
$$x = 4, \text{ or } -\frac{6}{5},$$

and
$$x + 2 = 6, \text{ or } \frac{4}{5}.$$

The second set of values is rejected.

46. Let r = the rate downstream in miles per hour.

Then
$$\frac{10}{2\frac{1}{2}} + \frac{10}{r} = 6.$$

Whence
$$r = 5.$$

Then
$$\frac{5 - 2\frac{1}{2}}{2} = 1\frac{1}{4}, \text{ the rate of the current,}$$

and
$$2\frac{1}{2} + 1\frac{1}{4} = 3\frac{3}{4}, \text{ the rate in still water.}$$

47. Let x = the number of sheep bought.

Then $x - 2$ = the number of sheep sold,

and
$$\frac{96}{x} = \frac{100}{x-2} - 2,$$

or
$$(x - 12)(x + 8) = 0.$$

Whence
$$x = 12, \text{ or } -8,$$

and
$$x - 2 = 10, \text{ or } -10.$$

The second set of values is rejected.

48. Let n = the number of yards bought.

Then $n - 4$ = the number of yards sold,

and $\frac{64}{n} = \frac{72}{n - 4} - 2$,

or $(x - 16)(x + 8) = 0$.

Whence $n = 16$,

and $\frac{64}{16} = 4$, the cost in dollars per yard.

49. Let r = the rate going in miles per hour.

Then $r + 5$ = the rate returning in miles per hour,

and $\frac{100}{r} - \frac{100}{r + 5} = 1$,

or $(r + 25)(r - 20) = 0$.

Whence $r = 20$, or -25 ,

and $r + 5 = 25$, or -20 .

The second set of values is rejected.

50. Let r = the rate going in miles per hour.

Then $r + 4$ = the rate returning in miles per hour,

and $\frac{120}{r} - \frac{120}{r + 4} = 5$,

or $(r + 12)(r - 8) = 0$.

Whence $r = 8$, or -12 ,

and $r + 4 = 12$, or -8 .

The second set of values is rejected.

51. Let r = the rate of slower automobile in miles per hour.

Then $r + 2$ = the rate of faster automobile in miles per hour,

and $\frac{72}{r} - \frac{72}{r + 2} = \frac{1}{2}$,

or $(r + 18)(r - 16) = 0$.

Whence $r = 16$, or -18 ,

and $r + 2 = 18$, or -16 .

The second set of values is rejected.

52. Let r = A's rate in miles per hour.

Then $r + 4$ = B's rate in miles per hour,

and $\frac{72}{r} = \frac{72}{r + 4} + 3$,

or $(r + 12)(r - 8) = 0$.

Whence $r = 8$, or -12 .

and $r + 4 = 12$, or -8 .

The second set of values is rejected.

53. Let

 $r =$ the rate of B in miles per hour.

Then

 $r + 1 =$ the rate of A in miles per hour,

$$\frac{172 - 28}{2}$$

and

$$2 + \frac{2}{r} = \frac{100}{r + 1},$$

or

$$2 + \frac{72}{r} = \frac{100}{r + 1},$$

or

$$(r - 4)(r - 9) = 0.$$

Whence

$$r = 9 \text{ or } 4,$$

and

$$r + 1 = 10 \text{ or } 5 \text{ respectively.}$$

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$$1. 5 : 10 = \frac{5}{10} = \frac{1}{2}.$$

$$2. 10 : 5 = \frac{10}{5} = 2.$$

$$3. 16a^2 : 8a = \frac{16a^2}{8a} = 2a.$$

$$4. 3\frac{1}{6} : 3\frac{1}{2} = \frac{\frac{49}{6}}{\frac{7}{2}} = \frac{7}{3}.$$

$$5. 8\frac{2}{3} : 5\frac{7}{9} = \frac{\frac{26}{3}}{\frac{52}{9}} = \frac{3}{2}.$$

$$6. 3 \text{ days} : 9 \text{ hours} = \frac{72}{9} = 8.$$

$$7. 150 \text{ pounds} : 1 \text{ ton} = \frac{150}{2000} = \frac{3}{40}.$$

$$8. (x^2 - y^2) : (x + y) = \frac{(x + y)(x - y)}{x + y} = x - y.$$

$$9. (a^3 + b^3) : (a + b) = \frac{a^3 + b^3}{a + b} = a^2 - ab + b^2.$$

$$10. \left(1 - \frac{1}{a^2}\right) : \left(1 + \frac{1}{a}\right) = \frac{(a^2 - 1)}{a^2} : \left(\frac{a + 1}{a}\right) = \frac{\frac{(a + 1)(a - 1)}{a^2}}{\frac{a + 1}{a}} = \frac{a - 1}{a}.$$

$$11. \left(2 + \frac{1}{x^2}\right) : \left(4 - \frac{1}{x^4}\right) = \frac{2x^2 + 1}{x^2} : \frac{4x^4 - 1}{x^4} = \frac{\frac{2x^2 + 1}{x^2}}{\frac{(2x^2 + 1)(2x^2 - 1)}{x^4}} = \frac{x^2}{2x^2 - 1}.$$

$$12. \frac{1}{x - 3} : \frac{1}{x^2 - 5x + 6} = \frac{\frac{1}{x - 3}}{\frac{1}{(x - 2)(x - 3)}} = x - 2.$$

$$13. (x^2 - xy + y^2) : (x^3 + y^3) = \frac{x^2 - xy + y^2}{(x + y)(x^2 - xy + y^2)} = \frac{1}{x + y}.$$

$$14. \left(x^2 - \frac{y^3}{x}\right) : \left(1 - \frac{y}{x}\right) = \frac{(x^3 - y^3)}{x} : \left(\frac{x - y}{x}\right) = \frac{(x - y)(x^2 + xy + y^2)}{x - y} \\ = x^2 + xy + y^2.$$

$$15. \left(\frac{16}{x} - x\right) : \left(\frac{24}{x^4} + \frac{10}{x^3} + \frac{1}{x^2}\right) = \left(\frac{16 - x^2}{x}\right) : \left(\frac{24 + 10x + x^2}{x^4}\right) \\ = \frac{(4 + x)(4 - x)}{x} = \frac{x^3(4 - x)}{(4 + x)(6 + x)x^4}.$$

16. Let	$2x =$ one part.	and	$6x =$ the third part.
Then	$3x =$ the other part.	$\therefore$	$4x + 5x + 6x = 135,$
	$\therefore 2x + 3x = 40,$	or	$x = 9,$
or	$x = 8,$		$4x = 36,$
	$2x = 16,$		$5x = 45,$
and	$3x = 24.$	and	$6x = 54.$

17. Let	$5x =$ one part.	21. Let	$x =$ the number.
Then	$3x =$ the other part.	Then	$\frac{5 + x}{8 + x} = \frac{21}{28}.$
	$\therefore 3x + 5x = 16,$	Transposing,	$7x = 28,$
or	$x = 2,$	or	$x = 4.$
	$5x = 10,$		
and	$3x = 6.$		

18. Let	$3x =$ one part.	22. Let	$x =$ the number.
Then	$11x =$ the other part.	Then	$\frac{13 - x}{27 - x} = \frac{2}{9}.$
	$\therefore 3x + 11x = 84,$	Transposing,	$-7x = -63,$
or	$x = 6,$	or	$x = 9.$
	$3x = 18,$		
and	$11x = 66.$		

19. Let	$2x =$ the first part.	23.	$\frac{4 + 2a}{4 + 3a} = \frac{16 + 24a + 8a^2}{(4 + 3a)(4 + 4a)},$
Then	$3x =$ the second part,		$\frac{4 + 3a}{4 + 4a} = \frac{16 + 24a + 9a^2}{(4 + 4a)(4 + 3a)}.$
and	$4x =$ the third part.	The second is the greater.	
	$\therefore 2x + 3x + 4x = 36,$		
or	$x = 4,$	24.	$\frac{a + 4b}{a + 5b} = \frac{a^2 + 11ab + 28b^2}{(a + 5b)(a + 7b)},$
	$2x = 8,$		$\frac{a + 6b}{a + 7b} = \frac{a^2 + 11ab + 30b^2}{(a + 7b)(a + 5b)}.$
	$3x = 12,$		
and	$4x = 16.$	The second is the greater.	

20. Let	$4x =$ the first part.	
Then	$5x =$ the second part,	

25. It is increased.

Page 179 (First set)

1. $\frac{3}{4} = \frac{6}{x} \therefore x = 8.$

2. $\frac{4}{7} = \frac{16}{x} \therefore x = 28.$

3. $\frac{3}{2} = \frac{x}{5} \therefore x = 7\frac{1}{2}.$

4. $3 : x = 7 : 9 \therefore x = 3\frac{6}{7}.$

5. $x : 4 = 3 : 6 \therefore x = 2.$

6. $\frac{4}{3} = 2 : \frac{1}{x} \therefore x = \frac{2}{3}.$

7. $\frac{2}{3} = \frac{\frac{1}{x}}{\frac{4}{4}} \therefore x = \frac{3}{8}.$

8. $\frac{2}{\frac{1}{x}} = \frac{3\frac{1}{3}}{4} \therefore x = \frac{5}{12}.$

9. $\frac{1}{x} : 2 = 3 : 4\frac{1}{2} \therefore x = \frac{3}{4}.$

10. $\frac{a}{b} = \frac{c}{x} \therefore x = \frac{bc}{a}.$

11. $\frac{a}{b} = \frac{c}{\frac{1}{x}} \therefore x = \frac{a}{bc}.$

12. $\frac{a}{x^2} = \frac{1}{x} : a^3 \therefore x = a^4.$

13. $4 : 3\frac{1}{3} = 3 : x - 3 \therefore x = 5\frac{1}{2}.$

14. $5 : x - 3 = 7 : 2x + 6 \therefore x = -17.$

Page 179 (Second set)

1. $m^2 = 1 \cdot 4, \text{ or } m = \pm 2.$

2. $m^2 = 4 \cdot 9, \text{ or } m = \pm 6.$

3. $m^2 = 16 \cdot 4, \text{ or } m = \pm 8.$

4. $m^2 = 3 \cdot 12, \text{ or } m = \pm 6.$

5. $m^2 = (a - b)^2 \cdot 4, \text{ or } m = \pm 2(a - b).$

6. $m^2 = \frac{1}{2} \cdot \frac{1}{8}, \text{ or } m = \pm \frac{1}{4}.$

7. $m^2 = \frac{1}{3} \cdot \frac{1}{27}, \text{ or } m = \pm \frac{1}{9}.$

8. $m^2 = \frac{4}{a^3} \cdot \frac{9}{a^2 x^2}, \text{ or } m = \pm \frac{6}{a^2 x}.$

9. $1 : 4 = 4 : t, \text{ or } t = 16.$

$4 : 9 = 9 : t, \text{ or } t = 20\frac{1}{4}.$

$16 : 4 = 4 : t, \text{ or } t = 1.$

$3 : 12 = 12 : t, \text{ or } t = 48.$

$(a - b)^2 : 4 = 4 : t,$

or $t = \frac{16}{(a - b)^2}.$

$\frac{1}{2} : \frac{1}{8} = \frac{1}{8} : t, \text{ or } t = \frac{1}{32}.$

$\frac{1}{3} : \frac{1}{27} = \frac{1}{27} : t, \text{ or } t = \frac{1}{243}.$

10. $1 : 2 = 3 : f, \text{ or } f = 6.$

11. $4 : 5 = 6 : f, \text{ or } f = \frac{15}{2}.$

12. $7 : 14 = 5 : f, \text{ or } f = 10.$

13. $5 : 12 = a : f, \text{ or } f = \frac{12a}{5}.$

14. $7 : 21x = 6x : f, \text{ or } f = 18x^2.$

15. $a : a^2 = a^3 : f, \text{ or } f = a^4.$

16. $a^3 : a^5 = a^4 : f, \text{ or } f = a^6.$

17. $(a + b) : (a - b) = (a^2 - b^2) : f,$
or $f = (a - b)^2.$

18. $\frac{a - b}{2} : \frac{1}{a + b} = (a^2 - b^2) : f,$

or $f = 2.$

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1. $3 : 6 = 2 : 4; 3 : 2 = 6 : 4; 6 : 3 = 4 : 2.$

2. $5 : 10 = 3 : 6; 10 : 5 = 6 : 3; 10 : 6 = 5 : 3.$

3. $3 : 2 = x : 6; 2 : 3 = 6 : x; 3 : x = 2 : 6.$

4. $a : b = c : d; a : c = b : d; b : a = d : c.$

5. $a - b : 2 = 3 : a + b; 2 : a - b = a + b : 3; a - b : 3 = 2 : a + b.$

$$6. a + b : a + 2 = a + 3 : a - b; a + 2 : a + b = a - b : a + 3;$$

$$a + b : a + 3 = a + 2 : a - b.$$

$$7. a - b : 2 = 3 : a + b.$$

$$8. a - b : 3 = 6 : a - b.$$

$$9. a - 2 : 4 = 2 : a - 3.$$

$$10. a - 4 : a + b = a + b : a - 3.$$

$$11. a - 3 : a - 2 = a - 8 : a - 3.$$

$$12. x - 2y : a + 5b$$

$$= a + 5b : x - 2y.$$

$$13. x : 2 = 2 : y.$$

$$14. x : 3 = 1 : y.$$

$$15. a : 1 = 1 : b.$$

$$16. ab : d = e : c.$$

$$28. \frac{4}{x} = \frac{3}{y}.$$

$$29. \frac{4}{6} = \frac{5}{x}.$$

$$30. \frac{3}{a} = \frac{1}{b}.$$

$$31. \frac{8}{4} = \frac{6}{3}.$$

$$32. \frac{2}{3} = \frac{b}{a}.$$

$$33. \frac{P_1}{P} = \frac{W}{W_1}.$$

$$17. mn : xy = z : p.$$

$$18. a : a + b = 1 : b.$$

$$19. 5 : 3 = 4 : x.$$

$$20. 4 : 9 = 7 : x.$$

$$21. c : b = a : x.$$

$$22. p : q = r : x.$$

$$23. ac : b = d : x.$$

$$24. a : 1 = 1 : x.$$

$$25. c : a = b : x.$$

$$26. y : y + 1 = 1 : x.$$

$$27. \frac{2}{4} = \frac{3}{6}.$$

$$34. \frac{V_2}{V_1} = \frac{P_1}{P_2}.$$

$$35. \frac{\frac{1}{x}}{3} = \frac{a}{2}.$$

$$36. \frac{\frac{1}{y}}{\frac{1}{x}} = \frac{z}{1}.$$

$$37. \frac{N_2}{N_1} = \frac{\sqrt{T_1}}{\sqrt{T_2}}.$$

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$$1. \frac{5}{3} = \frac{10}{6}.$$

$$2. \frac{16}{12} = \frac{32}{24}.$$

$$3. \frac{a+x}{x} = \frac{3}{2}.$$

$$6. \frac{-1}{3} = \frac{-2}{6}; \frac{-8}{12} = \frac{-16}{24}; \frac{a-x}{x} = \frac{-1}{2}; \frac{1}{3} = \frac{f-x}{x}.$$

$$7. \frac{5}{-1} = \frac{10}{-2}; \frac{16}{-8} = \frac{32}{-16}; \frac{a+x}{a-x} = \frac{3}{-1}; \frac{7}{1} = \frac{f+x}{f-x}.$$

$$8. \text{ If } \frac{a}{b} = \frac{c}{d},$$

$$\text{ then } \frac{b}{a} = \frac{d}{c},$$

$$\text{ and } \frac{b}{a} + 1 = \frac{d}{c} + 1.$$

$$\text{ Whence } \frac{b+a}{a} = \frac{d+c}{c}.$$

$$4. \frac{7}{3} = \frac{f+x}{x}.$$

$$5. \frac{A_1 + A_2}{A_2} = \frac{S_1^2 + S_2^2}{S_2^2}.$$

$$9. \text{ If } \frac{a}{b} = \frac{c}{d},$$

$$\text{ then } \frac{b}{a} = \frac{d}{c},$$

$$\text{ and } 1 - \frac{b}{a} = 1 - \frac{d}{c}.$$

$$\text{ Whence } \frac{a-b}{a} = \frac{c-d}{c}.$$

$$10. \text{ If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{2a}{b} = \frac{2c}{d}.$$

$$\text{ Whence } \frac{2a+b}{b} = \frac{2c+d}{d}.$$

11. If

$$\frac{a}{b} = \frac{c}{d},$$

then

$$\frac{a}{3b} = \frac{c}{3d}.$$

Whence

$$\frac{a + 3b}{3b} = \frac{c + 3d}{3d},$$

or

$$\frac{a + 3b}{b} = \frac{c + 3d}{d}.$$

12.

$$\frac{3x + 4}{3x - 4} = \frac{3 + 2}{3 - 2}.$$

13.

$$\frac{6x + 3}{6x - 3} = \frac{2a + b}{2a - b}.$$

Then

$$\frac{6x}{8} = \frac{6}{4},$$

Then

$$\frac{12x}{6} = \frac{4a}{2b},$$

or

$$x = 2.$$

or

$$x = \frac{a}{b}.$$

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1. From $\frac{1}{2} = \frac{3}{6} = \frac{5}{10}$, $\frac{9}{18} = \frac{1}{2}$, etc.2. From $\frac{1}{a} = \frac{2}{2a} = \frac{3b}{3ab}$, $\frac{3 + 3b}{3a + 3ab} = \frac{1}{a}$, etc.3. From $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$, $\frac{21}{28} = \frac{3}{4}$, etc.4. From $\frac{1}{x - y} = \frac{a}{ax - ay} = \frac{b}{bx - by}$, $\frac{1 + a + b}{(x - y)(1 + a + b)} = \frac{1}{x - y}$, etc.5. $\frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{1}{3}$. Also $\frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{13}{36} = \frac{1}{3}$. The perimeters are in the same ratio as the corresponding sides.6. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$. Multiply both terms of $\frac{a}{b}$ by 2, of $\frac{c}{d}$ by 3, and of $\frac{e}{f}$ by 4.

Then use (10) on p. 183 in text, which gives

$$\frac{2a + 3c + 4e}{2b + 3d + 4f} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}.$$

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1. If $\frac{a}{b} = \frac{4}{x}$ and $\frac{a}{b} = \frac{4}{\frac{1}{x}}$, then $\frac{4}{x} = \frac{4}{\frac{1}{x}}$, or $x = \pm 1$.2. If $\frac{a}{b} = \frac{9}{y}$ and $\frac{a}{b} = \frac{1}{\frac{1}{y}}$, then $\frac{9}{y} = \frac{1}{\frac{1}{y}}$, or $y = \pm 3$.3. If $\frac{p}{r} = \frac{7}{6}$ and $\frac{q}{r} = \frac{3}{5}$, then, by dividing, $\frac{p}{q} = \frac{35}{18}$.4. If $\frac{p}{q} = \frac{4}{a}$ and $\frac{q}{r} = \frac{a}{7}$, then, by multiplying, $\frac{p}{r} = \frac{4}{7}$.

5. Let the parts of 12 be x and $12 - x$ respectively.

$$\text{Then } \frac{8}{10} = \frac{x}{12 - x},$$

$$\text{or } x = 5\frac{1}{3}, \text{ and } 12 - x = 6\frac{2}{3}.$$

6. Let p = one part.

Then $21 - p$ = the other part,

$$\text{and } \frac{p}{21 - p} = \frac{18}{24},$$

$$\text{or } p = 9, \text{ and } 21 - p = 12.$$

7. Let

h = the height of the pole in feet.

Then

$$\frac{70}{12} : h = \frac{35}{12} : 36,$$

or

$$h = 72.$$

8. Let x and x_1 be the required sides respectively.

$$\text{Then } \frac{10}{12} = \frac{15}{x}.$$

$$\text{Also } \frac{10}{12} = \frac{20}{x_1}.$$

$$\text{Clearing, } 10x = 180. \quad \text{Clearing, } 10x_1 = 240.$$

$$x = 18. \quad x_1 = 24.$$

$$\frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{5}{6}.$$

Therefore they have the same ratio.

9. Let x , x_1 , and x_2 be the required sides respectively.

$$\text{Then } \frac{9}{x} = \frac{9 + 10 + 17}{108} = \frac{36}{108} = \frac{1}{3}. \quad \frac{10}{x_1} = \frac{1}{3}. \quad \frac{17}{x_2} = \frac{1}{3}.$$

$$\text{Clearing, } 108 = 4x. \quad 30 = x_1. \quad 51 = x_2.$$

$$x = 27.$$

$$10. (a) \quad \frac{AD}{DB} = \frac{AE}{EC}.$$

$$(c) \quad \frac{AD}{AB} = \frac{AE}{AC}.$$

$$\text{Substituting, } \frac{6}{4} = \frac{10}{EC}.$$

$$\text{Substituting, } \frac{8}{8 + 4} = \frac{AE}{10}.$$

$$\text{Clearing, } 6EC = 40.$$

$$\text{Clearing, } 80 = 12AE.$$

$$EC = 6\frac{2}{3}.$$

$$AE = 6\frac{2}{3}.$$

$$(b) \quad \frac{AD}{AB} = \frac{DE}{BC}.$$

$$(d) \quad \frac{AD}{AB} = \frac{AE}{AC}.$$

$$\text{Substituting, } \frac{8}{8 + 4} = \frac{6}{BC}.$$

$$\text{Substituting, } \frac{10}{16} = \frac{AE}{16}.$$

$$\text{Clearing, } 8BC = 72. \quad BC = 9.$$

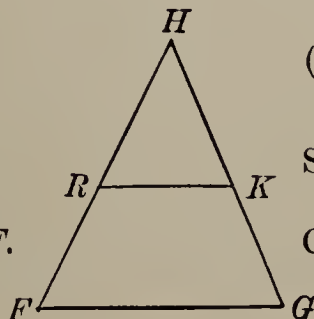
$$\text{Clearing, } AE = 10.$$

$$11. (a) \quad \frac{FG}{RK} = \frac{HF}{RH}.$$

$$\text{Subst., } \frac{15}{10} = \frac{HF}{8}.$$

$$\text{Clearing, } 120 = 10HF.$$

$$HF = 12.$$



$$(b) \quad \frac{FG - RK}{FG} = \frac{HF - HR}{HF}.$$

$$\text{Subst., } \frac{4}{20} = \frac{RF}{10}.$$

$$\text{Clearing, } 40 = 20RF.$$

$$RF = 2.$$

(c) $\frac{FG}{RK} = \frac{HR + RF}{HR}$. Substituting, $\frac{20}{15} = \frac{HR + 6}{HR}$. Clearing, $20 HR = 15 HR + 90$. Transposing, $5 HR = 90$. $HR = 18$.	(d) $\frac{FG}{RK} = \frac{HK + KG}{HK}$. Substituting, $\frac{18}{15} = \frac{KH + 4}{HK}$. Clearing, $18 HK = 15 HK + 60$. Transposing, $3 HK = 60$. $HK = 20$.
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12. Let x = the third side of the triangle.

Then $\frac{12}{24} = \frac{12}{x}$.

Clearing, $x = 24$.

Therefore the third side of the triangle is 24 centimeters.

13. Let x , x_1 , and x_2 be the required lengths in centimeters.

Then $\frac{5}{10} = \frac{x}{16}$, $\frac{6}{12} = \frac{x_1}{10}$, $\frac{8}{16} = \frac{x^2}{12}$.

$x = 8$, $x_1 = 5$, $x_2 = 6$.

14. (a) $\frac{AD}{BD} = \frac{BD}{DC}$. Substituting, $\frac{9}{6} = \frac{6}{DC}$. Clearing, $9 DC = 36$. $DC = 4$.	(b) $\frac{AD}{BD} = \frac{BD}{DC}$. Substituting, $\frac{4}{BD} = \frac{BD}{20 - 4}$. Clearing, $64 = \overline{BD}^2$. $BD = 8$.
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15. (a) $\frac{AD}{CD} = \frac{CD}{BD}$. Substituting, $\frac{2}{CD} = \frac{CD}{18}$. Clearing, $36 = \overline{CD}^2$. $CD = 6$.	(b) $\frac{AD}{CD} = \frac{CD}{BD}$. Substituting, $\frac{AD}{8} = \frac{8}{34 - AD}$. Clearing, $34 AD - \overline{AD}^2 = 64$. $\overline{AD}^2 - 34 AD + 64 = 0$. $(AD - 32)(AD - 2) = 0$. $AD = 32, 2$.
--	---

16. $\frac{DC}{BE} = \frac{AB + BC}{AB}$.

Substituting, $\frac{480}{420} = \frac{8}{7} = \frac{AB + 160}{AB}$.

Clearing, $8 AB = 7 AB + 1120$.

$AB = 1120$.

Therefore the computed value of AB was 1120 feet.

17. Let x , x_1 , and x_2 be the sides of the second triangle respectively.

$$\begin{array}{l} \text{Then} \quad \frac{45}{135} = \frac{11}{x}, \quad \frac{45}{135} = \frac{19}{x_1}, \quad \frac{45}{135} = \frac{45 - 11 - 19}{x_2}. \\ \quad \quad \quad x = 33. \quad \quad \quad x_1 = 57. \quad \quad \quad x_2 = 45. \end{array}$$

18. Let $2x$ = the rate of the first man in miles per hour.

Then $3x$ = the rate of the second man in miles per hour.

Therefore $10x + 15x = 100$,

or $x = 4$,

$$2x = 8,$$

and $3x = 12$.

19. Let $7x$ = the amount A contributed.

Then $8x$ = the amount B contributed.

Also $\frac{7x + 8x}{3}$, or $5x$ = the amount each ate.

Therefore $7x - 5x$, or $2x$ = the amount A should be paid for.

$8x - 5x$, or $3x$ = the amount B should be paid for.

Therefore $2x + 3x = 30$,

or $x = 6$,

$$2x = 12 \text{ dollars,}$$

and $3x = 18 \text{ dollars.}$

20. From February 21 to March 28 is 35 days.

Now $\frac{\text{true number of minutes in a day}}{\text{false number of minutes in a day}} = \frac{\text{true number of days}}{\text{false number of days}}.$

$$\text{Therefore} \quad \frac{1440}{1450} = \frac{x}{35}.$$

Whence $x = 34.758 \text{ days.}$

$$34.758 \text{ days} = 34 \text{ days, } 18 \text{ hours, } 12 + \text{ minutes.}$$

Therefore the correct time was 10.12 A.M. March 28.

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1. It begins at 80° and ends at 83° .

2. The highest temperature recorded is 88° ; the lowest, 54° .

3. The highest temperature was recorded at 7 P.M. Wednesday. The lowest temperature was recorded at 9.30 A.M. Thursday.

4. The instrument recorded 80° five times, 72° three times, 78° four times, and 62° twice.

5. The 80° records were made at 2 P.M. Wednesday, 10 P.M. Wednesday, 1 P.M. Thursday, 10 P.M. Thursday, 10.30 A.M. Friday.

The 72° records were made at 1 A.M. Thursday, 9 A.M. Thursday, 11 A.M. Thursday.

The 78° records were made at 10.30 P.M. Wednesday, 12.30 P.M. Thursday, 12 P.M. Thursday, 1 A.M. Friday, 9.30 A.M. Friday.

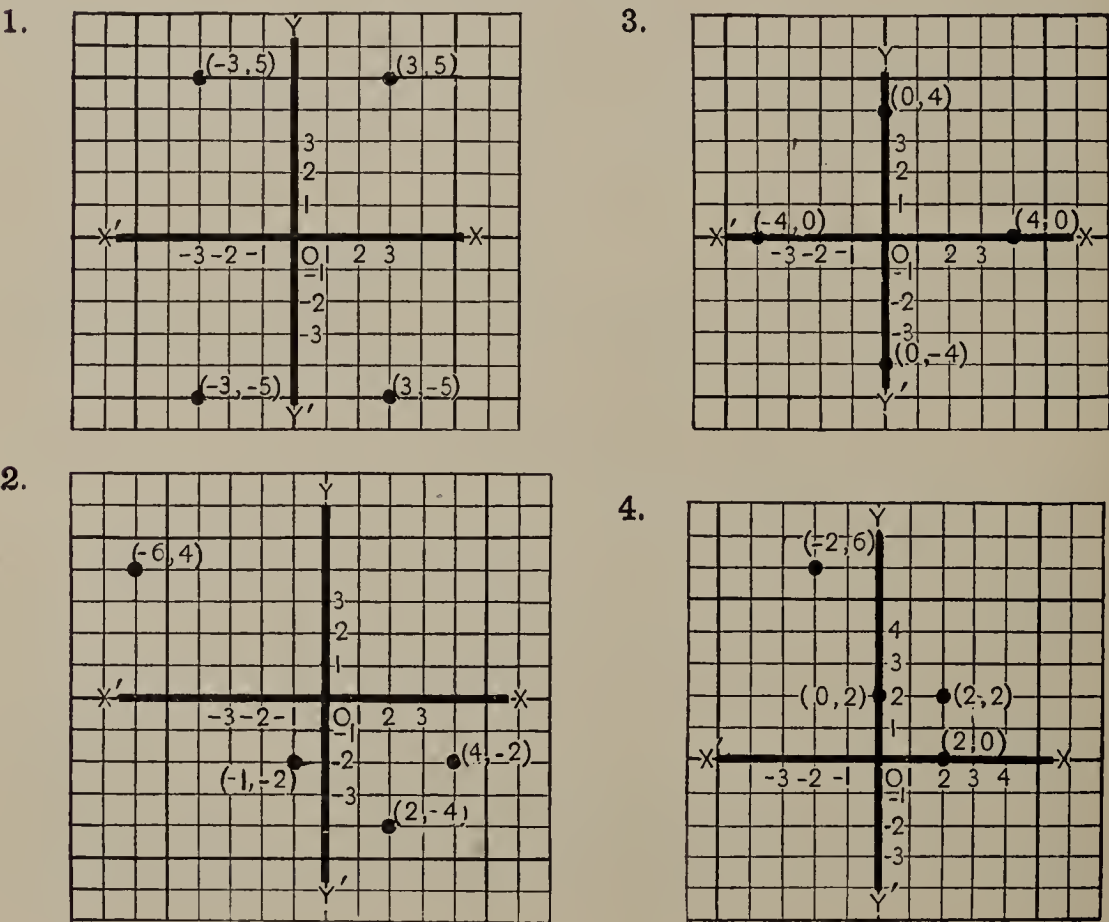
The 62° records were made at 9 A.M. Thursday, 9.30 A.M. Thursday.

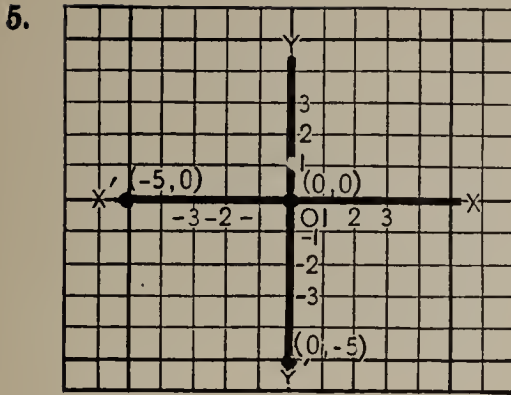
6. A graph such as the one here explained might be used as a check against the waste of coal in overheating a building when not in actual use. It might be used to test the care exercised in keeping a room or a car at a uniform temperature.

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- 1. The line through 8 seconds cuts the curve at a distance of 1024 feet, the line through 3 seconds cuts it at 144 feet, the one through 7 seconds cuts it at 784 feet, the one through $2\frac{1}{2}$ seconds cuts it at 100 feet, and the one through 6.2 seconds cuts it at 615 feet.
- 2. The line through 400 feet cuts the curve at the line for 5 seconds.
The line through 196 feet cuts the curve at the line for $3\frac{1}{2}$ seconds.
The line through 100 feet cuts the curve at the line for $2\frac{1}{2}$ seconds.
The line through 25 feet cuts the curve at the line for $1\frac{1}{4}$ seconds.
The line through 120 feet cuts the curve at the line for $2\frac{3}{4}$ seconds.
The line through 750 feet cuts the curve at the line for $6\frac{4}{5}$ seconds.

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6. If one coordinate of a point is zero, the point is located on one of the axes.

If both are zero, it is located at the origin.

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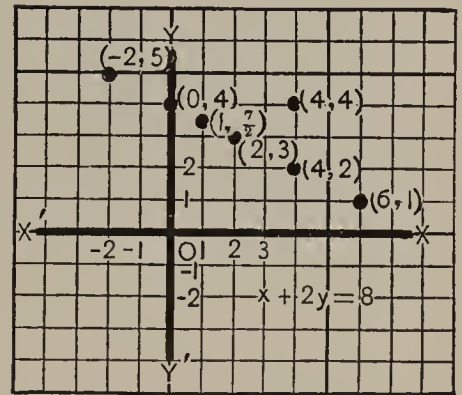
1 $x + 2y = 8$.

x	-2	0	1	2	4	6
y	5	4	$\frac{7}{2}$	3	2	1

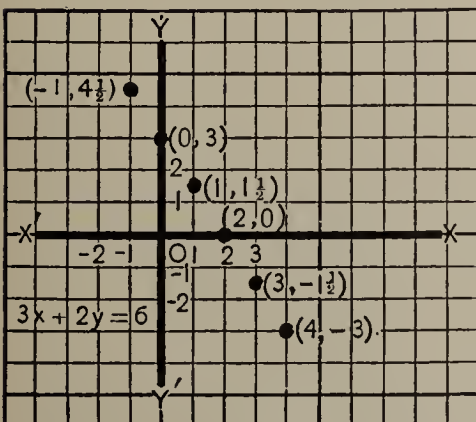
The points are in a straight line which cuts the x -axis at $(8, 0)$ and the y -axis at $(0, 4)$.

The graph of $x + 2y = 8$ is a straight line which passes through these points.

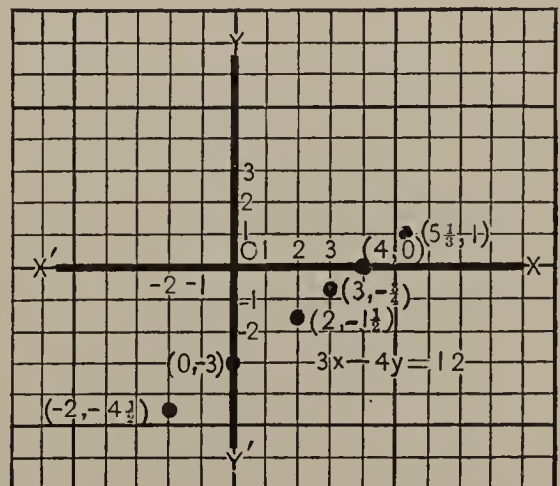
No, $(4, 4)$ does not satisfy the equation. The point $(4, 4)$ is not on the graph of the equation. If the x - and y -distances of a point satisfy the equation $x + 2y = 8$, the point is located on the graph of the equation. Moreover only such points are there located.



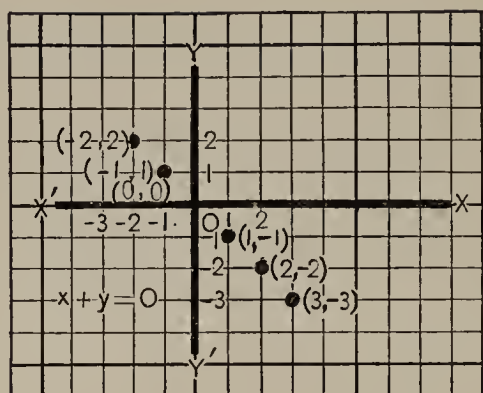
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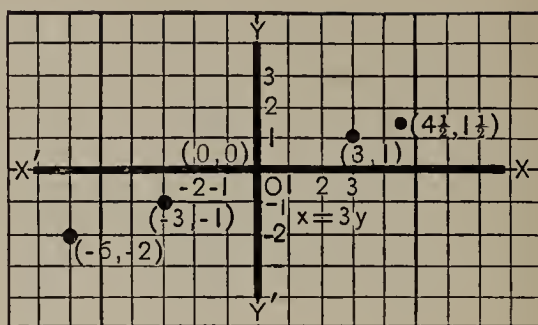
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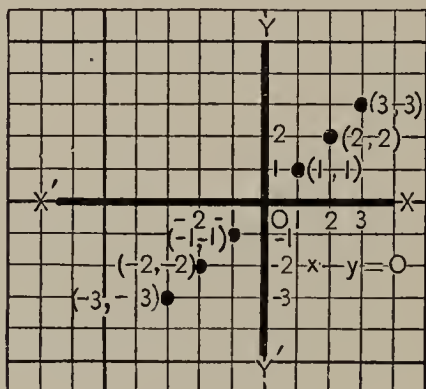
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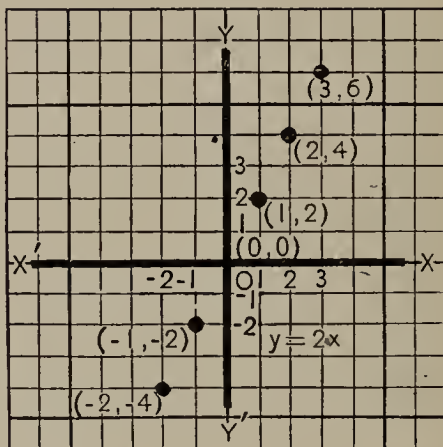
6.



5.



7.

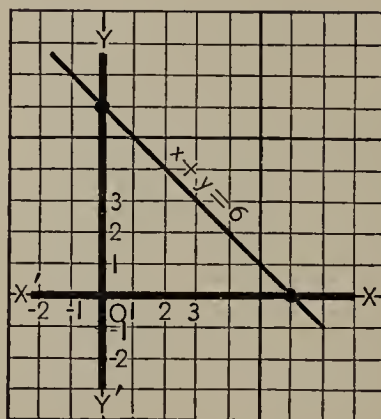


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1.

$$x + y = 6.$$

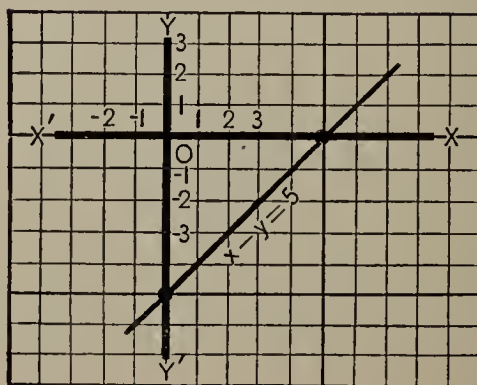
x	0	6
y	6	0



2.

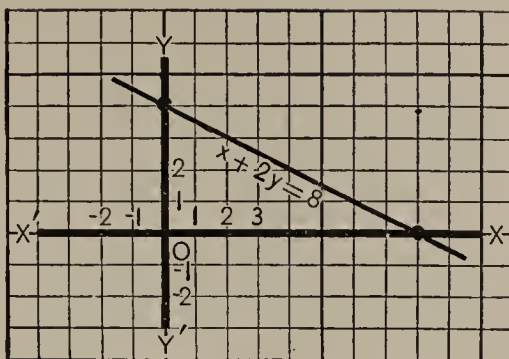
$$x - y = 5.$$

x	0	5
y	-5	0



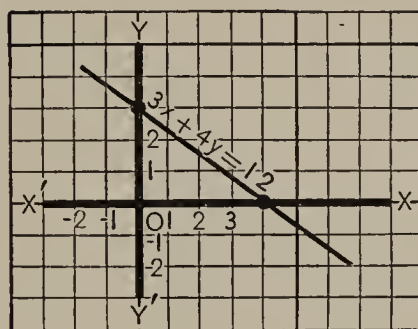
3. $x + 2y = 8.$

x	0	8
y	4	0



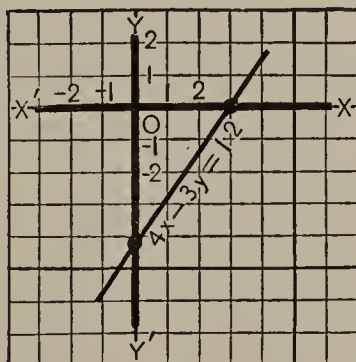
4. $3x + 4y = 12.$

x	0	4
y	3	0



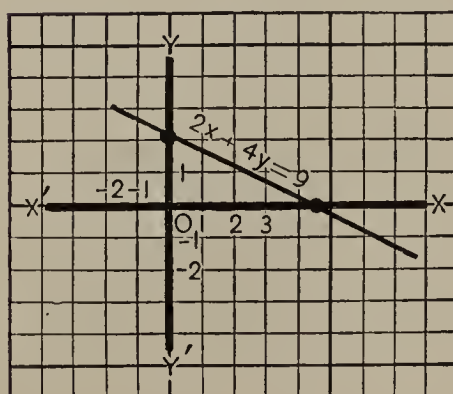
5. $4x - 3y = 12.$

x	0	3
y	-4	0



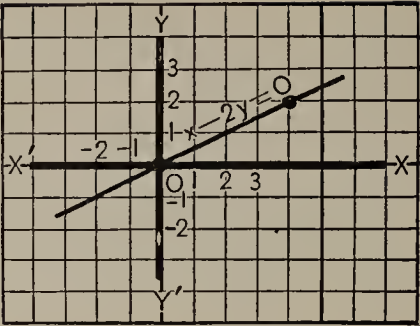
6. $2x + 4y = 9.$

x	0	$4\frac{1}{2}$
y	$2\frac{1}{4}$	0



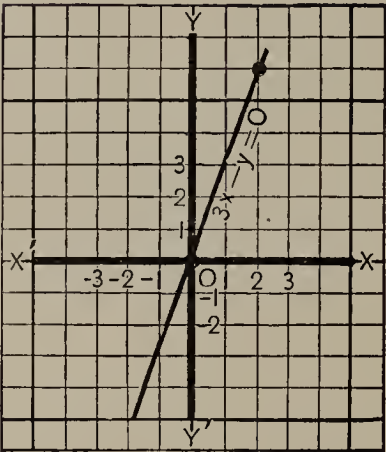
7. $x - 2y = 0.$

x	0	4
y	0	2

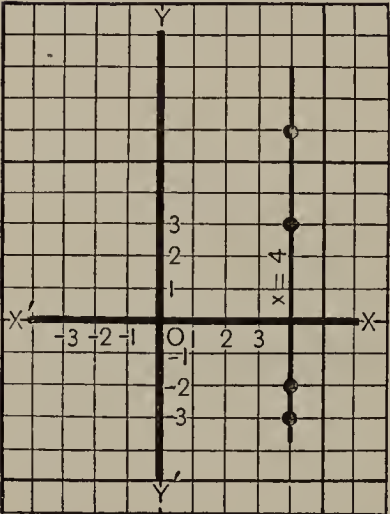


8. $3x - y = 0.$

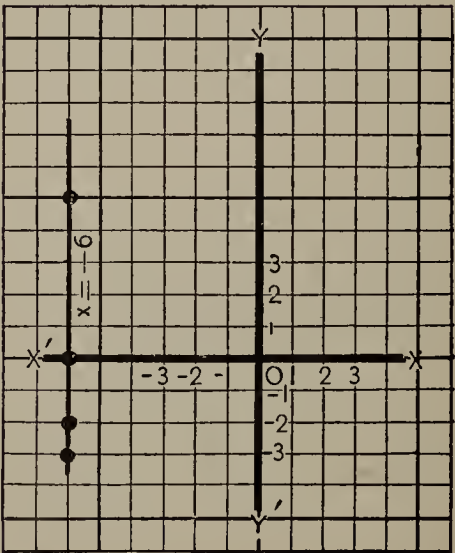
x	0	6
y	0	2



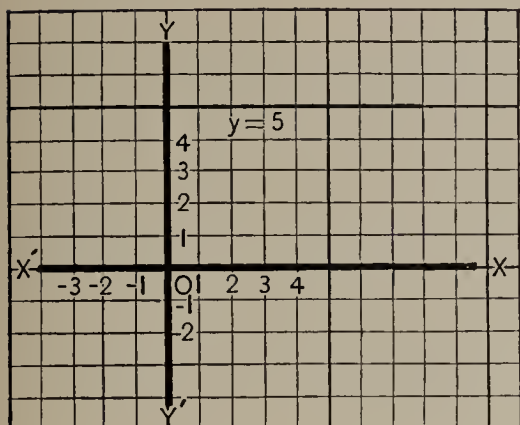
9. $x = 4$ for all values of $y.$



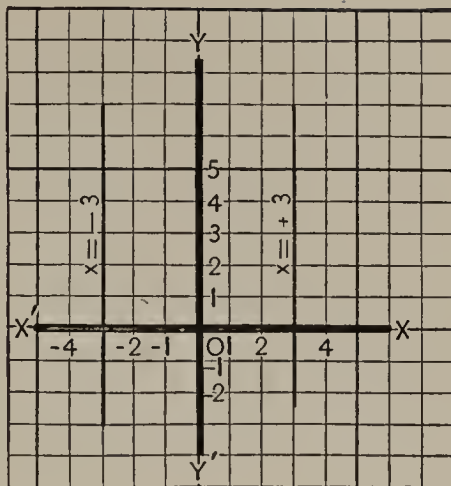
10. $x = -6$ for all values of $y.$



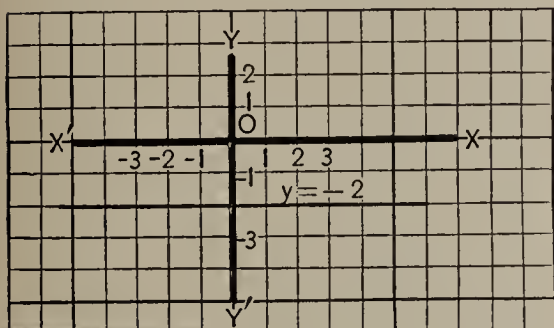
11. $y = 5$ for all values of x .



15.



12. $y = -2$ for all values of x .



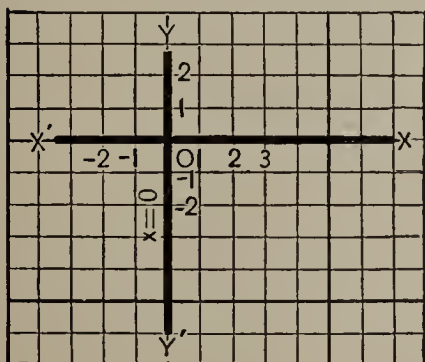
16. Yes.

17. Yes.

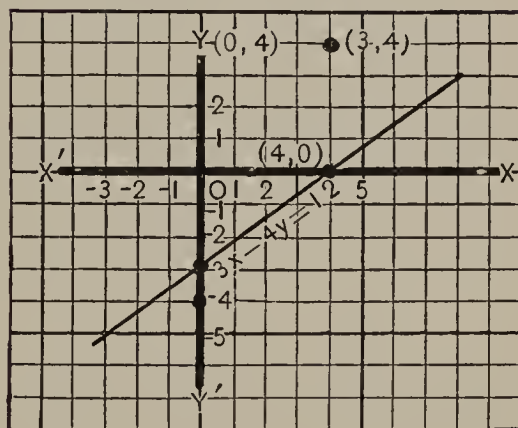
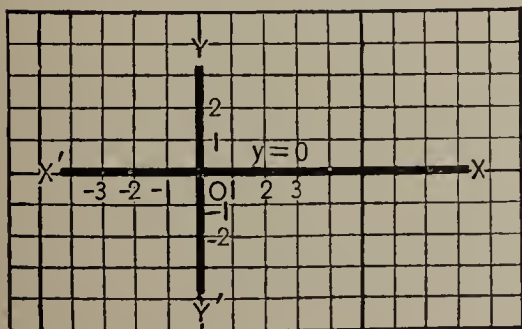
18. $3x - 4y = 12$.

x	0	4
y	-3	0

13. $x = 0$ for all values of y .



14. $y = 0$ for all values of x .



Point (3, 4) is not on the graph of $3x - 4y = 12$.

Point (0, 4) is not on the graph of $3x - 4y = 12$.

Point (4, 0) is on the graph of $3x - 4y = 12$.

19. Point (2, 6) will be on a graph if it satisfies the equation of that graph. It satisfies the equation of Exercise 8, and therefore lies on the graph of $3x - y = 0$.

20. The graphs of equations in Exercises 7, 8, 13, and 14 pass through the origin, for their constant terms are zero.

21. An equation whose graph passes through the origin has zero for its constant term. The constant term of an equation is never zero if the graph does not pass through the origin.

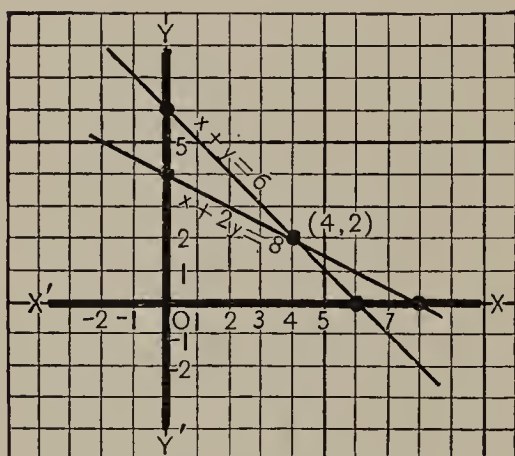
22. Yes. If the constant term of the equation is zero, and x is set equal to zero, then y equals zero also.

23. The graph of a linear equation in x and y will be parallel to the x -axis if the coefficient of y is zero.

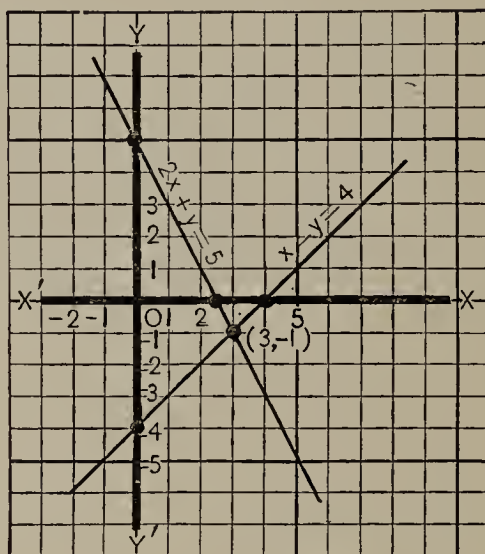
Likewise, the graph will be parallel to the y -axis if the coefficient of x is zero.

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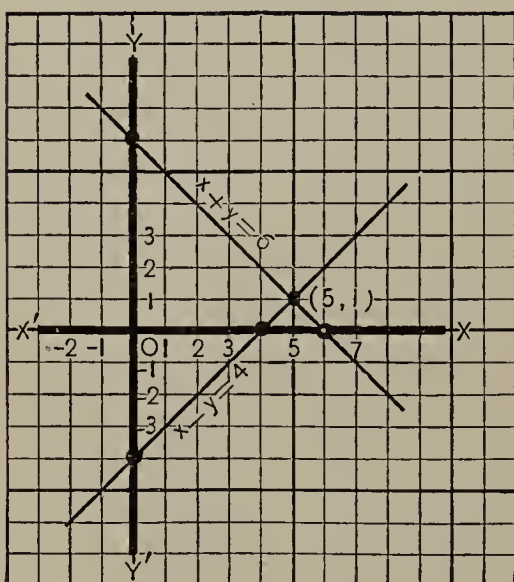
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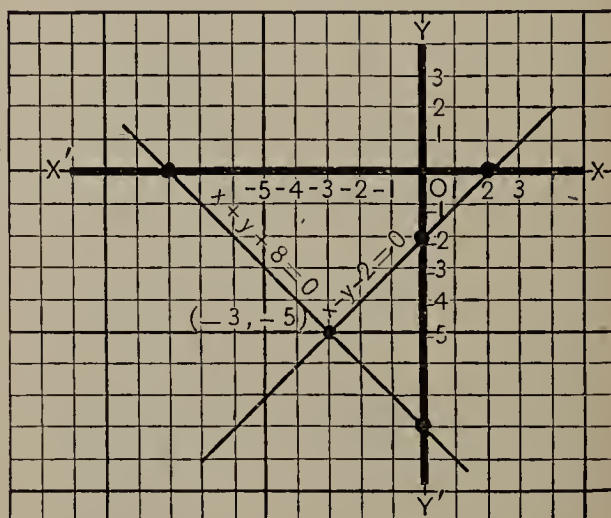
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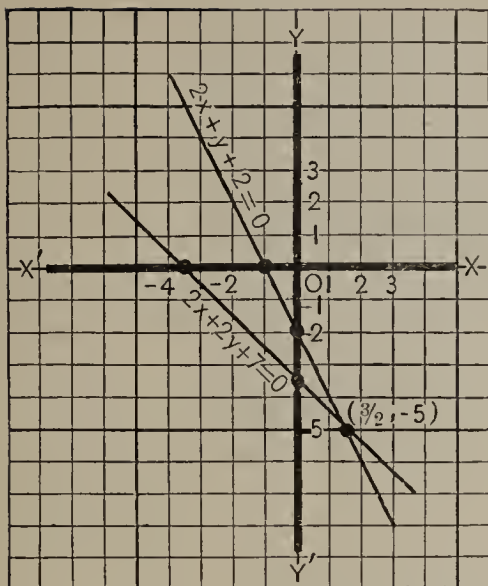
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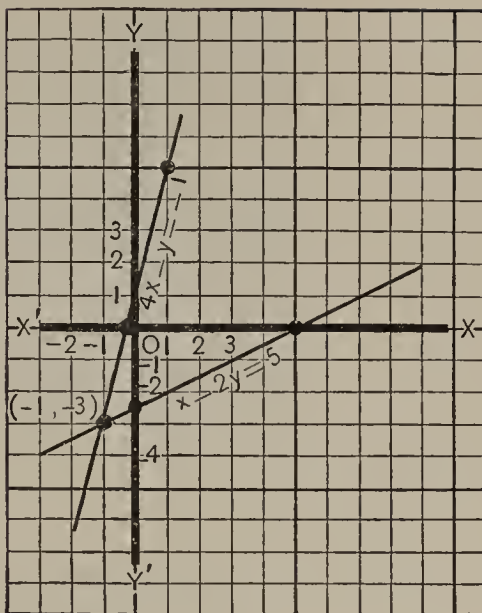
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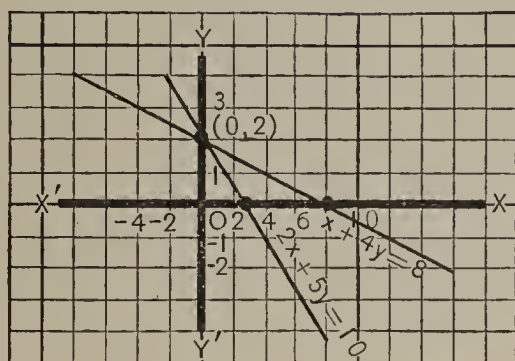
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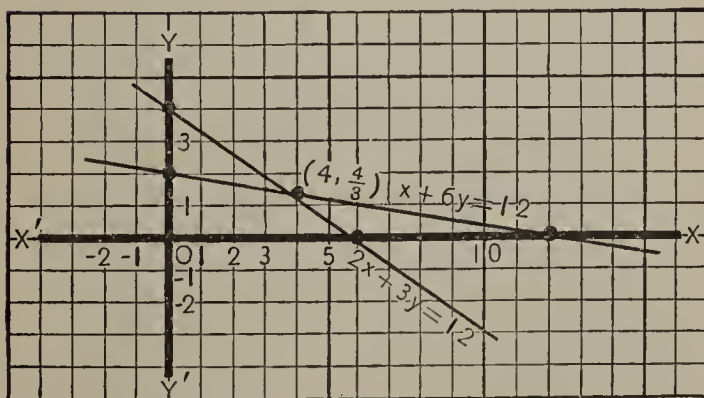
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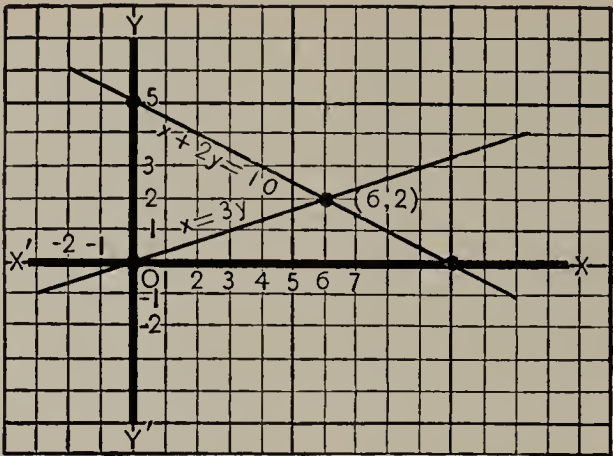
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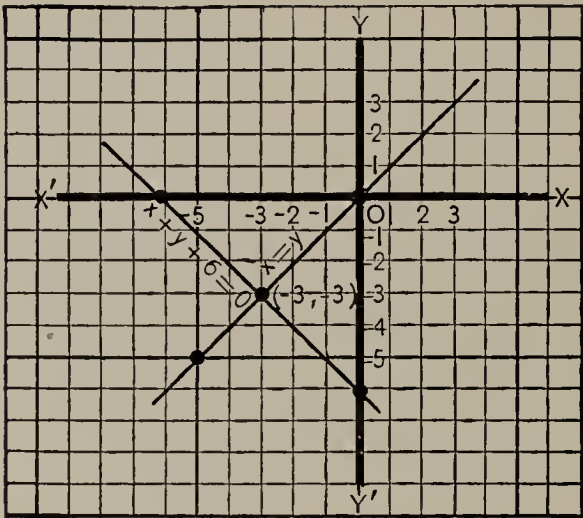
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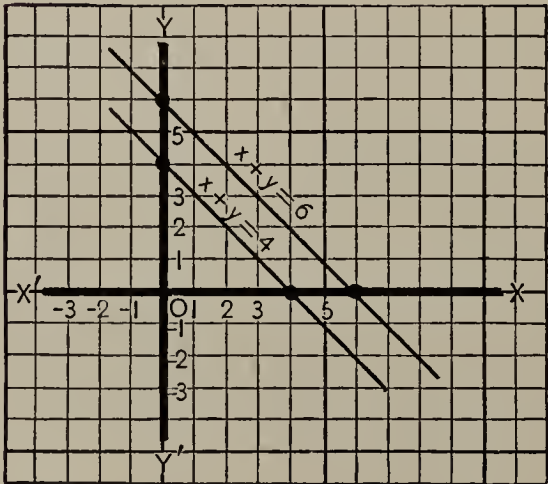
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10.

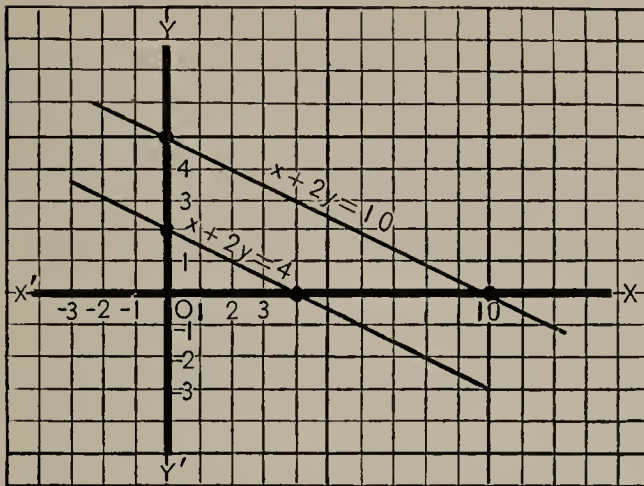


11.



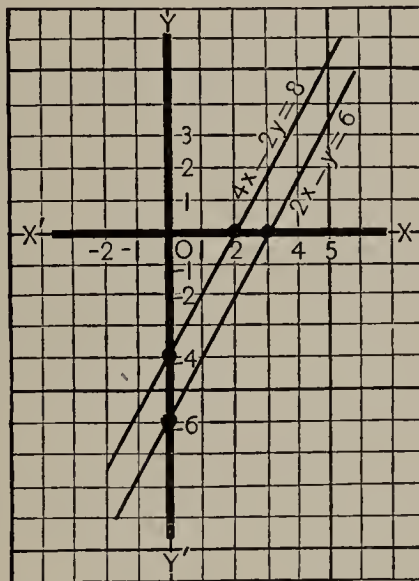
The system is inconsistent, for there is no graphical solution.

12.



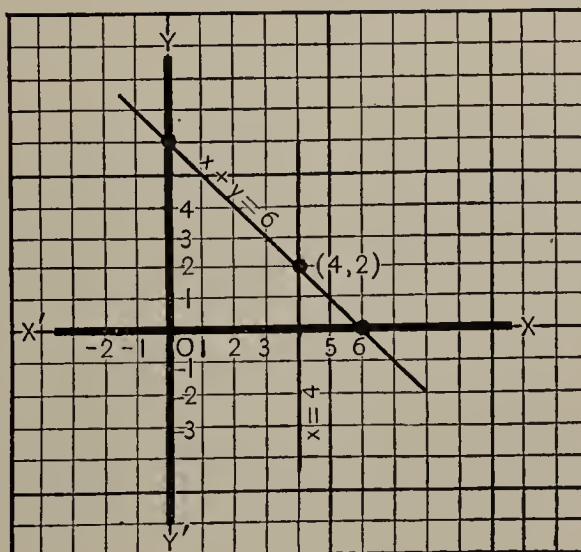
The system is inconsistent, for there is no graphical solution.

13.

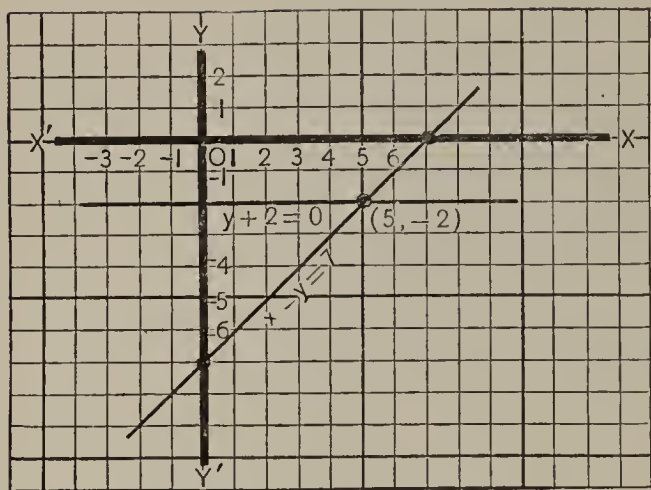


The system is inconsistent, for there is no graphical solution.

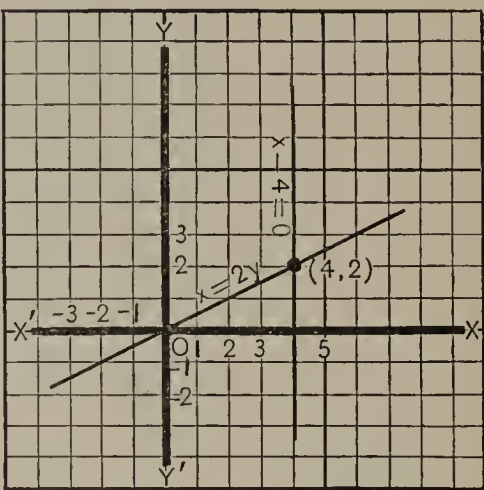
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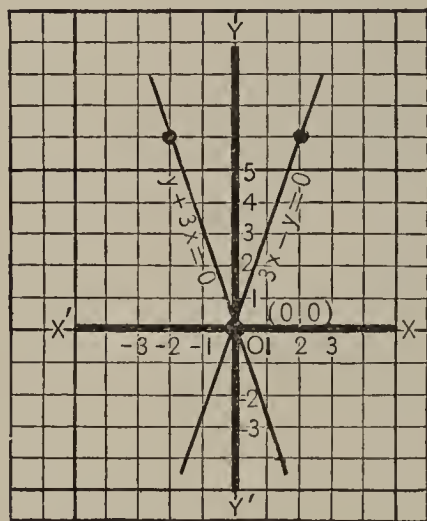
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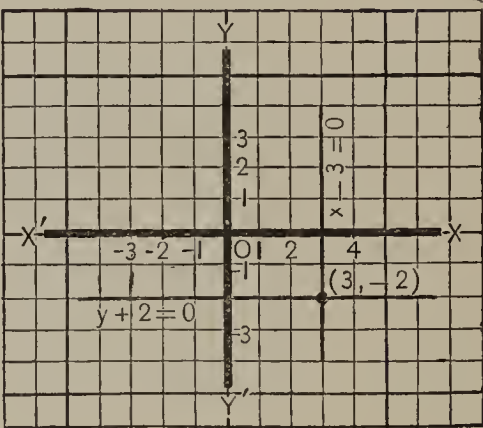
16.



17.

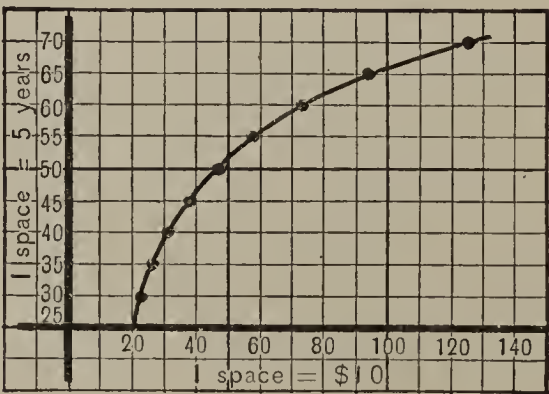


18.

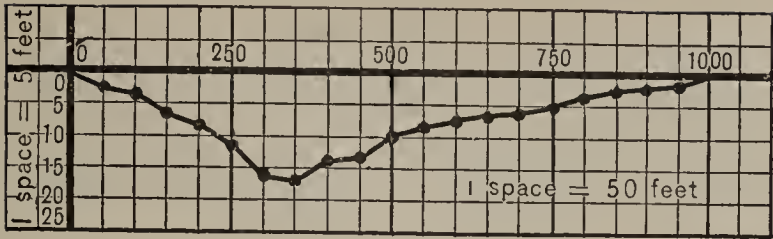


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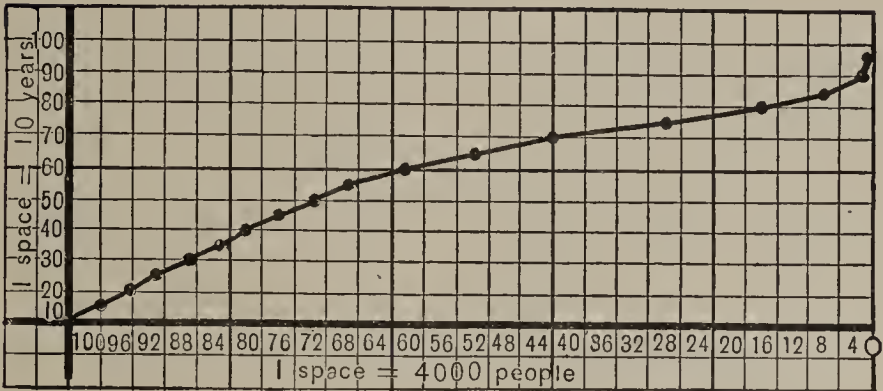
1.



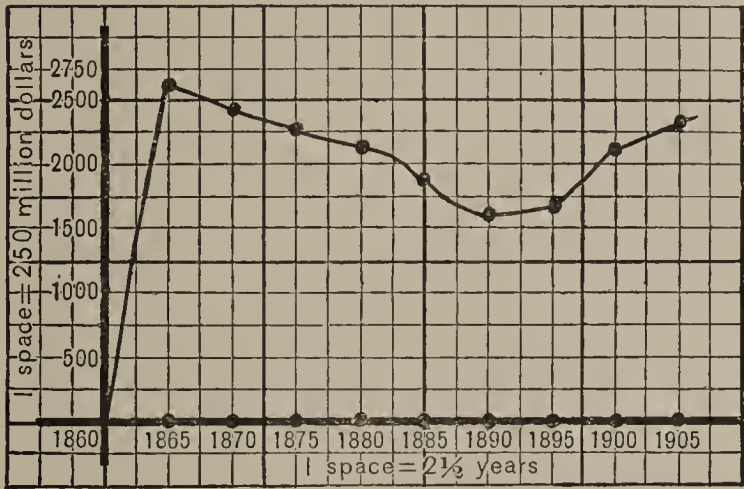
2.



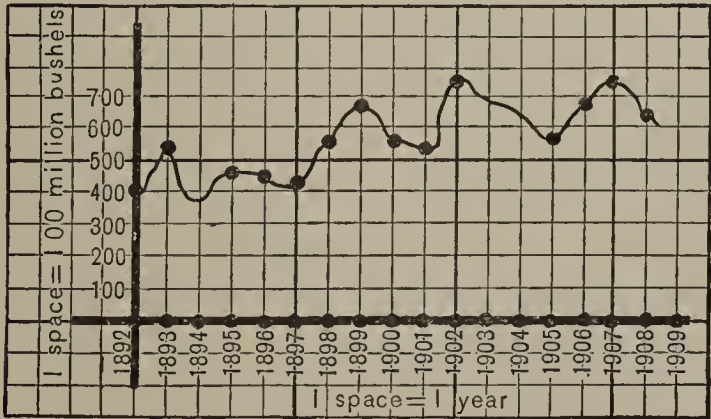
3.



4.



5.



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1. $x + 2y = 7,$ (1)
 $5x - 2y = 11.$ (2)
 $[(1) + (2)] \div 6,$ $x = 3.$
Then from (1), $y = 2.$
2. $2x + y = 4,$ (1)
 $3x - y = 21.$ (2)
 $[(1) + (2)] \div 5,$ $x = 5.$
Then from (1), $y = -6.$
3. $7m - n = 2,$ (1)
 $n - 2m = -3.$ (2)
 $[(1) + (2)] \div 5,$ $m = -\frac{1}{5}.$
Then $n = -\frac{17}{5}.$
4. $10h - k = -3,$ (1)
 $12h + 12k = 102.$ (2)
 $[(1) \cdot 12 + (2)] \div 132,$ $h = \frac{1}{2}.$
Then $k = 8.$
5. $7r - 8s = -30,$ (1)
 $r + 11s = 20.$ (2)
 $[(1) - (2) \cdot 7] \div -85,$ $s = 2.$
Then $r = -2.$
6. $8x + y = 7,$ (1)
 $11x + 2y = 28.$ (2)
 $[(1) \cdot 2 - (2)] \div 5,$ $x = -\frac{14}{5}.$
Then $y = \frac{147}{5}.$
7. $5l + 2p = 0,$ (1)
 $3l + p = 3.$ (2)
 $[(1) - (2) \cdot 2] \div -1,$ $l = 6.$
Then $p = -15.$
8. $10v + 2u = 22,$ (1)
 $u + 5v = 11.$ (2)
 $[(1) - (2) \cdot 2]$ gives $0 = 0.$
The system has an infinite number of sets of roots.
9. $12x + 5y = 14,$ (1)
 $3x - 10y = 8.$ (2)
 $[(1) \cdot 2 + (2)] \div 27,$ $x = \frac{4}{3}.$
Then $y = -\frac{2}{3}.$
10. $2x - y = -1,$ (1)
 $15x - 9y = 20.$ (2)
 $[(1) \cdot 9 - (2)] \div 3,$ $x = -\frac{29}{3}.$
Then $y = -\frac{55}{3}.$
11. $3s - t = 12,$ (1)
 $2t - 6s = 10.$ (2)
 $[(1) \cdot 2 + (2)]$ gives $0 = 14.$
Therefore the system is not consistent.
12. $5x - 3w = 2,$ (1)
 $15x + 12w = -5.$ (2)
 $[(1) \cdot 3 - (2)] \div -21,$
 $w = -\frac{11}{21}.$
Then $x = \frac{3}{5}.$
13. $x_1 - 6x_2 = 7,$ (1)
 $12x_2 - x_1 = 0.$ (2)
 $[(1) + (2)] \div 6,$ $x_2 = \frac{7}{6}.$
Then $x_1 = 14.$
14. $27h + 32k = 6,$ (1)
 $16k - 9h = 8.$ (2)
 $[(1) + (2) \cdot 3] \div 80,$ $k = \frac{3}{8}.$
Then $h = -\frac{2}{9}.$
15. $2r + 25r_1 = 15,$ (1)
 $3r = 10r_1 - 44.$ (2)
 $[(1) \cdot 3 - (2) \cdot 2] \div 95,$
 $r_1 = \frac{7}{5}.$
Then $r = -10.$
16. $12n - 2m = 18,$ (1)
 $3m = 18n + 10.$ (2)
 $[(1) \cdot 3 + (2) \cdot 2]$ gives
 $0 = 74.$
Therefore the system is not consistent.

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$$1. \quad x - 2y = 8, \quad (1)$$

$$3x + 2y = 7. \quad (2)$$

Substituting $2y + 8$ for x from (1) into (2),

$$3(2y + 8) + 2y = 7.$$

$$\text{Therefore } y = -\frac{17}{8}.$$

Then from (1),

$$x = \frac{15}{4}.$$

$$2. \quad x - 2y = -12, \quad (1)$$

$$4x - y = 1. \quad (2)$$

Substituting $2y - 12$ for x from (1) into (2),

$$8y - 48 - y = +1.$$

$$\text{Therefore } y = 7,$$

$$\text{and } x = 2.$$

$$3. \quad 14m - 2n = 1, \quad (1)$$

$$n - 6m = 0. \quad (2)$$

Substituting $6m$ for n from (2) into (1),

$$m = \frac{1}{2}.$$

$$\text{Then } n = 3.$$

$$4. \quad 6h + 10k = 19, \quad (1)$$

$$2k = 3h. \quad (2)$$

Substituting $\frac{3h}{2}$ for k from (2) into (1),

$$h = \frac{19}{21}.$$

$$\text{Then } k = \frac{19}{14}.$$

$$5. \quad 3s + 12 = 3 + t, \quad (1)$$

$$t = s + 1. \quad (2)$$

Substituting $s + 1$ for t in (1),

$$s = -4,$$

$$\text{and } t = -3.$$

$$11. \quad \frac{1}{7 + 2m_1} = \frac{-7}{m_2 - 1}, \quad (1)$$

$$m_1 = m_2 + 3. \quad (2)$$

Substituting $m_2 + 3$ for m_1 from (2) into (1) and solving,

$$m_2 = -6.$$

$$\text{Then } m_1 = -3.$$

$$6. \quad 18 + 2p = q, \quad (1)$$

$$p + q = -9. \quad (2)$$

Substituting $2p + 18$ for q from (1) into (2),

$$p = -9.$$

$$\text{Then } q = 0.$$

$$7. \quad 3r + 15s = 7, \quad (1)$$

$$12 + 5s = -r. \quad (2)$$

Substituting $-12 - 5s$ for r from (2) into (1),

$$36 = 7.$$

Therefore the system is inconsistent.

$$8. \quad 20y - 3z = 1, \quad (1)$$

$$z - 6y = 0. \quad (2)$$

Substituting $6y$ for z from (2) into (1),

$$y = \frac{1}{2}.$$

$$\text{Therefore } z = 3.$$

$$9. \quad .75p + 1.5q = 3, \quad (1)$$

$$q = p - 16. \quad (2)$$

Substituting $p - 16$ for q into (1) $\cdot 100$,

$$75p + 150p - 2400 = 300.$$

$$\text{Therefore } p = 12,$$

$$\text{and } q = -4.$$

$$10. \quad \frac{3x - 20}{2} = \frac{2x + 5y}{3}, \quad (1)$$

$$10 = x - y. \quad (2)$$

Substituting $10 + y$ for x from (2) into (1) $\cdot 6$,

$$y = -2.$$

$$\text{Then } x = 8.$$

$$12. \quad \frac{5R_1 + 2R_2}{2} = 2(R_2 + 2), \quad (1)$$

$$R_1 - \frac{2}{3}R_2 = 0. \quad (2)$$

Substituting $\frac{2R_2}{3}$ for R_1 from (2) into (1) and solving,

$$R_2 = 6.$$

Then

$$R_1 = 4.$$

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$$1. \quad \frac{2x}{3} + 4y = \frac{26}{3}, \quad (1)$$

$$3x - \frac{7y}{2} = -4. \quad (2)$$

$$(1) \cdot 3, \quad 2x + 12y = 26. \quad (3)$$

$$(2) \cdot 2, \quad 6x - 7y = -8. \quad (4)$$

$$[(3) \cdot 3 - (4)] \div 43, \quad y = 2.$$

$$\text{Whence} \quad x = 1.$$

$$2. \quad m - \frac{3n}{5} = \frac{18}{5}, \quad (1)$$

$$\frac{8n}{3} + 7m = -16. \quad (2)$$

$$(1) \cdot 5, \quad 5m - 3n = 18. \quad (3)$$

$$(2) \cdot 3, \quad 21m + 8n = -48. \quad (4)$$

$$[(3) \cdot 8 + (4) \cdot 3] \div 103,$$

$$m = 0.$$

$$\text{Whence} \quad n = -6.$$

$$3. \quad \frac{5R_1}{6} + \frac{R_2}{4} = 7, \quad (1)$$

$$\frac{2R_1}{3} - \frac{R_2}{8} = 3. \quad (2)$$

$$[(1) \cdot 12 + (2) \cdot 24] \div 26,$$

$$R_1 = 6.$$

$$\text{Whence} \quad R_2 = 8.$$

$$4. \quad 3k - \frac{12h}{5} = 18, \quad (1)$$

$$\frac{11k}{10} + \frac{17h}{2} = 53\frac{1}{2}. \quad (2)$$

$$(1) \cdot 5, \quad 15k - 12h = 90. \quad (3)$$

$$(2) \cdot 10, \quad 11k + 85h = 535. \quad (4)$$

$$[(3) \cdot 11 + (4) \cdot 15] \div -1407,$$

$$h = 5.$$

$$\text{Whence} \quad k = 10.$$

$$5. \quad \frac{2x}{9} - \frac{y}{2} = -1, \quad (1)$$

$$x = \frac{9y}{4} - 4\frac{1}{2}. \quad (2)$$

$$(1) \cdot 18, \quad 4x - 9y = -18, \quad (3)$$

$$(2) \cdot 4, \quad 4x - 9y = -18, \quad (4)$$

$$(3) - (4), \quad 0 = 0.$$

Therefore the system has an infinite number of sets of roots.

$$6. \quad .4x + .9y = 5.7, \quad (1)$$

$$2x - y = 1. \quad (2)$$

$$[(1) \cdot 10 - (2) \cdot 2] \div 11,$$

$$y = 5.$$

$$\text{Whence} \quad x = 3.$$

$$7. \quad 12m = \frac{11y}{9} + 17, \quad (1)$$

$$\frac{2m}{3} - \frac{49}{12} = \frac{5y}{12}. \quad (2)$$

$$(1) \cdot 9, \quad 108m - 11y = 153. \quad (3)$$

$$(2) \cdot 12, \quad 8m - 5y = 49. \quad (4)$$

$$[(3) \cdot 5 - (4) \cdot 11] \div 452,$$

$$m = \frac{1}{2}.$$

$$\text{Whence} \quad y = -9.$$

$$8. \quad 9s = \frac{t}{3} + \frac{23}{4}, \quad (1)$$

$$\frac{7t}{3} - \frac{9}{4} = \frac{-3s}{4}. \quad (2)$$

$$(1) \cdot 12, \quad 108s - 4t = 69. \quad (3)$$

$$(2) \cdot 12, \quad 28t + 9s = 27. \quad (4)$$

$$[(3) \cdot 7 + (4) \cdot 1] \div 765, \\ s = \frac{2}{3}.$$

$$\text{Whence} \quad t = \frac{3}{4}.$$

$$9. \quad .04m + .75n = 10, \quad (1)$$

$$.8m - 1.25n = 5. \quad (2)$$

$$(1) \cdot 100, \quad 4m + 75n = 1000. \quad (3)$$

$$(2) \cdot 100, \quad 80m - 125n = 500. \quad (4)$$

$$[(3) \cdot 20 - (4) \cdot 1] \div 1625, \\ n = 12.$$

$$\text{Whence} \quad m = 25.$$

$$10. \quad 28x - 16y = 56, \quad (1)$$

$$\frac{2x + 7}{2} - y = 4. \quad (2)$$

$$[(1) - (2) \cdot 16] \div 12, \quad x = 4.$$

$$\text{Whence} \quad y = \frac{7}{2}.$$

$$11. \quad \frac{r + 3}{7} - \frac{l}{5} = 0, \quad (1)$$

$$5r + 2 = 7l. \quad (2)$$

$$[(1) \cdot 35 - (2) \cdot 1] \text{ gives} \\ 13 = 0.$$

Therefore the system is inconsistent.

$$12. \quad \frac{x + 3}{5} + \frac{y + 4}{10} = \frac{3}{2}, \quad (1)$$

$$\frac{7x + 1}{3} - \frac{11y - 4}{7} = 4. \quad (2)$$

$$(1) \cdot 10, \quad 2x + y = 5. \quad (3)$$

$$(2) \cdot 21, \quad 49x - 33y = 65. \quad (4)$$

$$[(3) \cdot 33 + (4) \cdot 1] \div 115, \\ x = 2.$$

$$\text{Whence} \quad y = 1.$$

$$13. \quad \frac{20m + 9}{7} = \frac{n}{7} + 1, \quad (1)$$

$$m + \frac{29}{5} = n. \quad (2)$$

$$[(1) \cdot 7 - (2) \cdot 20] \div -19,$$

$$n = 6.$$

$$\text{Whence} \quad m = \frac{1}{5}.$$

$$14. \quad \frac{1}{x} + \frac{1}{y} = \frac{5}{6}, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}. \quad (2)$$

$$[(1) + (2)] \cdot x, \quad x = 2.$$

$$\text{Whence} \quad y = 3.$$

$$15. \quad \frac{1}{m_1} + \frac{1}{m_2} = \frac{2}{15}, \quad (1)$$

$$\frac{1}{m_1} - \frac{1}{m_2} = -\frac{1}{3}. \quad (2)$$

$$(1) + (2) \cdot 5 m_1, \quad m_1 = -10.$$

$$\text{Whence} \quad m_2 = \frac{30}{7}.$$

$$16. \quad \frac{6}{x} + \frac{7}{y} = \frac{3}{2}, \quad (1)$$

$$\frac{7}{x} - \frac{6}{y} + 5\frac{1}{3} = 0. \quad (2)$$

$$(1) \cdot 6 + (2) \cdot 7, \quad x = -3.$$

$$\text{Whence} \quad y = 2.$$

$$17. \quad \frac{1}{x} + \frac{1}{y} = 10, \quad (1)$$

$$\frac{3}{x} + \frac{3}{y} = 14. \quad (2)$$

$$(1) \cdot 3 - (2), \quad 0 = 16.$$

Therefore the system is inconsistent.

$$18. \quad \frac{5}{x} + 12 = 17, \quad (1)$$

$$\frac{2}{x} - 15y = 0. \quad (2)$$

$$[(1) \cdot 2 - (2) \cdot 5] \div 75, \quad y = \frac{2}{15}.$$

$$\text{Whence} \quad x = 1.$$

$$19. \quad \frac{4r + q}{6r + q} = \frac{2}{5}, \quad (1)$$

$$\frac{2r}{3} - q = 5. \quad (2)$$

$$(1) \cdot 5(6r + q), \quad 8r + 3q = 0. \quad (3)$$

$$(2) \cdot 3, \quad 2r - 3q = 15. \quad (4)$$

$$[(3) + (4)] \div 10, \quad r = \frac{3}{2}.$$

$$\text{Whence} \quad q = -4.$$

$$20. \quad \frac{1}{t_2} + \frac{2}{t_1} = \frac{40}{t_1 t_2}, \quad (1)$$

$$2 t_2 - 3 t_1 = 0. \quad (2)$$

$$[(1) \cdot 2 \cdot t_1 t_2 - (2)] \div 5,$$

$$t_1 = 10.$$

$$\text{Whence} \quad t_2 = 15.$$

$$21. \quad \frac{5m + 3n - 1}{3} = n + \frac{1}{3}, \quad (1)$$

$$\frac{25m}{10m + 2n} = 1. \quad (2)$$

$$(1) \cdot 3 \div 5, \quad m = \frac{2}{5}.$$

$$\text{Whence} \quad n = 3.$$

$$22. \quad \frac{k + 5}{k + 1} = \frac{l + 2}{l - 2}, \quad (1)$$

$$\frac{11k + 4}{l + k + 1} = 4. \quad (2)$$

$$\frac{[(1) \cdot (k + 1)(l - 2) + (2) \cdot (l + k + 1)]}{3},$$

$$k = 4.$$

$$\text{Whence} \quad l = 7.$$

$$23. \quad \frac{2x_1 - 3x_2}{x_2 - 4x_1} = 3, \quad (1)$$

$$\frac{8}{x_2 + x_1} + \frac{8}{x_1 - x_2} = 0. \quad (2)$$

$$(1) \cdot (x_2 - 4x_1),$$

$$14x_1 - 6x_2 = 0. \quad (3)$$

$$(2) \cdot (x_2 + x_1)(x_1 - x_2),$$

$$16x_1 = 0. \quad (4)$$

$$(4) \div 16, \quad x_1 = 0.$$

$$\text{Then} \quad x_2 = 0.$$

But this set makes the first member of (1) indeterminate, also the first member of (2) is not a number.

Therefore the system has no set of roots.

$$24. \quad 3x - 2y = 7, \quad (1)$$

$$\frac{2}{x - 5} = -\frac{5}{y - 4}. \quad (2)$$

$$[(1) \cdot 1 + (2) \cdot (x - y)(y - 4)] \div 8,$$

$$x = 5.$$

$$\text{Whence} \quad x = 4.$$

But this set does not satisfy (2).

Hence the system has no set of roots.

$$25. \quad \frac{2}{6 + s} - \frac{4}{4 - t} = 0, \quad (1)$$

$$\frac{t}{4} + \frac{s}{2} + 2 = 0. \quad (2)$$

$$(1) \cdot (6 + s)(4 - t),$$

$$-2t - 4s = 16. \quad (3)$$

$$(2) \cdot 4, \quad t + 2s = -8. \quad (4)$$

$$[(3) \cdot 1 + (4) \cdot 2] \text{ gives } 0 = 0.$$

Therefore the system has an infinite number of sets of roots.

$$26. \quad \frac{2}{n_1 + 2 + n_2} = \frac{1}{5n_2 + 3n_1 - 5}, \quad (1)$$

$$n_1 + 2n_2 = 4. \quad (2)$$

$$(1) \text{ cleared, } 5n_1 + 9n_2 = 12. \quad (3)$$

$$(2) \cdot 5, \quad 5n_1 + 10n_2 = 20. \quad (4)$$

$$(4) - (3), \quad n_2 = 8.$$

$$\text{Whence} \quad n_1 = -12.$$

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1. Let g = the greater number,

and l = the less.

$$\text{Then} \quad g - l = 25, \quad (1)$$

$$\text{and} \quad g + l = 46. \quad (2)$$

$$[(1) + (2)] \div 2,$$

$$g = \frac{71}{2}.$$

$$\text{Whence} \quad l = \frac{21}{2}.$$

2. Let g = the greater number, and l = the less.

$$\text{Then} \quad \frac{g}{l} = 6,$$

$$\text{or} \quad g = 6l, \quad (1)$$

$$\text{and} \quad g + l = 49. \quad (2)$$

$$[(1) - (2)] \div -7, \quad l = 7.$$

$$\text{Whence} \quad g = 42.$$

3. Let g = the greater number,
and l = the less.

$$\text{Then } g - l = 36, \quad (1)$$

$$\text{and } \frac{g}{l} = 3,$$

$$\text{or } g - 3l = 0. \quad (2)$$

$$[(1) - (2)] \div 2, \quad l = 18.$$

$$\text{Whence } g = 54.$$

4. Let $\frac{n}{d}$ = the fraction.

$$\text{Then } \frac{n}{d} = \frac{3}{5},$$

$$\text{or } 5n - 3d = 0, \quad (1)$$

$$\text{and } \frac{n+3}{d+1} = \frac{3}{4}.$$

$$\text{or } 4n - 3d = -9. \quad (2)$$

$$(1) - (2), \quad n = 9.$$

$$\text{Whence } d = 15,$$

$$\text{and } \frac{n}{d} = \frac{9}{15}.$$

5. Let g = the greater number,
and l = the less.

$$\text{Then } \frac{g}{l} = 3 + \frac{6}{l},$$

$$\text{or } g - 3l = 6, \quad (1)$$

$$\text{and } \frac{l}{g} = .3,$$

$$\text{or } 10l - 3g = 0. \quad (2)$$

$$(1) \cdot 3 + (2), \quad l = 18.$$

$$\text{Whence } g = 60.$$

6. Let n = the number of nickels,
and q = the number of quar-
ters.

$$\text{Then } n + q = 77, \quad (1)$$

$$\text{and } 5n + 25q = 985. \quad (2)$$

$$[(1) \cdot 5 - (2) \cdot 1] \div -20,$$

$$q = 30.$$

$$\text{Whence } n = 47.$$

7. Let $\frac{n}{d}$ = the fraction.

$$\text{Then } \frac{n - \frac{4}{3}}{d + \frac{1}{9}} = \frac{3}{5},$$

$$\text{or } 5n - 3d = 7, \quad (1)$$

$$\text{and } n + d = 19. \quad (2)$$

$$[(1) \cdot 1 + (2) \cdot 3] \div 8,$$

$$n = 8.$$

$$\text{Whence } d = 11,$$

$$\text{and } \frac{n}{d} = \frac{8}{11}.$$

8. Let $\frac{n}{d}$ = the fraction.

$$\text{Then } d - n = 24, \quad (1)$$

$$\text{and } \frac{n + \frac{3}{8}}{d - \frac{1}{4}} = \frac{3}{14},$$

$$\text{or } 14n - 3d = -6. \quad (2)$$

$$[(1) \cdot 3 + (2) \cdot 1] \div 11,$$

$$n = 6.$$

$$\text{Whence } d = 30,$$

$$\text{and } \frac{n}{d} = \frac{6}{30}.$$

9. Let w_1 and w_2 represent the
weights in pounds.

$$\text{Then } 12w_1 = 8w_2, \quad (1)$$

$$\text{and } 10(w_1 + 2) = 8w_2. \quad (2)$$

$$[(1) - (2)] \div 2, \quad w_1 = 10,$$

$$\text{and } w_2 = 15.$$

10. Let w_1 and w_2 represent the
weights in pounds.

$$\text{Then } 12w_1 = 18w_2, \quad (1)$$

$$\text{and } 12(w_1 - 12) = 15w_2. \quad (2)$$

$$[(1) - (2)] \div -3,$$

$$w_2 = 48.$$

$$\text{Whence } w_1 = 72.$$

11. Let w_1 and w_2 represent the
weights in pounds.

$$\text{Then } 15w_1 = 10w_2, \quad (1)$$

$$\text{and } 14(w_1 - 6) = 10(w_2 - 12). \quad (2)$$

$$(1) - (2), \quad w_1 = 36.$$

$$\text{Whence } w_2 = 54.$$

12. Let x = A's age in years,
and y = B's age in years.

Then $x - 2y = 0,$ (1)

and $y - 7 = \frac{x - 7}{3}.$ (2)

[(1) + (2)], $y = 14.$

Whence $x = 28.$

13. Let x = A's age now in years,
and y = B's age now in years.

Then $x + 5 = 2(y + 5),$ (1)

and $x - 5 = 3(y - 5). (2)$

(1) - (2), $y = 15.$

Whence $x = 35.$

14. Let l = the length,
and w = the width.

Then $2l + 2w = 232,$ (1)

$l - 2w = 8. (2)$

[(1) + (2)] $\div 3,$ $l = 80.$

Whence $w = 36.$

15. Let p_1 = the number of dol-
lars at 6%,
and p_2 = the number of dol-
lars at 5%.

Then $p_1 + p_2 = 1000,$ (1)

and $.06p_1 + .05p_2 = 54. (2)$

[(1) $\cdot 5$ - (2) $\cdot 100$] $\div -1,$

$p_1 = 400.$

Whence $p_2 = 600.$

16. Let x = the number of dol-
lars at $4\frac{1}{2}\%$,

and y = the number of dol-
lars at $3\frac{1}{2}\%$.

Then $x + y = 2000,$ (1)

and $.035y - .045x = 10. (2)$

[(1) $\cdot 35$ - (2) $\cdot 1000$] $\div 80,$

$x = 750.$

Whence $y = 1250.$

17. Let n_1 = the number of dol-
lars at 4%,

and n_2 = the number of dol-
lars at 6%.

Then $n_1 + n_2 = 5000,$ (1)

and

$5\left(\frac{4n_1}{100}\right) - 3\left(\frac{6n_2}{100}\right) = 126. (2)$

[(2) $\cdot 100$ + (1) $\cdot 18$] $\div 38,$

$n_1 = 2700,$

Whence $n_2 = 2300.$

18. Let r = the value of a ruble
in cents,

and m = the value of a mark
in cents.

Then

$20m - 10r = 10,$ (1)

and $12m - 4r = 100. (2)$

[(1) $\cdot 2$ + (2) $\cdot 5$] $\div 20,$

$m = 24.$

Whence $r = 47.$

19. Let f = the value of a franc
in cents,

and F = the value of a florin
in cents.

Then $5f - 2F = 19,$ (1)

and $3f + F = 95. (2)$

[(1) + (2) $\cdot 2$] $\div 11,$

$f = 19.$

Whence $F = 38.$

21. Let t = the digit in tens' place,
and u = the digit in units'
place.

Then $t + u = 7,$ (1)

and $10t + u + 27 = 10u + t,$

or $t - u = -3. (2)$

[(1) + (2)] $\div 2,$ $t = 2,$

and $u = 5.$

Therefore the number is 25.

22. Let t = the digit in tens' place,
and u = the digit in units'
place.

Then $t - 2u = 0$, (1)
 and $10t + u - 36 = 10u + t$,
 or $t - u = 4$. (2)
 $[(1) - (2)] \div -1$,
 $u = 4$,
 and $t = 8$.

Therefore the number is 84.

23. Let t = the digit in tens' place,
 and u = the digit in units'
 place.

Then $\frac{10t + u}{t + u} = 4$,
 or $6t - 3u = 0$; (1)
 and $10t + u + 36 = 10u + t$,
 or $3t - 3u = -12$. (2)
 $[(1) - (2)] \div 3$, $t = 4$,
 and $u = 8$.

Therefore the number is 48.

24. Let t = the digit in tens' place,
 and u = the digit in units'
 place.

Then $\frac{10t + u + 3}{t + u} = 9$,
 or $t - 8u = -3$; (1)
 and $\frac{10t + u}{3u} = 17$,
 or $t - 5u = 0$. (2)
 $[(1) - (2)] \div -3$,
 $u = 1$,
 and $t = 5$.

Therefore the number is 51.

25. Let t = the digit in tens' place,
 and u = the digit in units'
 place.

Then $\frac{10t + u}{t + u} = 7$,
 or $t - 2u = 0$; (1)
 and $\frac{10u + t}{t + u + 3} = 3$,

or $-2t + 7u = 9$. (2)
 $[(1) \cdot 2 + (2)] \div 3$,
 $u = 3$,
 and $t = 6$.

Therefore the number is 63.

26. Let n_1 = the first number,
 and n_2 = the second number.

Then $\frac{1}{n_1} + \frac{1}{n_2} = \frac{5}{24}$, (1)
 and $\frac{1}{n_1} - \frac{1}{n_2} = \frac{1}{24}$. (2)
 $[(1) + (2)]$ gives $n_1 = 8$.
 Whence $n_2 = 12$.

27. Let n_1 = the first number,
 and n_2 = the second number.

Then $\frac{1}{n_1} - \frac{1}{n_2} = \frac{7}{6}$, (1)
 and $\frac{n_2}{n_1} = \frac{15}{8}$. (2)

Substituting $\frac{15n_1}{8}$ for n_2 from (2)
 into (1),
 $\frac{1}{n_1} - \frac{8}{15n_1} = \frac{7}{6}$. (3)
 $[(3) \cdot 30n_1] \div 35$,
 $n_1 = \frac{2}{5}$.
 Then $n_2 = \frac{3}{4}$.

28. Let w_1 = the number of grams
 in first pan,
 and w_2 = the number of grams
 in second pan.

Then $w_1 - 15 = \frac{w_2 + 15}{2}$,
 or $2w_1 - w_2 = 45$; (1)
 and $w_2 - 85 = w_1 + 85$,
 or $w_2 - w_1 = 170$. (2)
 $(1) + (2)$, $w_1 = 215$.
 Then $w_2 = 385$.

29. Let a = the number of dollars A had,
 and b = the number of dollars B had.
 Then $2(a - 20) = b + 20,$
 or $2a - b = 60;$ (1)
 and $a - 20 + 75 = 5(b + 20 - 75),$
 or $a - 5b = -330.$ (2)
 $[(1) - (2) \cdot 2] \div 4,$ $b = 80.$
 Then $a = 70.$

30. Let c_1 = the number of feet in the circumference of the fore wheel,
 and c_2 = the number of feet in the circumference of the rear wheel.
 Then $c_2 - c_1 = 2,$ (1)
 and $\frac{155}{c_1} = \frac{186}{c_2},$
 or $5c_2 - 6c_1 = 0.$ (2)
 $[(1) \cdot 5 - (6)],$ $c_1 = 10.$
 Then $c_2 = 12.$

31. Let l = the number of feet in the length of the rectangle,
 and w = the number of feet in the width of the rectangle.
 Then $(l + 1)(w + 1) = lw + 18,$
 or $w + l = 17;$ (1)
 and $(l - 1)(w - 1) = lw - 16,$
 or $w + l = 17.$ (2)
 $(1) - (2),$ $0 = 0.$

Therefore an infinite number of sets of dimensions satisfy the problem.
 The sum of each set is 17 feet.

32. Let a = the number of days A requires, and b = the number of days B requires. Then $\frac{1}{a} + \frac{1}{b} = \frac{5}{12},$ (1) and $a - \frac{2b}{3} = 0.$ (2) Substituting $\frac{2b}{3}$ for a from (2) into (1), $\frac{3}{2b} + \frac{1}{b} = \frac{5}{12}.$ Whence $b = 6.$ Then $a = 4.$	33. Let a = the number of days A requires, and b = the number of days B requires. Then $\frac{1}{a} + \frac{1}{b} = \frac{5}{36},$ (1) and $\frac{5}{a} + \frac{5}{b} + \frac{11}{3a} = 1.$ (2) $[(1) \cdot 5 - (2)]$ gives $-\frac{11}{3a} = -\frac{11}{36},$ or $a = 12.$ Whence $b = 18.$
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34. Let l = the length in feet,
and w = the width in feet.

Then

$$(l + 4)(w - 2) = lw + 8,$$

$$\text{or } 2w - l = 8; \quad (1)$$

$$\text{and } (l - 1)(w + 3) = lw + 33,$$

$$\text{or } -w + 3l = 36. \quad (2)$$

$$[(1) + (2) \cdot 2] \div 5,$$

$$l = 16.$$

$$\text{Whence } w = 12.$$

$$\text{Area} = \frac{16 \cdot 12}{9} = 21\frac{1}{3} \text{ square yards.}$$

35. Let l = the length in feet,

and w = the width in feet.

$$\text{Then } lw = (l + 10)(w - 6),$$

$$\text{or } 5w - 3l = 30; \quad (1)$$

$$\text{and } lw = (l + 4)(w - 3),$$

$$\text{or } 4w - 3l = 12. \quad (2)$$

$$(1) - (2), \quad w = 18.$$

$$\text{Whence } l = 20.$$

36. Let

and

Then

x = the first number of the first pair,

y = the second number of the first pair.

$x + 2$ = the first number of the second pair,

$y - 15$ = the second number of the second pair,

$x + 3$ = the first number of the third pair,

and $y - 18$ = the second number of the third pair.

Therefore

$$xy = (x + 2)(y - 15),$$

$$\text{or } 2y - 15x = 30; \quad (1)$$

$$\text{and } xy = (x + 3)(y - 18),$$

$$\text{or } 3y - 18x = 54. \quad (2)$$

$$[(1) \cdot 3 - (2) \cdot 2] \div -9, \quad x = 2.$$

$$\text{Then } y = 30.$$

The pairs are 2, 30; 4, 15; and 5, 12 respectively.

37. Let

and

Then

x = the number of men,

y = the number of dollars each paid.

$$(x + 3)(y - 200) = xy,$$

$$\text{or } 3y - 200x = 600; \quad (1)$$

$$\text{and } (x - 4)(y + 500) = xy,$$

$$\text{or } -4y + 500x = 2000. \quad (2)$$

$$[(1) \cdot 4 + (2) \cdot 3] \div 700, \quad x = 12.$$

$$\text{Then } y = 1000,$$

$$\text{and } xy = 12,000.$$

38. Let x = the rate of the man in still water in miles per hour,

and y = the rate of the river in miles per hour.

$$\text{Then } \frac{10}{x + y} = 2,$$

$$\text{or } x + y = 5; \quad (1)$$

$$\text{and } \frac{10}{x - y} = 2\frac{1}{2},$$

$$\begin{array}{ll}
 \text{or} & x - y = 4. \\
 [(1) + (2)] 2, & x = \frac{9}{2}. \\
 \text{Then} & y = \frac{1}{2}.
 \end{array} \tag{2}$$

39. Let x = the rate of the boat in still water in miles per hour,
and y = the rate of the current in miles per hour.

$$\begin{array}{ll}
 \text{Then} & \frac{36}{x + y} = 3, \\
 \text{or} & x + y = 12;
 \end{array} \tag{1}$$

$$\begin{array}{ll}
 \text{and} & \frac{24}{x - y} = 3, \\
 \text{or} & x - y = 8.
 \end{array} \tag{2}$$

$$\begin{array}{ll}
 [(1) + (2)] \div 2, & x = 10. \\
 \text{Then} & y = 2.
 \end{array}$$

40. Let x = the rate of the current in miles per hour,
and y = the number of miles from A to B.

$$\begin{array}{ll}
 \text{Then} & \frac{y}{x + 8\frac{1}{2}} = 14, \\
 \text{or} & 14x - y = -119;
 \end{array} \tag{1}$$

$$\begin{array}{ll}
 \text{and} & \frac{\frac{y}{2}}{8\frac{1}{2} - x} = 10, \\
 \text{or} & 20x + y = 170. \\
 [(1) + (2)] \div 34, & x = \frac{3}{2}. \\
 \text{Then} & y = 140.
 \end{array} \tag{2}$$

41. Let x = the rate of the stream in miles per hour,
and y = the number of miles from A to C.

$$\begin{array}{ll}
 \text{Then} & \frac{y}{12 + x} = 7, \\
 \text{or} & 7x - y = -84;
 \end{array} \tag{1}$$

$$\begin{array}{ll}
 \text{and} & \frac{y - 36}{12 - x} = 5, \\
 \text{or} & 5x + y = 96.
 \end{array} \tag{2}$$

$$\begin{array}{ll}
 [(1) + (2)] \div 12, & x = 1. \\
 \text{Then} & y = 91.
 \end{array}$$

42. Let d = the distance from A to B in miles,
and r = the usual rate in miles per hour.

$$\begin{array}{ll}
 \text{Then} & \frac{\frac{d}{5r}}{\frac{d}{r}} = \frac{d}{r} - 1, \\
 & 4
 \end{array}$$

$$\text{or} \quad d - 5r = 0; \quad (1)$$

$$\text{and} \quad \frac{d}{24} = \frac{d}{r} - \frac{5}{6},$$

$$\text{or} \quad rd - 24d + 20r = 0. \quad (2)$$

Substituting $5r$ for d from (1) into (2),

$$5r^2 - 100r = 0.$$

$$\text{Whence} \quad r = 0 \text{ or } 20.$$

$$\text{Then, from (1),} \quad d = 0 \text{ or } 100.$$

The set $\begin{cases} r = 0 \\ d = 0 \end{cases}$ is rejected.

43. Let d = the distance in miles from A to B,
and r = the usual rate in miles per hour.

$$\text{Then} \quad \frac{d}{6r} = \frac{d}{r} - \frac{2}{5},$$

$$\text{or} \quad 5d - 12r = 0; \quad (1)$$

$$\text{and} \quad \frac{15}{r} + \frac{d - 15}{\frac{6r}{5}} = \frac{d}{r} - \frac{3}{20},$$

$$\text{or} \quad 5d - 9r = 75. \quad (2)$$

$$[(1) - (2)] \div -3, \quad r = 25.$$

$$\text{Whence} \quad d = 60.$$

44. Let x = the number of feet in the length of the passenger train,
and y = the number of feet in the length of the freight train.

$$\text{Then} \quad 2x - y = 0, \quad (1)$$

$$\text{and} \quad \frac{x + y}{66 + 44} = 15,$$

$$\text{or} \quad x + y = 1650. \quad (2)$$

$$[(1) + (2)] \div 3, \quad x = 550.$$

$$\text{Whence} \quad y = 1100.$$

45. Let p = the number of feet in the length of the passenger train,
and f = the number of feet in the length of the freight train.

$$\text{Then} \quad f - p = 350, \quad (1)$$

$$\text{and} \quad \frac{p + f}{5280(45 - 15)} = \frac{75}{60 \cdot 60},$$

$$\text{or} \quad p + f = 1650. \quad (2)$$

$$[(1) + (2)] \div 2, \quad f = 1000.$$

$$\text{Whence} \quad p = 650.$$

46. Let p = the rate of the passenger train in feet per minute,
and f = the rate of the freight train in feet per minute.

$$\begin{aligned}
 &\text{Then} \quad \frac{1540 + 660}{p + f} = \frac{1}{3}, \\
 &\text{or} \quad p + f = 6600; \quad (1) \\
 &\text{and} \quad \frac{1540 + 660}{p - f} = \frac{5}{3}, \\
 &\text{or} \quad p - f = 1320. \quad (2) \\
 &[(1) + (2)] \div 2, \quad p = 3960, \text{ feet per minute or 45 miles per hour,} \\
 &\text{and} \quad f = 2640, \text{ feet per minute or 30 miles per hour.}
 \end{aligned}$$

47. Let R_1 = the rate of faster in miles per hour,
and R_2 = the rate of slower in miles per hour.

$$\begin{aligned}
 &\text{Then} \quad \frac{220}{3600} (R_1 + R_2) = \frac{1}{4}, \\
 &\text{or} \quad R_1 + R_2 = \frac{900}{22}; \quad (1) \\
 &\text{and} \quad \frac{220 R_1 - 220 R_2}{3600} = \frac{1}{4}, \\
 &\text{or} \quad R_1 - R_2 = \frac{90}{22}. \quad (2) \\
 &[(1) + (2)] \div 2, \quad R_1 = 22\frac{1}{2}. \\
 &\text{Whence} \quad R_2 = 18\frac{9}{22}.
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad &3x + 7y = 17a, \quad (1) \\
 &10x - 4y = 2a. \quad (2) \\
 &[(1) \cdot 10 - (2) \cdot 3] \div 82, \\
 &\quad y = 2a. \\
 &\text{Whence} \quad x = a. \\
 2. \quad &3x - y = 10b, \quad (1) \\
 &4x + 9y = 3b. \quad (2) \\
 &[(1) \cdot 9 + (2)] \div 31, \quad x = 3b. \\
 &\text{Whence} \quad y = -b. \\
 3. \quad &5m - 4n = 10a - 4, \quad (1) \\
 &m - 2na = 0. \quad (2) \\
 &[(1) - (2) \cdot 5] \div (10a - 4), \\
 &\quad n = 1. \\
 &\text{Whence} \quad m = 2a. \\
 4. \quad &11h + 5k = 33c, \quad (1) \\
 &\quad \frac{h}{c} - \frac{k}{2c} = 3. \quad (2) \\
 &[(1) + (2) \cdot 10c] \div 21, \\
 &\quad h = 3c. \\
 &\text{Whence} \quad k = 0. \\
 5. \quad &12R_1 - 11R_2 = a + 12b, \quad (1) \\
 &\quad R_1 + R_2 = 2a + b. \quad (2) \\
 &[(1) + (2) \cdot 11] \div 23, \\
 &\quad R_1 = a + b. \\
 &\text{Whence} \quad R_2 = a. \\
 6. \quad &8p + 9q = 4a + 9a_1, \quad (1) \\
 &\quad \frac{p}{2} - 3q = \frac{a - 12a_1}{4}. \quad (2) \\
 &[(1) - (2) \cdot 16] \div 57, \\
 &\quad q = a_1. \\
 &\text{Whence} \quad p = \frac{a}{2}. \\
 7. \quad &\frac{7cx}{3} - \frac{5y}{2} = -3c, \quad (1) \\
 &\quad x + \frac{11y}{4} = 11c + 3. \quad (2) \\
 &[(1) \cdot 12 - (2) \cdot 28c] \div (-30 - 77c), \\
 &\quad y = 4c. \\
 &\text{Whence} \quad x = 3.
 \end{aligned}$$

$$8. \quad 7.5x + 3y = 6a, \quad (1)$$

$$.25x + .5y = 0. \quad (2)$$

$$[(1) \cdot 10 - (2) \cdot 300] \div -120,$$

$$y = \frac{-a}{2}.$$

$$\text{Whence} \quad x = a.$$

$$9. \quad \frac{h}{2a} - \frac{2k}{a} = -5, \quad (1)$$

$$\frac{3h}{a} - \frac{7k}{4a} = \frac{3}{4}. \quad (2)$$

$$[(1) \cdot 24a - (2) \cdot 4a] \div -41,$$

$$k = 3a.$$

$$\text{Whence} \quad h = 2a.$$

$$10. \quad dr + 3s = 1 - d, \quad (1)$$

$$7dr + 36s = 7 - 12d. \quad (2)$$

$$[(1) \cdot 7 - (2)] \div -15,$$

$$s = -\frac{d}{3}.$$

$$\text{Whence} \quad r = \frac{1}{d}.$$

$$11. \quad h - k = 0, \quad (1)$$

$$4 - \frac{h+k}{5c} = \frac{h-k}{2c}. \quad (2)$$

Substituting h for k from (1) into (2),

$$h = 10c.$$

$$\text{Whence} \quad k = 10c.$$

$$12. \quad (a+b)n = 1 - cm, \quad (1)$$

$$(a+b)m - 1 = -cn. \quad (2)$$

$$[(1) \cdot (a+b) - (2) \cdot c] \div [(a+b)^2 - c^2],$$

$$n = \frac{1}{a+b+c}.$$

$$\text{Whence} \quad m = \frac{1}{a+b+c}.$$

$$13. \quad \frac{m}{b_1 - b_2} + \frac{n}{b_1 + b_2} = 2, \quad (1)$$

$$m + n = 2b_1. \quad (2)$$

Substituting $2b_1 - n$ for m from (2) into (1) and solving for n ,

$$\begin{aligned} n &= b_1 + b_2, \\ m &= b_1 - b_2. \end{aligned}$$

$$14. \quad \frac{1}{x} + \frac{1}{y} = 2b, \quad (1)$$

$$\frac{2}{x} - \frac{3}{y} = 5c - b. \quad (2)$$

$[(1) \cdot 3 + (2)]$ solved for x gives

$$x = \frac{1}{b+c}.$$

$$\text{Whence} \quad y = \frac{1}{b-c}.$$

$$15. \quad \frac{1}{x+a_1} + \frac{1}{y} = \frac{a_1+a_2}{2a_1a_2}, \quad (1)$$

$$\frac{a_1}{x+a_1} - \frac{a_2}{y} = 0. \quad (2)$$

$[(1) \cdot a_1 - (2)]$ solved gives

$$y = 2a_2.$$

$$\text{Whence} \quad x = a_1.$$

$$16. \quad \frac{m+2a}{n-a} = 1,$$

$$\text{or} \quad m - n = -3a; \quad (1)$$

$$\frac{\frac{6}{5}n - a}{3m - 2a} - \frac{2n}{5m} = 0,$$

$$\text{or} \quad -5am + 4an = 0. \quad (2)$$

$$[(1) \cdot 5a + (2)] \div -a,$$

$$n = 15a.$$

$$\text{Whence} \quad m = 12a.$$

$$17. \quad kx - ry = 0, \quad (1)$$

$$x + y - h = 0. \quad (2)$$

$$[(1) + (2) \cdot r] \div (k+r),$$

$$x = \frac{rh}{r+k}.$$

$$\text{Whence} \quad y = \frac{hk}{r+k}.$$

$$18. \quad x + dy = 3, \quad (1)$$

$$d(x-3) - y = 0. \quad (2)$$

$$[(1) + (2) \cdot d] \div (d^2 + 1),$$

$$x = 3.$$

$$\text{Whence} \quad y = 0.$$

$$19. \quad ax - by = c, \quad (1)$$

$$x + y = b. \quad (2)$$

$$[(1) + (2) \cdot b] \div (a + b),$$

$$x = \frac{b^2 + c}{a + b}.$$

$$\text{Whence} \quad y = \frac{ab - c}{a + b}.$$

$$20. \quad ax + by = c, \quad (1)$$

$$dx + ey = f. \quad (2)$$

$$[(1) \cdot d - (2) \cdot a] \div (bd - ae),$$

$$y = \frac{cd - af}{bd - ae}.$$

$$\text{Whence} \quad x = \frac{bf - ec}{bd - ae}.$$

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1. ac dollars.

2. $\frac{b}{a}$ dollars; $\frac{bc}{a}$ dollars.

3. (a) $2a + 2b$, ab ;

(b) $6a + 6b$, $9ab$;

(c) 3;

(d) 9.

$$4. \quad \frac{8 \cdot 10}{2} = 40.$$

$$5. \quad \frac{8 \cdot b}{2} = 4b.$$

$$6. \quad \frac{a \cdot b}{2}.$$

$$7. \quad \frac{(a + b)(a - b)}{2} = \frac{a^2 - b^2}{2}.$$

$$8. \quad \frac{(x - 2y)(x + 2y)}{2} = \frac{x^2}{2} - 2y^2.$$

$$9. \quad \frac{2k}{b}.$$

10. Let x be the required number of inches the base must be increased.

$$\text{Then} \quad \frac{(a - 2)(10 + x)}{2} = \frac{a \cdot 10}{2}.$$

$$\text{Whence} \quad x = \frac{20}{a - 2}.$$

11. Let x be the required number of feet the base must be decreased.

$$\text{Then} \quad \frac{(a + h)(b - x)}{2} = \frac{ab}{2}.$$

$$\text{Whence} \quad x = \frac{bh}{a + h}.$$

12. Let x = the greater number, and y = the less number.

$$\text{Then} \quad x + y = s, \quad (1)$$

$$\text{and} \quad x - y = d. \quad (2)$$

$$[(1) + (2)] \div 2, \quad x = \frac{s + d}{2}.$$

$$\text{Whence} \quad y = \frac{s - d}{2}.$$

13. Let f = the first number, and s = the second number.

$$\text{Then} \quad f - as = 0, \quad (1)$$

$$\text{and} \quad f - s = b. \quad (2)$$

$$[(1) - (2)] \div (1 - a),$$

$$s = \frac{b}{a - 1}.$$

$$\text{Whence} \quad f = \frac{ab}{a - 1}.$$

14. Let x = the first number, and y = the second number.

$$\text{Then} \quad x + y = b, \quad (1)$$

$$\text{and} \quad \frac{x}{y} = a. \quad (2)$$

$$[(1) - (2) \cdot y] \div (a + 1),$$

$$y = \frac{b}{a + 1}.$$

$$\text{Whence} \quad x = \frac{ab}{a + 1}.$$

15. Let $\frac{n}{d}$ = the fraction.

$$\text{Then} \quad \frac{n + a}{d} = 2,$$

or $n - 2d = -a$; (1) and $y =$ the number of feet
the b -pound boy is
and $\frac{n}{d+b} = 1$, from the fulcrum.

or $n - d = b$. (2) Then $ax - by = 0$, (1)
and $x + y = l$. (2)
[(1) - (2)] $\div -1$,
 $d = b + a$.
[(1) + (2) $\cdot b$] $\div (a + b)$,

Whence $n = a + 2b$.

The fraction is $\frac{a+2b}{a+b}$.

$$x = \frac{bl}{a+b}.$$

Whence $y = \frac{al}{a+b}.$

16. Let $n =$ the numerator,
and $d =$ the denominator.

Then $\frac{n+1}{d} = x$,

or $n - dx = -1$; (1)

and $\frac{n}{d+2} = y$,

or $n - dy + 2y = 0$. (2)

[(1) - (2)] $\div (y - x)$,

$$d = \frac{2y-1}{y-x}.$$

Whence $n = \frac{2xy-y}{y-x}.$

17. Let $n =$ the numerator,
and $d =$ the denominator.

Then $\frac{n}{d} = b$,

or $n - db = 0$; (1)

and $\frac{n+2}{d} = c$,

or $n - cd = -2$. (2)

[(1) - (2)] $\div (c - b)$,

$$d = \frac{2}{c-b}.$$

Whence $n = \frac{2b}{c-b}.$

18. Let $x =$ the number of feet
the a -pound boy is
from the fulcrum,

19. Let $x =$ the number of books
at 80 cents each,
and $y =$ the number of books
at 110 cents each.

Then

$$80x + 110y = 100h, \quad (1)$$

and $110x + 80y = 100k$. (2)

[(1) $\cdot 11 - (2) \cdot 8$] $\div 570$,

$$y = \frac{110h - 80k}{57}.$$

Whence $x = \frac{110k - 80h}{57}.$

20. Let $x =$ the cost of the first
book in cents,

and $y =$ the cost of the second
book in cents.

Then $x - y = d$, (1)

and $x + y = 100c$. (2)

[(1) + (2)] $\div 2$, $x = \frac{d + 100c}{2}.$

Whence $y = \frac{100c - d}{2}.$

21. Let $x =$ the number of dol-
lars A has,

and $y =$ the number of dol-
lars B has.

Then $x + y = k$, (1)

and $x - h = y + h$. (2)

$$[(1) + (2)] \div 2, \quad x = \frac{2h + k}{2}.$$

Whence $y = \frac{k - 2h}{2}.$

22. Let x = the number of dollars A has,
and y = the number of dollars B has.

Then $x - h = y + h, \quad (1)$

and $x + k = 2y - 2k. \quad (2)$

$[(1) - (2)], \quad y = 3k + 2h.$

Whence $x = 3k + 4h.$

23. Let x = the number of dollars A has,
and y = the number of dollars B has.

Then $x - 10 + h = y + 10, \quad (1)$

and $x + k = 3(y - k). \quad (2)$

$[(1) - (2)] \div 2, \quad x = 30 + 2k - \frac{3h}{2}.$

Whence $y = 10 + 2k - \frac{h}{2}.$

24. Let x = the number of dollars A has,
and y = the number of dollars B has.

Then $x + y = 40, \quad (1)$

and $x - h + k = y + h - k. \quad (2)$

$[(1) - (2)] \div 2, \quad y = 20 - h + k.$

Whence $x = 20 + h - k.$

25. Let x = the number of dollars A had at first,
and y = the number of dollars B had at first.

Then $x - r = \frac{y + r}{2},$

or $2x - y = 3r; \quad (1)$

and $\frac{3}{4}(x - r + 8) = y + r - 8,$

or $-3x + 4y = -7r + 56. \quad (2)$

$[(1) \cdot 3 + (2) \cdot 2] \div 5,$

$y = \frac{112 - 5r}{5}.$

Whence $x = \frac{56 + 5r}{5}.$

26. Let x = the number of dollars at $a\%$,
and y = the number of dollars at $b\%$.

Then $x + y = 1000, \quad (1)$

and $\frac{ax}{100} + \frac{by}{100} = c. \quad (2)$

$[(1) \cdot a - (2) \cdot 100] \div (a - b),$

$y = \frac{1000a - 100c}{a - b}.$

Then $x = \frac{1000b - 100c}{b - a}.$

27. Let m = the number of dollars at 5% ,
and n = the number of dollars at 4% .

Then $m + n = x, \quad (1)$

and $\frac{5m}{100} + \frac{4n}{100} = y. \quad (2)$

$[(1) \cdot 5 - (2) \cdot 100],$

$n = 5x - 100y.$

Whence $m = 100y - 4x.$

28. Let x = the number of days A requires,
and y = the number of days B requires.

Then $3x - y = 0, \quad (1)$

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{c}. \quad (2)$

Substituting $3x$ for y from (1) into (2),

$x = \frac{4c}{3}.$

Whence $y = 4c.$

29. Let x = the number of days

A requires,

and y = the number of days

B requires.

Then $hx - y = 0$, (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$. (2)

Substituting hx for y from (1) into (2),

$$x = \frac{4h + 4}{h}.$$

Whence $y = 4h + 4$.

30. Let x = the number of days

A requires,

and y = the number of days

B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{h}$, (1)

and $\frac{6}{x} = \frac{2}{3}$. (2)

From (2), $x = 9$.

Whence $y = \frac{9h}{9 - h}$.

31. Let x = the number of days

A requires,

and y = the number of days

B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{5}$, (1)

and $\frac{k}{x} = \frac{2}{5}$. (2)

From (2), $x = \frac{5k}{2}$.

Whence $y = \frac{5k}{k - 2}$.

32. Let x = the number of days

A requires,

and y = the number of days

B requires.

Then $2x - y = 0$, (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$. (2)

Substituting $2x$ for y from (1) into (2),

$$x = \frac{3n}{2}.$$

Whence $y = 3n$.

33. Let x = the number of days

A requires,

and y = the number of days

B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{p}$, (1)

and $qx - y = 0$. (2)

Substituting qx for y from (2) into (1),

$$x = \frac{pq + p}{q}.$$

Whence $y = pq + p$.

34. Let h represent the required number of hours.

Then $\frac{n}{3} + \frac{n}{4} = h$,

and $h = \frac{7n}{12}$.

35. Let h represent the required number of hours for traveling.

Then $3h = A$'s distance,

and $5h = B$'s distance.

Hence $3h + 5h = k$,

and $h = \frac{k}{8}$.

Whence $3h = \frac{3k}{8}$,

and $5h = \frac{5k}{8}$.

36. Then $\frac{n}{p} + \frac{n}{q} = h$,

and $h = \frac{nq + np}{pq}$.

37. Let x represent the number of hours which must pass.

Then

$$3(x - h) + 5x = k,$$

and
$$x = \frac{k + 3h}{8}.$$

Whence

$$\text{A's distance} = \frac{3k - 15h}{8} \text{ miles,}$$

and B's distance $= \frac{5k + 15h}{8}$ miles.

38. Let n represent the required number of hours.

Then $pn + qn = c.$

Whence
$$n = \frac{c}{p + q}.$$

39. A travels $\frac{pc}{p + q}$ and B travels $\frac{qc}{p + q}.$

40. Let r represent the rate of carriage in miles per hour.

Then
$$\frac{d}{r} + h = \frac{d}{3}.$$

Whence
$$r = \frac{3d}{d - 3h}.$$

41. Then
$$\frac{d}{r} + h = \frac{d}{c}.$$

Whence
$$r = \frac{cd}{d - ch}.$$

42. Let r represent the rate of riding in miles per hour.

Then
$$\frac{p}{r} + \frac{p}{q} = t.$$

Whence
$$r = \frac{pq}{qt - p}.$$

43. Let a = the rate of A in miles per hour,

and b = the rate of B in miles per hour.

Then $an + bn = 50, \quad (1)$

and $a - b = 2. \quad (2)$

$[(1) + (2) \cdot n] \div 2n,$

$$a = \frac{25 + \bar{n}}{n}.$$

Whence
$$b = \frac{25 - \bar{n}}{n}.$$

44. Then

$$an + bn = h, \quad (1)$$

and $a - b = k. \quad (2)$

$[(1) + (2) \cdot n] \div 2n,$

$$a = \frac{h + kn}{2n}.$$

Whence
$$b = \frac{h - kn}{2n}.$$

45. Let d represent the distance in miles.

Then
$$\frac{d}{p} + \frac{d}{q} = t.$$

Whence
$$d = \frac{tpq}{p + q}.$$

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1. $m + n - 2p = 13, \quad (1)$

$m - 3n - p = -3, \quad (2)$

$m - n + 4p = -17. \quad (3)$

$(1) - (2), \quad 4n - p = 16. \quad (4)$

$(1) - (3), \quad 2n - 6p = 30. \quad (5)$

$[(4) - (5) \cdot 2] \div 11, \quad p = -4.$

Whence $n = 3,$

and $m = 2.$

2. $x + y + 3z = \frac{7}{2}, \quad (1)$

$x - 2y + 4z = 7, \quad (2)$

$2x - 11y - 24z = 5. \quad (3)$

$(1) - (2), \quad 3y - z = -\frac{7}{2}. \quad (4)$

$(1) \cdot 2 - (3), \quad 13y + 30z = 2. \quad (5)$

$[(4) \cdot 30 + (5)] \div 103, \quad y = -1.$

Whence $z = \frac{1}{2},$

and $x = 3.$

$$\begin{aligned}
 3. \quad & x + y + z = -1, \quad (1) \\
 & 3x - y - 5z = 13, \quad (2) \\
 & 5x + 3y + 2z = 1. \quad (3) \\
 (1) + (2), \quad & 4x - 4z = 12. \quad (4) \\
 (1) \cdot 3 - (3), \quad & -2x + z = -4. \quad (5) \\
 [(4) + (5) \cdot 2] \div -2, \quad & z = -2. \\
 \text{Whence} \quad & x = 1, \\
 \text{and} \quad & y = 0.
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & 2h + 3k - 4l = -26, \quad (1) \\
 & 3h - k + 27l = 87\frac{1}{2}, \quad (2) \\
 & h + 5k + 33l = 74\frac{1}{2}. \quad (3) \\
 [(1) - (3) \cdot 2] \div -7, \quad & \\
 & k + 10l = 25. \quad (4) \\
 [(2) \cdot 1 - (3) \cdot 3] \div -8, \quad & \\
 & 2k + 9l = 17. \quad (5) \\
 [(4) \cdot 2 - (5)] \div 11, \quad & l = 3. \\
 \text{Whence} \quad & k = -5, \\
 \text{and} \quad & h = \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & 2m + 3n - 4p = -3, \quad (1) \\
 & m + n + 3p = -9, \quad (2) \\
 & m + 2n - 7p = 6. \quad (3) \\
 (1) - (2) &= (3).
 \end{aligned}$$

Hence the system has an infinite number of sets of roots.

$$\begin{aligned}
 6. \quad & x + 8y + 5z = 1, \quad (1) \\
 & 3x + 10z + 4y = -5, \quad (2) \\
 & x + 4z = 0. \quad (3) \\
 [(1) - (2) \cdot 2], \quad & 5x + 15z = -11. \quad (4) \\
 [(4) - (3) \cdot 5] \div -5, \quad & z = 2\frac{1}{5}. \\
 \text{Whence} \quad & x = -8\frac{4}{5}, \\
 \text{and} \quad & y = -\frac{3}{20}.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & 2h - 3l + 4k - 2 = 0, \quad (1) \\
 & 3h - 3l - 15 = 0, \quad (2) \\
 & 7h - 4k - 31 = 0. \quad (3) \\
 [(1) - (2)] \div -1, \quad & \\
 & h - 4k = 13. \quad (4) \\
 [(3) - (4)] \div 6, \quad & h = 3. \\
 \text{Whence} \quad & k = -\frac{5}{2}, \\
 \text{and} \quad & l = -2.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & 4r - 10s = 5, \quad (1) \\
 & 6r - t = 3, \quad (2) \\
 & 5s + 2t = -\frac{3}{2}. \quad (3) \\
 [(1) \cdot 3 - (2) \cdot 2], \quad & \\
 & -30s + 2t = 9. \quad (4) \\
 [(4) - (3)] \div -35, \quad & s = -\frac{3}{10}. \\
 \text{Whence} \quad & r = \frac{1}{2}, \\
 \text{and} \quad & t = 0.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & 2a_1 - 3a_2 = 4, \quad (1) \\
 & 3a_1 + a_3 = 5, \quad (2) \\
 & a_2 - 2a_3 = 2. \quad (3) \\
 [(1) \cdot 3 - (2) \cdot 2] \div -1, \quad & \\
 & 9a_2 + 2a_3 = -2. \quad (4) \\
 [(4) + (3)] \div 10, \quad & a_2 = 0. \\
 \text{Whence} \quad & a_1 = 2, \\
 \text{and} \quad & a_3 = -1.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & 3r_1 + 5r_2 = 74, \quad (1) \\
 & r_1 - 2r_3 = -16, \quad (2) \\
 & 7r_3 - 4r_2 = 44. \quad (3) \\
 [(1) - (2) \cdot 3], \quad & \\
 & 5r_2 + 6r_3 = 122. \quad (4) \\
 [(3) \cdot 5 + (4) \cdot 4] \div 59, \quad & \\
 & r_3 = 12. \\
 \text{Whence} \quad & r_2 = 10, \\
 \text{and} \quad & r_1 = 8.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{1}{m} + \frac{1}{n} - \frac{1}{p} = 1, \quad (1) \\
 & \frac{1}{m} + \frac{1}{n} + \frac{1}{p} = \frac{2}{3}, \quad (2) \\
 & \frac{1}{m} - \frac{1}{n} + \frac{1}{p} = 0. \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 [(1) + (2)] \div 2, \quad & \\
 & \frac{1}{m} + \frac{1}{n} = \frac{5}{6}. \\
 [(2) - (3)] \div 2, \quad & \frac{1}{n} = \frac{1}{3}. \\
 \text{Whence} \quad & n = 3. \\
 \text{Then} \quad & m = 2, \\
 \text{and} \quad & p = -6.
 \end{aligned}$$

$$12. \quad \frac{2}{m} - \frac{3}{n} + \frac{10}{p} = -3, \quad (1)$$

$$\frac{4}{m} + \frac{5}{p} + \frac{6}{n} = 15, \quad (2)$$

$$\frac{1}{m} - \frac{1}{n} + \frac{5}{p} = -\frac{1}{2}. \quad (3)$$

$$[(1) - (2) \cdot 2] \div -3,$$

$$\frac{2}{m} + \frac{5}{n} = 11. \quad (4)$$

$$(2) - (3), \quad \frac{3}{m} + \frac{7}{n} = \frac{31}{2}. \quad (5)$$

$$[(4) \cdot 3 - (5) \cdot 2], \quad \frac{1}{n} = 2.$$

$$\text{Whence} \quad n = \frac{1}{2}.$$

$$\text{Then} \quad m = 2,$$

$$\text{and} \quad p = 5.$$

$$13. \quad \frac{1}{A} + \frac{1}{B} = 2, \quad (1)$$

$$\frac{1}{A} + \frac{1}{C} = 3, \quad (2)$$

$$\frac{1}{B} + \frac{1}{C} = 4. \quad (3)$$

$$[(1) - (2)] + (3), \quad \frac{2}{B} = 3.$$

$$\text{Whence} \quad B = \frac{2}{3}.$$

$$\text{Then} \quad A = 2, \text{ and } C = \frac{2}{5}.$$

$$14. \quad r + s + t + u = 2.8, \quad (1)$$

$$r - s + t - u = 7.2, \quad (2)$$

$$r + 2s + 3t - 5u = 7, \quad (3)$$

$$r + s - 8t + u = -1.7. \quad (4)$$

$$(3) - (1), s + 2t - 6u = 4.2. \quad (5)$$

$$(3) - (2),$$

$$3s + 2t - 4u = -.2. \quad (6)$$

$$(4) - (2),$$

$$2s - 9t + 2u = -8.9. \quad (7)$$

$$[(7) - (5) \cdot 2],$$

$$-13t + 14u = -17.3. \quad (8)$$

$$[(6) - (5) \cdot 3],$$

$$-4t + 14u = -12.8. \quad (9)$$

$$[(9) - (8)] \div 9, \quad t = \frac{1}{2}.$$

$$\text{Whence} \quad u = -\frac{54}{70},$$

$$s = -\frac{10}{7},$$

$$\text{and} \quad r = \frac{9}{2}.$$

$$15. \quad \frac{2}{x} + \frac{3}{y} = 26, \quad (1)$$

$$\frac{4}{y} - \frac{10}{z} = 3, \quad (2)$$

$$\frac{1}{x} + \frac{5}{z} = \frac{25}{2}. \quad (3)$$

$$[(3) \cdot 2 + (2)], \quad \frac{2}{x} + \frac{4}{y} = 28. \quad (4)$$

$$(4) - (1), \quad \frac{1}{y} = 2.$$

$$\text{Whence} \quad y = \frac{1}{2}.$$

$$\text{Then} \quad x = \frac{1}{10},$$

$$\text{and} \quad z = 2.$$

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1. Let x , y , and z represent the first, the second, and the third numbers respectively.

$$\text{Then} \quad x + y = 76, \quad (1)$$

$$y + z = 54, \quad (2)$$

$$\text{and} \quad x + z = 58. \quad (3)$$

$$[(1) - (2) + (3)] \div 2,$$

$$x = 40.$$

$$\text{Whence} \quad y = 36,$$

$$\text{and} \quad z = 18.$$

2. Let x , y , and z represent the first, the second, and the third numbers respectively.

$$\text{Then} \quad x + y + z = 58, \quad (1)$$

$$x + y = 24, \quad (2)$$

$$\text{and} \quad \frac{x}{y} = 2. \quad (3)$$

$$(1) - (2), \quad z = 34.$$

$$[(2) - (3) \cdot y] \div 3, \quad y = 8.$$

$$\text{Then} \quad x = 16.$$

3. Let a , b , and c represent the sides of the triangle in feet.

$$\text{Then } a + b + c = 64, \quad (1)$$

$$a - b = 0, \quad (2)$$

$$\text{and } a - c = -10. \quad (3)$$

$$[(1) + (2) + (3)] \div 3,$$

$$a = 18.$$

$$\text{Whence } b = 18,$$

$$\text{and } c = 28.$$

4. Let a , b , and c represent the sides of the triangle in feet.

$$\text{Then } a + b = 52, \quad (1)$$

$$a - b = 12, \quad (2)$$

$$\text{and } a + b + c = 93. \quad (3)$$

$$[(1) + (2)] \div 2, \quad a = 32.$$

$$\text{Whence } b = 20,$$

$$\text{and } c = 41.$$

5. Let a , b , and c represent the sides of the triangle in feet.

$$\text{Then } a + b = 41, \quad (1)$$

$$a + c = 48, \quad (2)$$

$$\text{and } b + c = 43. \quad (3)$$

$$[(1) - (2) + (3)] \div 2,$$

$$b = 18.$$

$$\text{Whence } a = 23,$$

$$\text{and } c = 25.$$

6. Let x , y , and z represent the numbers.

$$\text{Then } x + y + z = 26, \quad (1)$$

$$\frac{x}{y} = 9,$$

$$\text{or } x - 9y = 0. \quad (2)$$

$$\text{Also } \frac{x + y}{z} = \frac{10}{3},$$

$$3x + 3y - 10z = 0. \quad (3)$$

$$[(1) \cdot 3 - (3) \cdot 1] \div 13,$$

$$z = 6.$$

$$\text{Whence } x = 18,$$

$$\text{and } y = 2.$$

7. Let x , y , and z represent the angles respectively in degrees.

$$\text{Then } x - y = 0, \quad (1)$$

$$x + y - z = 0, \quad (2)$$

$$x + y + z = 180. \quad (3)$$

$$[(2) + (3) + (1) \cdot 2] \div 4,$$

$$x = 45.$$

$$\text{Whence } y = 45,$$

$$\text{and } z = 90.$$

8. Let x , y , and z represent the angles respectively in degrees.

$$\text{Then } x - y = 0, \quad (1)$$

$$2x + 2y - z = 0, \quad (2)$$

$$x + y + z = 180. \quad (3)$$

$$[(2) + (3) + (1) \cdot 3] \div 6,$$

$$x = 30.$$

$$\text{Whence } y = 30,$$

$$\text{and } z = 120.$$

9. Let x , y , and z represent the number of degrees in A , B , and C respectively.

$$\text{Then } x - y = 17, \quad (1)$$

$$y - z = 20, \quad (2)$$

$$\text{and } x + y + z = 180. \quad (3)$$

$$(1) + (2), \quad x - z = 37. \quad (4)$$

$$(3) - (2), \quad x + 2z = 160. \quad (5)$$

$$[(5) - (4)] \div 3, \quad z = 41.$$

$$\text{Whence } x = 78,$$

$$\text{and } y = 61.$$

10. Let x , y , and z represent the angles respectively in degrees.

$$\text{Then } x + y - z = 36, \quad (1)$$

$$6x - 6y - z = 0, \quad (2)$$

$$\text{and } x + y + z = 180. \quad (3)$$

$$(1) + (3), \quad 2x + 2y = 216. \quad (4)$$

$$(2) + (3), \quad 7x - 5y = 180. \quad (5)$$

$$[(4) \cdot 5 + (5) \cdot 2] \div 24,$$

$$x = 60.$$

$$\text{Whence } y = 48,$$

$$\text{and } z = 72.$$

11. Let x , y , and z represent the time in hours required by A, B, and C respectively.

$$\text{Then } \frac{1}{x} + \frac{1}{y} = \frac{1}{2}, \quad (1)$$

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{3}, \quad (2)$$

$$\text{and } \frac{1}{y} + \frac{1}{z} = \frac{1}{4}. \quad (3)$$

$$(1) - (2) + (3), \quad \frac{2}{y} = \frac{5}{12}.$$

$$\text{Whence } y = \frac{24}{5}.$$

$$\text{Then } x = 3\frac{3}{7},$$

$$\text{and } z = 24.$$

12. Let x , y , and z represent the number of hours required by the pumps respectively.

$$\text{Then } \frac{1}{x} + \frac{1}{y} = \frac{1}{4}, \quad (1)$$

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{6}, \quad (2)$$

$$\text{and } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{13}{48}. \quad (3)$$

$$(3) - (1), \quad \frac{1}{z} = \frac{1}{48},$$

$$\text{or } z = 48.$$

$$\text{Then } y = 9\frac{3}{5},$$

$$\text{and } x = 6\frac{6}{7}.$$

13. Let n_1 = the first numerator,
 n_2 = the second numerator,
tor,

and d = each denominator.

$$\text{Then } \frac{n_1}{d} + \frac{n_2}{d} = 6,$$

$$\text{or } n_1 + n_2 - 6d = 0; \quad (1)$$

$$\frac{n_1 + 1}{d} - \frac{n_2 - 1}{d} = 0,$$

$$\text{or } n_1 - n_2 = -2, \quad (2)$$

$$\text{and } \frac{n_1}{d + 22} + \frac{n_2}{d + 22} = \frac{1}{2},$$

$$\text{or } 2n_1 + 2n_2 - d = 22. \quad (3)$$

$$[(3) - (1) \cdot 2] \div 11, \quad d = 2.$$

$$\text{Whence } n_1 = 5,$$

$$\text{and } n_2 = 7.$$

Therefore the fractions are $\frac{5}{2}$
and $\frac{7}{2}$.

14. Let x , y , and z be the first, second, and third digits respectively.

$$\text{Whence } x + y + z = 15, \quad (1)$$

$$z - y = 5, \quad (2)$$

$$100x + 10y + z + 396 = 100z + 10y + x,$$

$$\text{or } x - z = -4. \quad (3)$$

$$[(1) + (2) - (3)] \div 3, \quad z = 8.$$

$$\text{Whence } y = 3,$$

$$\text{and } x = 4.$$

Therefore the number is 438.

15. Let x , y , and z be the hundreds', tens', and units' digits respectively.

$$\text{Then } 100x + 10z + y = 100x + 10y + z + 54,$$

$$\text{or } z - y = 6; \quad (1)$$

$$100y + 10x + z = 100x + 10y + z - 360,$$

$$x - y = 4, \quad (2)$$

$$\text{or } x + y + z = 10. \quad (3)$$

$$(3) - (2), \quad 2y + z = 6. \quad (4)$$

$$[(4) - (1)] \div 3, \quad y = 0.$$

Whence $z = 6$,
and $x = 4$.

Therefore the number is 406.

16. Let a , b , c , and d be the thousands', hundreds', tens', and units' digits respectively.

Then $1000a + 100b + 10c + d + 2997 = 1000d + 100c + 10b + a$,
or $111a + 10b - 10c - 111d = -333$; (1)

$$2a - d = 0, \quad (2)$$

$$b - c = 0, \quad (3)$$

and $a + b + c + d = 9. \quad (4)$

$$[(1) - (3) \cdot 10] \div 111, \quad a - d = -3. \quad (5)$$

$$(2) - (5), \quad a = 3.$$

$$\text{Whence} \quad b = 0,$$

$$c = 0,$$

$$\text{and} \quad d = 6.$$

Therefore the number is 3006.

17. Let a , b , c , and d be the number of degrees in the angles respectively.

$$\text{Then } a + b + c + d = 360, \quad (1)$$

$$a + b = 200, \quad (2)$$

$$b + c = 180, \quad (3)$$

$$\text{and } b + c + d = 255. \quad (4)$$

$$(1) - (4), \quad a = 105.$$

$$\text{Then } b = 95,$$

$$c = 85,$$

$$\text{and } d = 75.$$

18. Let a , b , c , and d be the number of degrees in the angles respectively.

$$\text{Then } a + c = 180, \quad (1)$$

$$a - c = 30, \quad (2)$$

$$b - d = 36, \quad (3)$$

$$\text{and } a + b + c + d = 360. \quad (4)$$

$$(4) - (1), \quad b + d = 180. \quad (5)$$

$$[(5) + (3)] \div 2, \quad b = 108.$$

$$\text{Then } d = 72,$$

$$a = 105,$$

$$\text{and } c = 75.$$

19. Let a , b , c , and d be the sides taken in order.

$$\text{Then } a + c = 30, \quad (1)$$

$$b + d = 35, \quad (2)$$

$$a - b = 0, \quad (3)$$

$$\text{and } a + b - c - d + 17 = 0. \quad (4)$$

$$(1) + (2) + (3) + (4),$$

$$3a + b = 48. \quad (5)$$

$$[(5) + (3)] \div 4, \quad a = 12.$$

$$\text{Then } b = 12,$$

$$c = 18,$$

$$\text{and } d = 23.$$

20. Let x , y , and z be the number of dollars A, B, and C have respectively.

$$\text{Then } 2(x - y - z) = 100, \quad (1)$$

$$y + y - (x - y - z) - 2z = 100, \quad (2)$$

$$2(z + z) = 100. \quad (3)$$

$$(1) \div 2, \quad x - y - z = 50. \quad (4)$$

$$\text{From (2), } -x + 3y - z = 100. \quad (5)$$

$$\text{From (3), } z = 25.$$

$$\text{Whence } x = 175,$$

$$\text{and } y = 100.$$

21. Let a , b , and c be the number of dollars A, B, and C had at first respectively.

$$\begin{array}{l} \text{Then } a - (b + c) + (a - b - c) + (2a - 2b - 2c) = 64, \\ \text{or } a - b - c = 16; \end{array} \quad (1)$$

$$\begin{array}{l} b + b - (2c + a - b - c) + (3b - a - c) = 64, \\ \text{or } 3b - a - c = 32; \end{array} \quad (2)$$

$$\begin{array}{l} c + c + 2c - (2a - 2b - 2c) - (3b - a - c) = 64, \\ \text{or } 7c - a - b = 64. \end{array} \quad (3)$$

$$[(1) + (2)] \div 2, \quad b - c = 24. \quad (4)$$

$$[(1) + (3)] \div 2, \quad -b + 3c = 40. \quad (5)$$

$$[(4) + (5)] \div 2, \quad c = 32.$$

$$\begin{array}{l} \text{Then } b = 56, \\ \text{and } a = 104. \end{array}$$

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$$\begin{array}{l} 1. \quad a^4 + 2a^3 + 3a^2 + 2a + 1 \mid a^2 + a + 1 \\ (a^2)^2 = a^4 \\ \begin{array}{r} 2(a^2) = 2a^2 \\ 2a^2 + a \end{array} \quad \begin{array}{r} 2a^3 + 3a^2 \\ 2a^3 + a^2 \end{array} = (2a^2 + a)a \\ 2(a^2 + a) = 2a^2 + 2a \quad \begin{array}{r} 2a^2 + 2a + 1 \\ 2a^2 + 2a + 1 \end{array} = (2a^2 + a + 1) \cdot 1 \end{array}$$

Therefore the square roots are $\pm (a^2 + a + 1)$.

$$\begin{array}{l} 2. \quad x^4 - 8x^3 + 24x^2 - 32x + 16 \mid x^2 - 4x + 4 \\ (x^2)^2 = x^4 \\ \begin{array}{r} 2(x^2) = 2x^2 \\ 2x^2 - 4x \end{array} \quad \begin{array}{r} -8x^3 + 24x^2 \\ -8x^3 + 16x^2 \end{array} = (2x^2 - 4x)(-4x) \\ 2(x^2 - 4x) = 2x^2 - 8x \quad \begin{array}{r} 8x^2 - 32x + 16 \\ 8x^2 - 32x + 16 \end{array} = (2x^2 - 8x + 4) \cdot 4 \end{array}$$

Therefore the square roots are $\pm (x^2 - 4x + 4)$.

$$\begin{array}{l} 3. \quad c^4 - 10c^3 + 21c^2 + 20c + 4 \mid c^2 - 5c - 2 \\ (c^2)^2 = c^4 \\ \begin{array}{r} 2c^2 \\ 2c^2 - 5c \end{array} \quad \begin{array}{r} -10c^3 + 21c^2 \\ -10c^3 + 25c^2 \end{array} = (2c^2 - 5c)(-5c) \\ 2(c^2 - 5c) = 2c^2 - 10c \quad \begin{array}{r} -4c^2 + 20c + 4 \\ -4c^2 + 20c + 4 \end{array} = (2c^2 - 10c - 2)(-2) \end{array}$$

Therefore the square roots are $\pm (c^2 - 5c - 2)$.

4. $n^6 - 6n^4 + 10n^3 + 9n^2 - 30n + 25 \mid n^3 - 3n + 5$
 $(n^3)^2 = n^6$

$2n^3$	$-6n^4 + 10n^3 + 9n^2$	
$2n^3 - 3n$	$-6n^4 \qquad + 9n^2$	$= (2n^3 - 3n)(-3n)$
$2(n^3 - 3n) = 2n^3 - 6n$	$10n^3 \qquad - 30n + 25$	
$2n^3 - 6n + 5$	$10n^3 \qquad - 30n + 25$	$= (2n^3 - 6n + 5)5$

Therefore the square roots are $\pm (n^3 - 3n + 5)$.

5. $4a^6 + 4a^5 - 11a^4 + 14a^3 + 19a^2 - 30a + 25 \mid 2a^3 + a^2 - 3a + 5$
 $(2a^3)^2 = 4a^6$

$4a^3$	$4a^5 - 11a^4$	
$4a^3 + a^2$	$4a^5 + \qquad a^4$	$= (4a^3 + a^2)a^2$
$4a^3 + 2a^2$	$-12a^4 + 14a^3 + 19a^2$	
$4a^3 + 2a^2 - 3a$	$-12a^4 - 6a^3 + 9a^2$	$= (4a^3 + 2a^2 - 3a)(-3a)$
$4a^3 + 2a^2 - 6a$	$20a^3 + 10a^2 - 30a + 25$	
$4a^3 + 2a^2 - 6a + 5$	$20a^3 + 10a^2 - 30a + 25$	$= (4a^3 + 2a^2 - 6a + 5)5$

Therefore the square roots are $\pm (2a^3 + a^2 - 3a + 5)$.

6. $c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \mid c^2 - 2cd + d^2$
 $(c^2)^2 = c^4$

$2c^2$	$-4c^3d + 6c^2d^2$	
$2c^2 - 2cd$	$-4c^3d + 4c^2d^2$	
$2c^2 - 4cd$	$2c^2d^2 - 4cd^3 + d^4$	
$2c^2 - 4cd + d^2$	$2c^2d^2 - 4cd^3 + d^4$	

Therefore the square roots are $\pm (c^2 - 2cd + d^2)$.

7. $25y^4 + 30xy^3 - 11x^2y^2 - 12x^3y + 4x^4 \mid 5y^2 + 3xy - 2x^2$
 $(5y^2)^2 = 25y^4$

$10y^2$	$30xy^3 - 11x^2y^2$	
$10y^2 + 3xy$	$30xy^3 + 9x^2y^2$	
$10y^2 + 6xy$	$-20x^2y^2 - 12x^3y + 4x^4$	
$10y^2 + 6xy - 2x^2$	$-20x^2y^2 - 12x^3y + 4x^4$	

Therefore the square roots are $\pm (5y^2 + 3xy - 2x^2)$.

8. $9a^6 - 36a^4x - 24a^3x^2 + 36a^2x^2 + 48ax^3 + 16x^4 \mid 3a^3 - 6ax - 4x^2$
 $(3a^3)^2 = 9a^6$

$6a^3$	$-36a^4x - 24a^3x^2 + 36a^2x^2$	
$6a^3 - 6ax$	$-36a^4x \qquad + 36a^2x^2$	
$6a^3 - 12ax$	$-24a^3x^2 \qquad + 48ax^3 + 16x^4$	
$6a^3 - 12ax - 4x^2$	$-24a^3x^2 \qquad + 48ax^3 + 16x^4$	

Therefore the square roots are $\pm (3a^3 - 6ax - 4x^2)$.

$$\begin{array}{r}
 9. \quad 9c^4 - 12abc^3 - 2a^2b^2c^2 + 4a^3b^3c + a^4b^4 \quad | \quad 3c^2 - 2abc - a^2b^2 \\
 (3c^2)^2 = 9c^4 \\
 \begin{array}{r|l}
 6c^2 & -12abc^3 - 2a^2b^2c^2 \\
 6c^2 - 2abc & -12abc^3 + 4a^2b^2c^2 \\
 \hline
 6c^2 - 4abc & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\
 6c^2 - 4abc - a^2b^2 & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4
 \end{array}
 \end{array}$$

Therefore the square roots are $\pm (3c^2 - 2abc - a^2b^2)$.

$$\begin{array}{r}
 10. \quad c^6 - 4xc^3 + 4x^2 + 2a^2xc^3 - 4a^2x^2 + a^4x^2 \quad | \quad c^3 - 2x + a^2x \\
 (c^3)^2 = c^6 \\
 \begin{array}{r|l}
 2c^3 & -4xc^3 + 4x^2 \\
 2c^3 - 2x & -4xc^3 + 4x^2 \\
 \hline
 2c^3 - 4x & 2a^2xc^3 - 4a^2x^2 + a^4x^2 \\
 2c^3 - 4x + a^2x & 2a^2xc^3 - 4a^2x^2 + a^4x^2
 \end{array}
 \end{array}$$

Therefore the square roots are $\pm (c^3 - 2x + a^2x)$.

$$\begin{array}{r}
 11. \quad 4 - \frac{4c^2}{5} + \frac{c^4}{25} \quad | \quad 2 - \frac{c^2}{5} \\
 2^2 = 4 \\
 \begin{array}{r|l}
 4 & -\frac{4c^2}{5} + \frac{c^4}{25} \\
 4 - \frac{c^2}{5} & -\frac{4c^2}{5} + \frac{c^4}{25}
 \end{array}
 \end{array}$$

Therefore the square roots are $\pm \left(2 - \frac{c^2}{5}\right)$.

$$\begin{array}{r}
 12. \quad \frac{9}{a^2} - 3 + \frac{a^2}{4} \quad | \quad \frac{3}{a} - \frac{a}{2} \\
 \left(\frac{3}{a}\right)^2 = \frac{9}{a^2} \\
 \begin{array}{r|l}
 \frac{6}{a} - \frac{a}{2} & -3 + \frac{a^2}{4} \\
 & -3 + \frac{a^2}{4}
 \end{array}
 \end{array}$$

Therefore the square roots are $\pm \left(\frac{3}{a} - \frac{a}{2}\right)$.

13.
$$x^4 - 4x^3 + 5x^2 - 2x + \frac{1}{4} \left| x^2 - 2x + \frac{1}{2} \right.$$

x^4	
$2x^2$	$-4x^3 + 5x^2$
$2x^2 - 2x$	$-4x^3 + 4x^2$
$2x^2 - 4x$	$x^2 - 2x + \frac{1}{4}$
$2x^2 - 4x + \frac{1}{2}$	$x^2 - 2x + \frac{1}{4}$

Therefore the square roots are $\pm (x^2 - 2x + \frac{1}{2})$.

15.
$$x^4 + 6x^3 + \frac{29x^2}{3} + 2x + \frac{1}{9} \left| x^2 + 3x + \frac{1}{3} \right.$$

$(x^2)^2 = x^4$	
$2x^2$	$6x^3 + \frac{29x^2}{3}$
$2x^2 + 3x$	$6x^3 + 9x^2$
$2x^2 + 6x$	$\frac{2x^2}{3} + 2x + \frac{1}{9}$
$2x^2 + 6x + \frac{1}{3}$	$\frac{2x^2}{3} + 2x + \frac{1}{9}$

Therefore the square roots are $\pm (x^2 + 3x + \frac{1}{3})$.

16.
$$4a^4 + \frac{4a^3}{3} - \frac{35a^2}{9} - \frac{2a}{3} + 1 \left| 2a^2 + \frac{a}{3} - 1 \right.$$

$(2a^2)^2 = 4a^4$	
$4a^2$	$\frac{4a^3}{3} - \frac{35a^2}{9}$
$4a^2 + \frac{a}{3}$	$\frac{4a^3}{3} + \frac{a^2}{9}$
$4a^2 + \frac{2a}{3}$	$-4a^2 - \frac{2a}{3} + 1$
$4a^2 + \frac{2a}{3} - 1$	$-4a^2 - \frac{2a}{3} + 1$

Therefore the square roots are $\pm \left(2a^2 + \frac{a}{3} - 1 \right)$.

17.
$$\frac{25m^4}{4} + \frac{10m^3}{3} - \frac{127m^2}{18} - 2m + \frac{9}{4} \left[\frac{5m^2}{2} + \frac{2m}{3} - \frac{3}{2} \right]$$

$\frac{25m^4}{4}$	$\frac{10m^3}{3} - \frac{127m^2}{18}$
$5m^2$	$\frac{10m^3}{3} - \frac{127m^2}{18}$
$5m^2 + \frac{2m}{3}$	$\frac{10m^3}{3} + \frac{4m^2}{9}$
$5m^2 + \frac{4m}{3}$	$-\frac{15m^2}{2} - 2m + \frac{9}{4}$
$5m^2 + \frac{4m}{3} - \frac{3}{2}$	$-\frac{15m^2}{2} - 2m + \frac{9}{4}$

Therefore the square roots are $\pm \left(\frac{5m^2}{2} + \frac{2m}{3} - \frac{3}{2} \right)$.

18.
$$9c^4 - 12c^3 + 4c^2 + 6 - \frac{4}{c} + \frac{1}{c^4} \left[3c^2 - 2c + \frac{1}{c^2} \right]$$

$9c^4$	$-12c^3 + 4c^2$
$6c^2$	$-12c^3 + 4c^2$
$6c^2 - 2c$	$6c^2 - 4c$
$6c^2 - 4c$	$+6 - \frac{4}{c} + \frac{1}{c^4}$
$6c^2 - 4c + \frac{1}{c^2}$	$+6 - \frac{4}{c} + \frac{1}{c^4}$

Therefore the square roots are $\pm \left(3c^2 - 2c + \frac{1}{c^2} \right)$.

19.
$$\frac{m^2}{n^2} - \frac{5m}{n} + \frac{17}{4} + \frac{5n}{m} + \frac{n^2}{m^2} \left[\frac{m}{n} - \frac{5}{2} - \frac{n}{m} \right]$$

$\frac{m^2}{n^2}$	$-\frac{5m}{n} + \frac{17}{4}$
$\frac{2m}{n}$	$-\frac{5m}{n} + \frac{25}{4}$
$\frac{2m}{n} - \frac{5}{2}$	$-\frac{5m}{n} + \frac{25}{4}$
$\frac{2m}{n} - 5$	$-2 + \frac{5n}{m} + \frac{n^2}{m^2}$
$\frac{2m}{n} - 5 - \frac{n}{m}$	$-2 + \frac{5n}{m} + \frac{n^2}{m^2}$

Therefore the square roots are $\pm \left(\frac{m}{n} - \frac{5}{2} - \frac{n}{m} \right)$.

20.
$$\frac{a^4}{25c^4} + \frac{2a^2}{c^3} + \frac{117}{5c^2} - \frac{40}{a^2c} + \frac{16}{a^4} \left| \frac{a^2}{5c^2} + \frac{5}{c} - \frac{4}{a^2} \right|$$

$$\begin{array}{r|l} \frac{a^4}{25c^4} & \\ \hline \frac{2a^2}{5c^2} & \frac{2a^2}{c^3} + \frac{117}{5c^2} \\ \frac{2a^2}{5c^2} + \frac{5}{c} & \frac{2a^2}{c^3} + \frac{25}{c^2} \\ \hline \frac{2a^2}{5c^2} + \frac{10}{c} & -\frac{8}{5c^2} - \frac{40}{a^2c} + \frac{16}{a^4} \\ \frac{2a^2}{5c^2} + \frac{10}{c} - \frac{4}{a^2} & -\frac{8}{5c^2} - \frac{40}{a^2c} + \frac{16}{a^4} \end{array}$$

Therefore the square roots are $\pm \left(\frac{a^2}{5c^2} + \frac{5}{c} - \frac{4}{a^2} \right)$.

21.
$$\frac{a^2}{4c^4} - \frac{3a}{c^2} + 9 + \frac{2}{5ac} - \frac{12c}{5a^2} + \frac{4c^2}{25a^4} \left| \frac{a}{2c^2} - 3 + \frac{2c}{5a^2} \right|$$

$$\begin{array}{r|l} \frac{a^2}{4c^4} & \\ \hline \frac{a}{c^2} & -\frac{3a}{c^2} + 9 \\ \frac{a}{c^2} - 3 & -\frac{3a}{c^2} + 9 \\ \hline \frac{a}{c^2} - 6 & +\frac{2}{5ac} - \frac{12c}{5a^2} + \frac{4c^2}{25a^4} \\ \frac{a}{c^2} - 6 + \frac{2c}{5a^2} & +\frac{2}{5ac} - \frac{12c}{5a^2} + \frac{4c^2}{25a^4} \end{array}$$

Therefore the square roots are $\pm \left(\frac{a}{2c^2} - 3 + \frac{2c}{5a^2} \right)$.

22.
$$\frac{x^4 - 8x^3 + 24x^2 - 32x + 16}{x^4} \left| \frac{x^2 - 4x + 4}{x^2} \right|$$

$$\begin{array}{r|l} 2x^2 & -8x^3 + 24x^2 \\ 2x^2 - 4x & -8x^3 + 16x^2 \\ \hline 2x^2 - 8x & 8x^2 - 32x + 16 \\ 2x^2 - 8x + 4 & 8x^2 - 32x + 16 \end{array} \quad \begin{array}{r|l} x^2 - 4x + 4 & x - 2 \\ \hline x^2 & \\ \hline 2x & -4x + 4 \\ 2x - 2 & -4x + 4 \end{array}$$

Therefore the fourth roots are $\pm (x - 2)$.

$$\begin{array}{r}
 c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \overline{) c^2 - 2cd + d^2} \\
 \underline{c^4} \\
 2c^2 \\
 \underline{2c^2 - 2cd} \\
 2c^2 - 4cd \\
 \underline{2c^2 - 4cd + d^2} \\
 2c^2d^2 - 4cd^3 + d^4 \\
 \underline{2c^2d^2 - 4cd^3 + d^4} \\
 0
 \end{array}$$

Therefore the fourth roots are $\pm (c - d)$.

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The square root of:

1. $6241 = \pm 79$.

2. $16129 = \pm 127$.

3. $223,729 = \pm 473$.

4. $2 = \pm 1.414+$.

5. $5 = \pm 2.236+$.

6. $7.135 = \pm 2.671+$.

7. $.6279 = \pm .792+$.

8. $.0451 = \pm .212+$.

9. $.0035 = \pm .059+$.

10. $\frac{11}{5} = \pm 1.483+$.

11. $1\frac{6}{7} = \pm 1.362+$.

12. $\frac{2}{21} = \pm .308+$.

13. $\sqrt{(84)^2 + (13)^2} = 85$, hypotenuse; $\frac{84 \cdot 13}{2} = 546$, area.

14. $\sqrt{(133)^2 + (156)^2} = 205$, hypotenuse; $\frac{133 \cdot 156}{2} = 10,374$, area.

15. $\sqrt{(645)^2 + (812)^2} = 1037$, hypotenuse; $\frac{645 \cdot 812}{2} = 261,870$, area.

16. $\sqrt{(65)^2 - (56)^2} = 33$, leg; $\frac{56 \cdot 33}{2} = 924$, area.

17. $\sqrt{(397)^2 - (325)^2} = 228$, leg; $\frac{325 \cdot 228}{2} = 37,050$, area.

18. $\sqrt{(143)^2 + (24)^2} = 145$ feet.

19. $\sqrt{(401)^2 - (399)^2} = 40$, side; $40 \cdot 399 = 15,960$, area.

20. $\sqrt{(677)^2 - (52)^2} = 675$, side. Then 1454 is the perimeter.

21. Let $s =$ one side in feet.

Then $s + 21 =$ the other side in feet.

Therefore $2s + 2(s + 21) = 102$.

Whence $s = 15$,

and $s + 21 = 36$.

$\sqrt{(15)^2 + (36)^2} = 39$, the diagonal in feet.

22. Let $s =$ the number of meters in the side.

Then $2s^2 = (74)^2$.

Whence $s = 52.325+$ meters.

23. $\sqrt{(52)^2 + (52)^2} = 73.539+$ inches.

24. Let s = the width.
 Then $2.4s$ = the length.
 Therefore
 $(s)^2 + (2.4s)^2 = (52)^2$.
 Whence $s = 20$,
 and $2.4s = 48$, the length.

25. Let l = the length.
 Then $\frac{3l}{4}$ = the width.

Whence
 $l^2 + \left(\frac{3l}{4}\right)^2 = (100)^2$.
 Then $l = 80$,
 and $\frac{3l}{4} = 60$.
 The area = $60 \cdot 80 = 4800$.

26. Let w = the width.
 Then
 $(2w)^2 - (w)^2 = (10)^2$.
 Whence $w = 5.773+$.

27. $\sqrt{6^2 - 3^2} = 5.196+$, BD .
 Then
 the area = $15.588+$.

28. $\sqrt{(10)^2 - (5)^2} = 8.660+$, BD .
 Then the area = $43.30+$.

29. $(AB)^2 - \left(\frac{AB}{2}\right)^2 = (10)^2$.
 Whence $AB = 11.546+$.
 Area = $57.734+$.

30. Let a = the altitude.
 Then
 $(12)^2 - (6)^2 = a^2$,
 or $a = 10.392+$.

31. Let s = the side in
 centimeters.
 Then $s^2 - \frac{s^2}{4} = (25)^2$,
 or $s = 28.86+$.

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1. $x^{\frac{4}{3}} = \sqrt[3]{x^4}$.
2. $x^{\frac{5}{2}} = \sqrt{x^5}$.
3. $(cd)^{\frac{3}{2}} = \sqrt{(cd)^3}$.
4. $(2x)^{\frac{1}{3}} = \sqrt[3]{2x}$.
5. $2x^{\frac{1}{3}} = 2\sqrt[3]{x}$.
6. $5a^{\frac{3}{4}} = 5\sqrt[4]{a^3}$.
7. $(5a)^{\frac{3}{4}} = \sqrt[4]{(5a)^3}$.
8. $3rx^{\frac{2}{3}} = 3r\sqrt[3]{x^2}$.
9. $h^{\frac{2}{3}}k^{\frac{2}{3}} = \sqrt[3]{h^2k^2}$.
10. $7s^{\frac{2}{3}}(t+w)^{\frac{1}{2}} = 7\sqrt[3]{s^2} \cdot \sqrt{t+w}$.
11. $3a^{\frac{5}{2}}(bc)^{\frac{1}{3}} = 3\sqrt{a^5} \cdot \sqrt[3]{bc}$.
12. $5^{\frac{1}{2}}x^{\frac{2}{3}}k^{\frac{5}{4}} = \sqrt{5} \cdot \sqrt[3]{x^2} \cdot \sqrt[4]{k^5}$.
13. $4^{\frac{1}{3}}t^{\frac{1}{4}} = \sqrt[3]{4} \cdot \sqrt[4]{t}$.
14. $2x^{\frac{1}{2}}by^{\frac{c}{d}} = 2\sqrt{x} \cdot \sqrt[d]{y^c}$.

15. $25^{\frac{1}{2}} = 5$.
16. $27^{\frac{1}{3}} = 3$.
17. $16^{\frac{1}{4}} = 2$.
18. $4^{\frac{3}{2}} = 8$.
19. $64^{\frac{2}{3}} = 16$.
20. $125^{\frac{2}{3}} = 25$.
21. $(-8)^{\frac{4}{3}} = 16$.
22. $32^{\frac{1}{5}} = 2$.
23. $81^{\frac{3}{4}} = 27$.
24. $(-216)^{\frac{2}{3}} = 36$.
25. $(\frac{1}{16})^{\frac{1}{2}} = \frac{1}{4}$.
26. $(\frac{1}{25})^{\frac{1}{2}} = \frac{1}{5}$.
27. $(\frac{1}{16})^{\frac{1}{4}} = \frac{1}{2}$.
28. $25^{\frac{1}{2}} \cdot 4^{\frac{5}{2}} = 160$.

29. $4^{\frac{1}{2}} \cdot (\frac{1}{4})^{\frac{1}{2}} = 1.$
 30. $(-32)^{\frac{4}{5}} (-64)^{\frac{1}{3}} = -64.$
 31. $36^{\frac{1}{2}} \cdot (\frac{1}{9})^{\frac{1}{2}} = 2.$
 32. $9^{\frac{1}{2}} \cdot (\frac{1}{27})^{\frac{1}{3}} = 1.$
 33. $2(\frac{4}{9})^{\frac{1}{2}} \cdot (\frac{1}{8})^{\frac{1}{3}} = \frac{2}{3}.$
 34. $\sqrt[5]{32} \cdot 4^{\frac{1}{2}} = 4.$
 35. $\sqrt[3]{8^2} \cdot 25^{\frac{3}{2}} = 500.$
 36. $121^{\frac{1}{2}} \cdot \sqrt{\frac{25}{121}} = 5.$
 37. $(-343)^{\frac{1}{3}} \cdot \sqrt{\frac{1}{49}} = -1.$
 38. $\frac{2}{3}(\frac{169}{144})^{\frac{1}{2}} \div (\frac{1}{216})^{\frac{1}{3}} = \frac{13}{3}.$
 39. $\sqrt[2]{a^3} = a^{\frac{3}{2}}.$
 40. $\sqrt[2]{ax^4} = a^{\frac{1}{2}}x^2.$
 41. $3\sqrt{2x^5} = 3 \cdot 2^{\frac{1}{2}}x^{\frac{5}{2}}.$
 42. $\sqrt{9x} = 3x^{\frac{1}{2}}.$
 43. $5\sqrt{16ax^2} = 20a^{\frac{1}{2}}x.$
 44. $\sqrt[3]{a^2} = a^{\frac{2}{3}}.$
 45. $\sqrt[3]{ax^4} = a^{\frac{1}{3}}x^{\frac{4}{3}}.$
 46. $2\sqrt[3]{2x^2} = 2^{\frac{4}{3}}x^{\frac{2}{3}}.$
 47. $3\sqrt[3]{8x^4} = 6x^{\frac{4}{3}}.$
 48. $4\sqrt[3]{27ax^3} = 12a^{\frac{1}{3}}x.$
 49. $2\sqrt[4]{a^2x^3} = 2a^{\frac{1}{2}}x^{\frac{3}{4}}.$
 50. $4\sqrt[4]{16x} = 8x^{\frac{1}{4}}.$
 51. $7\sqrt[5]{rs^7} = 7r^{\frac{1}{5}}s^{\frac{7}{5}}.$
 52. $5a\sqrt[5]{32m^2} = 10am^{\frac{2}{5}}.$
 53. $12x^2\sqrt[3]{ax^3} = 12a^{\frac{1}{3}}x^3.$
 54. $c\sqrt[2]{(de)^3} = cd^{\frac{3}{2}}e^{\frac{3}{2}}.$
 55. $uv\sqrt[2]{(u+v)^5} = uv(u+v)^{\frac{5}{2}}.$
 56. $3\sqrt[2]{a^3} \cdot \sqrt[3]{x^2} = 3a^{\frac{3}{2}}x^{\frac{2}{3}}.$
 57. $2a\sqrt[5]{ax^2} \cdot \sqrt[3]{2m} = 2^{\frac{4}{3}}a^{\frac{6}{5}}x^{\frac{2}{5}}m^{\frac{1}{3}}.$
 58. $\sqrt[n]{x^a} \cdot \sqrt[n]{y^b} = x^{\frac{a}{n}}y^{\frac{b}{n}}.$
 59. $\sqrt[n]{x^a} \cdot \sqrt[n]{x^{3a}} = x^{\frac{4a}{n}}.$

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1. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}.$
 2. $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}.$
 3. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}.$
 4. $2\sqrt{50} = 2\sqrt{25 \cdot 2} = 10\sqrt{2}.$
 5. $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = 2\sqrt[3]{5}.$
 6. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}.$
 7. $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}.$
 8. $\sqrt[3]{3000} = \sqrt[3]{1000 \cdot 3} = 10\sqrt[3]{3}.$
 9. $2\sqrt[3]{24} = 2\sqrt[3]{8 \cdot 3} = 4\sqrt[3]{3}.$
 10. $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{1}{2}\sqrt{2}.$
 11. $\sqrt{\frac{2}{5}} = \sqrt{\frac{20}{50}} = \frac{1}{5}\sqrt{10}.$
 12. $\sqrt{\frac{3}{8}} = \sqrt{\frac{6}{16}} = \frac{1}{4}\sqrt{6}.$
 13. $3\sqrt{\frac{9}{2}} = 9\sqrt{\frac{2}{4}} = \frac{9}{2}\sqrt{2}.$
 14. $5\sqrt{\frac{3}{5}} = 5\sqrt{\frac{15}{25}} = \frac{5}{5}\sqrt{15} = \sqrt{15}.$
 15. $\sqrt[3]{\frac{1}{2}} = \sqrt[3]{\frac{4}{8}} = \frac{1}{2}\sqrt[3]{4}.$
 16. $4\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{\frac{2}{8}} = \frac{4}{2}\sqrt[3]{2} = 2\sqrt[3]{2}.$
 17. $8\sqrt[3]{\frac{7}{4}} = 8\sqrt[3]{\frac{14}{8}} = \frac{8}{2}\sqrt[3]{14} = 4\sqrt[3]{14}.$
 18. $\sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}.$
 19. $\sqrt{1 + (\frac{1}{3})^2} = \sqrt{\frac{10}{9}} = \frac{1}{3}\sqrt{10}.$
 20. $\sqrt{2 + (\frac{3}{5})^2} = \sqrt{\frac{59}{25}} = \frac{1}{5}\sqrt{59}.$
 21. $\sqrt[3]{1 - (\frac{1}{2})^2} = \sqrt[3]{\frac{3}{4}} = \sqrt[3]{\frac{6}{8}} = \frac{1}{2}\sqrt[3]{6}.$
 22. $\sqrt[3]{4 + (\frac{2}{3})^2} = \sqrt[3]{\frac{40}{9}} = 2\sqrt[3]{\frac{5}{9}} = \frac{2}{3}\sqrt[3]{15}.$
 23. $\sqrt[3]{1 + (\frac{1}{3})^3} = \sqrt[3]{\frac{28}{27}} = \frac{1}{3}\sqrt[3]{28}.$
 24. $\sqrt[4]{25a^2} = \sqrt[4]{5^2 \cdot a^2} = \sqrt{5a}.$
 25. $3\sqrt{4x^5} = 3\sqrt{4x^4} \cdot \sqrt{x} = 6x^2\sqrt{x}.$
 26. $a\sqrt[6]{8x^3} = a\sqrt[6]{2^3x^3} = a\sqrt{2x}.$
 27. $x\sqrt[4]{49a^2x^2} = x\sqrt[4]{(7ax)^2} = x\sqrt{7ax}.$
 28. $\sqrt[6]{125x^3ay^3m} = \sqrt[6]{5^3 \cdot x^3a \cdot y^3m} = \sqrt{5x^ay^3m}.$

$$29. \sqrt{\frac{a^3}{27}} = \sqrt{3a \cdot \frac{a^2}{81}} = \frac{a}{9} \sqrt{3a}.$$

$$30. a \sqrt[3]{\frac{5}{2a^2}} = a \sqrt[3]{\frac{20a}{8a^3}} = \frac{a}{2a} \sqrt[3]{20a} = \frac{1}{2} \sqrt[3]{20a}.$$

$$31. \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3R^2}{4}} = \frac{R}{2} \sqrt{3}.$$

$$32. \sqrt{R^2 + \left(\frac{R}{2}\right)^2} = \sqrt{\frac{5R^2}{4}} = \frac{R}{2} \sqrt{5}.$$

$$33. \sqrt{\left(a + \frac{a}{3}\right)\left(a - \frac{a}{3}\right)} = \sqrt{a^2 - \frac{a^2}{9}} = \sqrt{\frac{8a^2}{9}} = \frac{2a}{3} \sqrt{2}.$$

$$34. \sqrt[3]{R^3 - \left(\frac{R}{2}\right)^3} = \sqrt[3]{\frac{7R^3}{8}} = \frac{R}{2} \sqrt[3]{7}.$$

$$35. (x-y) \sqrt{\frac{x+y}{x-y}} = (x-y) \sqrt{\frac{(x^2-y^2)}{(x-y)^2}} = \frac{(x-y)}{(x-y)} \sqrt{x^2-y^2} = \sqrt{x^2-y^2}.$$

$$36. \sqrt{4 + 4\sqrt{2}} = \sqrt{4(1 + \sqrt{2})} = 2\sqrt{1 + \sqrt{2}}.$$

$$37. \sqrt{8 - 4\sqrt{2}} = \sqrt{4(2 - \sqrt{2})} = 2\sqrt{2 - \sqrt{2}}.$$

$$38. \sqrt{18 + 9\sqrt{3}} = \sqrt{9(2 + \sqrt{3})} = 3\sqrt{2 + \sqrt{3}}.$$

$$39. \sqrt{25\sqrt{5} - 100} = \sqrt{25(\sqrt{5} - 4)} = 5\sqrt{\sqrt{5} - 4}.$$

$$40. \sqrt[3]{16 + 8\sqrt{2}} = \sqrt[3]{8(2 + \sqrt{2})} = 2\sqrt[3]{2 + \sqrt{2}}.$$

$$41. \sqrt[3]{81 + 3\sqrt[2]{243}} = \sqrt[3]{81 + 27\sqrt{3}} = \sqrt[3]{27(3 + \sqrt{3})} = 3\sqrt[3]{3 + \sqrt{3}}.$$

$$42. \sqrt{R^2 - 2R^2\sqrt{2}} = \sqrt{R^2(1 - 2\sqrt{2})} = R\sqrt{1 - 2\sqrt{2}}.$$

$$43. \sqrt[4]{32 - 64\sqrt{3}} = \sqrt[4]{16(2 - 4\sqrt{3})} = 2\sqrt[4]{2 - 4\sqrt{3}}.$$

$$44. \sqrt{\frac{R^2 + R^2\sqrt{3}}{2}} = \sqrt{\frac{2R^2(1 + \sqrt{3})}{4}} = \frac{R}{2} \sqrt{2 + 2\sqrt{3}}.$$

$$45. \sqrt{R^2 - \left(\frac{R}{2}\right)^2\sqrt{2}} = \sqrt{\frac{4R^2 - R^2\sqrt{2}}{4}} = \sqrt{\frac{R^2}{4}(4 - \sqrt{2})} = \frac{R}{2} \sqrt{4 - \sqrt{2}}.$$

$$46. \sqrt{R^2\sqrt{2} - \frac{R^2}{2}} = \sqrt{\frac{4R^2\sqrt{2} - 2R^2}{4}} = \frac{1}{2} \sqrt{R^2(4\sqrt{2} - 2)} = \frac{R}{2} \sqrt{4\sqrt{2} - 2}.$$

$$47. \sqrt{\frac{R^2}{3} - R^2\sqrt{3}} = \sqrt{\frac{3R^2 - 9R^2\sqrt{3}}{9}} = \frac{1}{3} \sqrt{R^2(3 - 9\sqrt{3})} = \frac{R}{3} \sqrt{3 - 9\sqrt{3}}.$$

$$48. 2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}.$$

$$49. 3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{45}.$$

$$50. 3\sqrt[3]{4} = \sqrt[3]{27} \cdot \sqrt[3]{4} = \sqrt[3]{108}.$$

$$51. 4\sqrt[3]{2} = \sqrt[3]{64} \sqrt[3]{2} = \sqrt[3]{128}.$$

$$52. 3\sqrt[3]{\frac{1}{3}} = \sqrt[3]{27} \sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{27}{3}} = \sqrt[3]{9}.$$

$$53. x^2 \sqrt{x^3} = \sqrt{x^4} \sqrt{x^3} = \sqrt{x^7}.$$

$$54. 2x^2 \sqrt[3]{x^2} = \sqrt[3]{8x^6} \cdot \sqrt[3]{x^2} = \sqrt[3]{8x^8}.$$

$$55. \frac{a}{2} \sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a^3}{8}} \cdot \sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a}{2}}.$$

$$56. (a+2) \sqrt{\frac{1}{a^2-4}} = \sqrt{(a+2)^2} \sqrt{\frac{1}{a^2-4}} = \sqrt{\frac{a+2}{a-2}}.$$

$$57. \frac{x+3}{ax} \sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{(x+3)^3}{(ax)^3}} \sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{x+3}{ax}}.$$

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$$1. \sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$$

$$2. \sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}.$$

$$3. 2\sqrt{200} - 3\sqrt{8} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}.$$

$$4. \sqrt[3]{56} + 2\sqrt[3]{189} = 2\sqrt[3]{7} + 6\sqrt[3]{7} = 8\sqrt[3]{7}.$$

$$5. 2\sqrt[3]{320} - \sqrt[3]{50} = 8\sqrt[3]{5} - 5\sqrt[3]{5}.$$

$$6. \sqrt[4]{25} - \sqrt[4]{50} = \sqrt{5} - 5\sqrt{2}.$$

$$7. \sqrt[4]{32} + 5\sqrt[4]{162} = 2\sqrt[4]{2} + 15\sqrt[4]{2} = 17\sqrt[4]{2}.$$

$$8. 5\sqrt{\frac{1}{2}} - \frac{3}{2}\sqrt{2} = \frac{5}{2}\sqrt{2} - \frac{3}{2}\sqrt{2} = \sqrt{2}.$$

$$9. \sqrt{\frac{1}{3}} + 2\sqrt{\frac{4}{3}} - 3\sqrt{\frac{2}{3}} = \frac{1}{3}\sqrt{3} + \frac{4}{3}\sqrt{3} - \sqrt{6} = \frac{5}{3}\sqrt{3} - \sqrt{6}.$$

$$10. \sqrt{\frac{3}{10}} - \sqrt{120} - 2\sqrt{\frac{6}{5}} = \frac{1}{10}\sqrt{30} - 2\sqrt{30} - \frac{2}{5}\sqrt{30} = -\frac{23}{10}\sqrt{30}.$$

$$11. \sqrt{\frac{5}{18}} + 2\sqrt{\frac{3}{5}} - \sqrt{\frac{10}{9}} = \frac{1}{6}\sqrt{10} + \frac{8}{5}\sqrt{10} - \frac{1}{3}\sqrt{10} = \frac{43}{30}\sqrt{10}.$$

$$12. \sqrt[3]{\frac{6}{8}} - \sqrt[3]{12} + \sqrt[3]{6} = \frac{1}{2}\sqrt[3]{6} - \sqrt[3]{12} + \sqrt[3]{6} = \frac{3}{2}\sqrt[3]{6} - \sqrt[3]{12}.$$

$$13. a\sqrt[4]{4} + \sqrt{8a^2} = a\sqrt{2} + 2a\sqrt{2} = 3a\sqrt{2}.$$

$$14. R - \sqrt{\frac{3R^2}{4}} = R - \frac{R}{2}\sqrt{3}.$$

$$15. 2x\sqrt[3]{54x} - 3\sqrt[3]{16x^4} + \sqrt[6]{4x^2} = 6x\sqrt[3]{2x} - 6x\sqrt[3]{2x} + \sqrt[3]{2x} = \sqrt[3]{2x}.$$

$$16. \sqrt[3]{81x^7} + x\sqrt[3]{375x^4} - \sqrt[12]{16x^4} = 3x^2\sqrt[3]{3x} + 5x^2\sqrt[3]{3x} - \sqrt[3]{2x} \\ = 8x^2\sqrt[3]{3x} - \sqrt[3]{2x}.$$

$$17. \sqrt{a^3bc} - a\sqrt{abc} + ac\sqrt{\frac{b}{ac}} = a\sqrt{abc} - a\sqrt{abc} + \sqrt{abc} = \sqrt{abc}.$$

$$18. \sqrt{(m+n)^3} - n\sqrt[4]{(m+n)^2} = (m+n)\sqrt{m+n} - n\sqrt{(m+n)} \\ = m\sqrt{m+n}.$$

$$19. \sqrt[3]{(a+b)^4} - \sqrt[3]{8a^3(a+b)} + \sqrt[6]{a^2+2ab+b^2} \\ = (a+b)\sqrt[3]{a+b} - 2a\sqrt[3]{a+b} + \sqrt[3]{a+b} \\ = b\sqrt[3]{a+b} - a\sqrt[3]{a+b} + \sqrt[3]{a+b} \\ = (b-a+1)\sqrt[3]{a+b}.$$

$$20. \sqrt[2]{a^3 + 4a^2 + 4a} - \sqrt[2]{a^3} - \frac{2}{a^3} \sqrt{a^7} = (a + 2) \sqrt{a} - a \sqrt{a} - 2 \sqrt{a} = 0.$$

$$21. \sqrt{x^3 y^3} - x^2 y^2 \sqrt{\frac{1}{xy}} + xy \sqrt{2 + \frac{x^2 + y^2}{xy}}$$

$$= xy \sqrt{xy} - xy \sqrt{xy} + \sqrt{(x^2 + 2xy + y^2)xy}$$

$$= (x + y) \sqrt{xy}.$$

$$22. rs \sqrt[3]{rs} + \sqrt[3]{\frac{1}{r^2 s^2}} - 2 \sqrt[3]{r^4 s^4} = rs \sqrt[3]{rs} + \frac{1}{rs} \sqrt[3]{rs} - 2 rs \sqrt[3]{rs}$$

$$= \frac{1}{rs} \sqrt[3]{rs} - rs \sqrt[3]{rs}$$

$$= \sqrt[3]{rs} \left(\frac{1}{rs} - rs \right)$$

$$= \frac{1 - r^2 s^2}{rs} \sqrt[3]{rs}.$$

$$23. 2 \sqrt{\frac{y}{x}} - \sqrt{\frac{x}{y}} + \sqrt{2 + \frac{x^2 + y^2}{xy}} = \frac{2}{x} \sqrt{xy} - \frac{1}{y} \sqrt{xy} + \frac{x + y}{xy} \sqrt{xy}$$

$$= \sqrt{xy} \left(\frac{2}{x} - \frac{1}{y} + \frac{x + y}{xy} \right) = \frac{3}{x} \sqrt{xy}.$$

$$24. \sqrt{3x^2 - 18x + 27} - \sqrt{27(x^2 + 2x + 1)} = (x - 3) \sqrt{3} - (3x + 3) \sqrt{3}$$

$$= -2x \sqrt{3} - 6 \sqrt{3} = \sqrt{3}(-2x - 6).$$

$$25. \sqrt[3]{(a - 3)^2(5a - 15)} + \sqrt[3]{40} = (a - 3) \sqrt[3]{5} + 2 \sqrt[3]{5}$$

$$= a \sqrt[3]{5} - \sqrt[3]{5} = \sqrt[3]{5}(a - 1).$$

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$$1. \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4.$$

$$2. \sqrt{3} \cdot \sqrt{27} = \sqrt{81} = 9.$$

$$3. 5^{\frac{1}{2}} \cdot 20^{\frac{1}{2}} = 100^{\frac{1}{2}} = 10.$$

$$4. 18^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = 144^{\frac{1}{2}} = 12.$$

$$5. \sqrt[3]{16} \cdot \sqrt[3]{4} = \sqrt[3]{64} = 4.$$

$$6. \sqrt[3]{4} \cdot \sqrt[3]{12} = \sqrt[3]{48} = 2 \sqrt[3]{6}.$$

$$7. (100)^{\frac{1}{3}} (30)^{\frac{1}{3}} = 3000^{\frac{1}{3}} = 10 \sqrt[3]{3}.$$

$$8. \sqrt[4]{8} \cdot \sqrt[4]{32} = \sqrt[4]{2^3 \cdot 2^5} = \sqrt[4]{2^8}$$

$$= 2^2 = 4.$$

$$9. \sqrt{\frac{12}{5}} \cdot \sqrt{75} = \sqrt{36} = 6.$$

$$17. \sqrt{75a} \cdot (45a)^{\frac{1}{2}} = \sqrt{3 \cdot 5^2 \cdot 3^2 \cdot 5a^2} = 15a \sqrt{15}.$$

$$18. \sqrt{2u} \cdot \sqrt{4v} \cdot \sqrt{6uv} = \sqrt{48u^2v^2} = 4uv \sqrt{3}.$$

$$10. \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{27}{8}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$$

$$11. \sqrt{11} \cdot \sqrt{\frac{1}{11}} = \sqrt{1} = 1.$$

$$12. a^{\frac{1}{2}} \cdot (bc)^{\frac{1}{2}} = \sqrt{abc}.$$

$$13. 2 \sqrt{x} \cdot \sqrt[2]{4x^3} = 2 \sqrt{4x^4} = 4x^2.$$

$$14. 5 \sqrt[3]{2a} \cdot 3 \sqrt[3]{16a} = 15 \sqrt[3]{32a^2}$$

$$= 30 \sqrt[3]{4a^2}.$$

$$15. 2 \sqrt{Rs} \cdot 7 \sqrt{r^3 s^3 t^2} = 14 \sqrt{r^3 s^4 t^2 R}$$

$$= 14 rs^2 t \sqrt{Rr}.$$

$$16. \sqrt{\frac{a}{x}} \cdot \sqrt{\frac{4x}{a}} = \sqrt{4} = 2.$$

$$19. 5 \sqrt{3m} \cdot 5 \sqrt{3m} = 25 \sqrt{9m^2} = 75m.$$

$$20. (3 \sqrt{3x})^2 = 9 \cdot 3x = 27x.$$

$$21. \sqrt[3]{2} \cdot \sqrt{2} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{2}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}.$$

$$22. \sqrt{3} \cdot \sqrt[3]{3} = 3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}} = 3^{\frac{5}{6}} = \sqrt[6]{3^5} = \sqrt[6]{243}.$$

$$23. \sqrt[3]{2} \cdot \sqrt{3} = 2^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} = 2^{\frac{2}{6}} \cdot 3^{\frac{3}{6}} = \sqrt[6]{2^2 \cdot 3^3} = \sqrt[6]{108}.$$

$$24. \sqrt{2} \cdot \sqrt[3]{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{3}} = 2^{\frac{3}{6}} \cdot 3^{\frac{2}{6}} = \sqrt[6]{2^3 \cdot 3^2} = \sqrt[6]{72}.$$

$$25. \sqrt[3]{4} \cdot \sqrt{2} = 2^{\frac{2}{3}} \cdot 2^{\frac{1}{2}} = 2^{\frac{7}{6}} = 2 \sqrt[6]{2}.$$

$$26. \sqrt{8} \cdot \sqrt[3]{8} = 2^{\frac{3}{2}} \cdot 2 = 4 \sqrt{2}.$$

$$27. \sqrt{3} \cdot \sqrt[3]{24} = 3^{\frac{1}{2}} \cdot 2 \cdot 3^{\frac{1}{3}} = 2 \cdot 3^{\frac{5}{6}} = 2 \sqrt[6]{243}.$$

$$28. \sqrt{a} \cdot \sqrt[3]{a} = a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}.$$

$$29. \sqrt{a^3} \cdot \sqrt[3]{a^2} = a^{\frac{3}{2}} \cdot a^{\frac{2}{3}} = a^{\frac{13}{6}} = a^2 \sqrt[6]{a}.$$

$$30. \sqrt{2a} \cdot \sqrt[3]{2a} = (2a)^{\frac{1}{2}} (2a)^{\frac{1}{3}} = (2a)^{\frac{5}{6}} = \sqrt[6]{(2a)^5} = \sqrt[6]{32a^5}.$$

$$31. \sqrt[3]{4x^2} \cdot \sqrt{2x} = 2^{\frac{2}{3}} \cdot x^{\frac{2}{3}} \cdot 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = 2^{\frac{4}{6}} \cdot x^{\frac{4}{6}} \cdot 2^{\frac{3}{6}} \cdot x^{\frac{3}{6}} = 2^{\frac{7}{6}} x^{\frac{7}{6}} = 2x \sqrt[6]{2x}.$$

$$32. \sqrt[3]{\frac{5x}{a}} \cdot \sqrt{\frac{a^3}{5x^2}} = \left(\frac{5x}{a}\right)^{\frac{1}{3}} \left(\frac{a}{5}\right)^{\frac{1}{2}} \cdot \frac{a}{x} = \frac{a}{x} \left(\frac{5x}{a}\right)^{\frac{2}{6}} \left(\frac{a}{5}\right)^{\frac{3}{6}} = \frac{a}{x} \sqrt[6]{\left(\frac{5x}{a}\right)^2 \left(\frac{a}{5}\right)^3}$$

$$= \frac{a}{x} \sqrt[6]{\frac{x^2 a}{5}} = \frac{a}{5x} \sqrt[6]{3125 ax^2}.$$

$$33. (x + a \sqrt{m}) \sqrt{m^3} = x \sqrt{m^3} + a \sqrt{m^4} = mx \sqrt{m} + am^2.$$

$$34. (\sqrt{5a} - \sqrt{10a^2})(\sqrt{5a}) = 5a - \sqrt{50a^3} = 5a - 5a \sqrt{2a}.$$

$$35. (\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = 2 - 3 = -1.$$

$$36. \begin{array}{r} 5\sqrt{3} - 4 \\ 3\sqrt{3} + 8 \\ \hline 15 \cdot 3 - 12\sqrt{3} \\ 40\sqrt{3} - 32 \\ \hline 45 + 28\sqrt{3} - 32 = 13 + 28\sqrt{3}. \end{array}$$

$$37. \begin{array}{r} \sqrt{5} - \sqrt{3} - \sqrt{2} \\ \sqrt{5} + \sqrt{3} + \sqrt{2} \\ \hline 5 - \sqrt{15} - \sqrt{10} \\ \sqrt{15} - 3 - \sqrt{6} \\ + \sqrt{10} - \sqrt{6} - 2 \\ \hline 5 - 3 - 2\sqrt{6} - 2 = -2\sqrt{6}. \end{array}$$

$$38. \quad \frac{3\sqrt{2} + 2\sqrt{3} + \sqrt{30}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$$

$$\frac{3 \cdot 2 + 2\sqrt{6} + 2\sqrt{15}}{}$$

$$\frac{\begin{array}{r} 3\sqrt{6} \\ - 5\sqrt{6} - 2\sqrt{15} \end{array} + \begin{array}{r} 2 \cdot 3 + 3\sqrt{10} \\ - 3\sqrt{10} \end{array}}{6 + 6} = 12.$$

$$39. \quad \frac{6a + b\sqrt{2}}{2} \cdot \frac{6a + b\sqrt{2}}{2} = \frac{36a^2 + 12ab\sqrt{2} + 2b^2}{4}$$

$$= 9a^2 + 3ab\sqrt{2} + \frac{b^2}{2}.$$

$$40. \quad R - \frac{R}{2}\sqrt{3}$$

$$\frac{2R + \frac{3R}{2}\sqrt{3}}{2R^2 - R^2\sqrt{3}}$$

$$\frac{\frac{3R^2}{2}\sqrt{3} - \frac{3R^2}{4} \cdot 3}{2R^2 + \frac{R^2}{2}\sqrt{3} - \frac{9R^2}{4}} = \frac{R^2}{2}\sqrt{3} - \frac{R^2}{4} = \frac{R^2}{2}(\sqrt{3} - \frac{1}{2}).$$

$$41. \quad \sqrt{a+b} \cdot \sqrt{a-b} \cdot \sqrt{2a^2-2b^2} = (a^2-b^2)\sqrt{2}.$$

$$42. \quad \frac{\begin{array}{r} 2\sqrt{x-a} + \sqrt{a} \\ - 3\sqrt{x-a} - 5\sqrt{a} \end{array}}{-6(x-a) - 3\sqrt{xa-a^2}}$$

$$\frac{-10\sqrt{xa-a^2} - 5a}{-6x + 6a - 13\sqrt{ax-a^2} - 5a} = a - 6x - 13\sqrt{ax-a^2}.$$

$$43. \quad (3\sqrt[3]{x-a})(-5\sqrt[3]{x-a}) = -15\sqrt[3]{x^2-2ax+a^2}.$$

$$44. \quad \sqrt{2-\sqrt{2}} \cdot \sqrt{2+\sqrt{2}} = \sqrt{4-2} = \sqrt{2}.$$

$$45. \quad \sqrt{a-\sqrt{b}} \cdot \sqrt{2a-2\sqrt{b}} = \sqrt{a-\sqrt{b}} \cdot \sqrt{2(a-\sqrt{b})} = (a-\sqrt{b})\sqrt{2}.$$

$$46. \quad \sqrt{\frac{R}{2}}\sqrt{5} + R \cdot \sqrt{\frac{R}{2}}\sqrt{5} - R = \sqrt{\frac{R^2 \cdot 5}{4} - R^2} = \sqrt{\frac{R^2}{4}} = \frac{R}{2}.$$

$$47. \quad (\sqrt[3]{2})^2 = \sqrt[3]{4}.$$

$$49. \quad (2\sqrt[3]{12})^2 = 4\sqrt[3]{144} = 8\sqrt[3]{18}.$$

$$48. \quad (2\sqrt[3]{3})^2 = 4\sqrt[3]{9}.$$

$$50. \quad (\sqrt{2-\sqrt{2}})^2 = 2-\sqrt{2}.$$

$$51. \quad (3\sqrt{x+\sqrt{3}})^2 = 9(x+\sqrt{3}) = 9x+9\sqrt{3}.$$

$$52. \quad [(2\sqrt[3]{3})^2]^2 = (4\sqrt[3]{9})^2 = 16\sqrt[3]{81} = 48\sqrt[3]{3}.$$

$$53. (2a\sqrt[3]{8x})^2 = 4a^2\sqrt[3]{64x^2} = 16a^2\sqrt[3]{x^2}.$$

$$54. (\sqrt[3]{2} + \sqrt[3]{3})^2 = \sqrt[3]{4} + 2\sqrt[3]{6} + \sqrt[3]{9}.$$

$$55. (\sqrt[3]{4 + 4\sqrt{3}})^2 = \sqrt[3]{16 + 32\sqrt{3} + 48} = \sqrt[3]{8(8 + 4\sqrt{3})} = 2\sqrt[3]{8 + 4\sqrt{3}}.$$

$$56. \left(\frac{1}{3}\sqrt[3]{9 - 9\sqrt{2}}\right)^2 = \frac{1}{9}\sqrt[3]{81 - 162\sqrt{2} + 162} = \frac{1}{9}\sqrt[3]{27(9 - 6\sqrt{2})}$$

$$= \frac{1}{3}\sqrt[3]{9 - 6\sqrt{2}}.$$

$$57. (\sqrt{2} + \sqrt[3]{3})^2 = 2 + 2\sqrt{2}\sqrt[3]{3} + \sqrt[3]{9} = 2 + 2\sqrt[6]{72} + \sqrt[3]{9}.$$

$$58. (2\sqrt[4]{3 - \sqrt{2}})^2 = 4\sqrt[2]{3 - \sqrt{2}}. \quad 60. (3\sqrt{2})^3 = 27\sqrt{8} = 54\sqrt{2}.$$

$$59. (2\sqrt[3]{3})^3 = 8\sqrt{27} = 24\sqrt{3}. \quad 61. [(\sqrt{2})^2]^3 = (2)^3 = 8.$$

$$62. [(\sqrt[2]{3})^3]^3 = (3^{\frac{3}{2}})^3 = 3^{\frac{9}{2}} = 81\sqrt{3}.$$

$$63. (\sqrt{3} - \sqrt{6})^3 = (\sqrt{3})^3 - 3(\sqrt{3})^2\sqrt{6} + 3(\sqrt{3})(\sqrt{6})^2 - (\sqrt{6})^3$$

$$= 21\sqrt{3} - 15\sqrt{6}.$$

$$64. (2\sqrt{2} + \sqrt{3})^3 = (2\sqrt{2})^3 + 3(2\sqrt{2})^2\sqrt{3} + 3(2\sqrt{2})(\sqrt{3})^2 + (\sqrt{3})^3$$

$$= 34\sqrt{2} + 27\sqrt{3}.$$

$$65. (\sqrt[3]{3} - \sqrt[3]{2})^3 = (\sqrt[3]{3})^3 - 3(\sqrt[3]{3})^2\sqrt[3]{2} + 3(\sqrt[3]{3})(\sqrt[3]{2})^2 - (\sqrt[3]{2})^3$$

$$= 1 - 3\sqrt[3]{18} + 3\sqrt[3]{12}.$$

$$66. (\sqrt{2} + \sqrt{2})^3 = (2 + \sqrt{2})\sqrt{2} + \sqrt{2}.$$

$$67. (\sqrt[3]{2} - \sqrt{3})^3 = (\sqrt[3]{2})^3 - 3(\sqrt[3]{2})^2\sqrt{3} + 3(\sqrt[3]{2})(\sqrt{3})^2 - (\sqrt{3})^3$$

$$= 2 - 3\sqrt[6]{432} + 9\sqrt[3]{2} - 3\sqrt{3}.$$

$$68. R^2 + \left(\frac{R}{2}\sqrt{2 - \sqrt{2}}\right)^2 = R^2 + \left[\frac{R^2}{4}(2 - \sqrt{2})\right]$$

$$= R^2 + \frac{R^2}{2} - \frac{R^2\sqrt{2}}{4}$$

$$= \frac{3R^2}{2} - \frac{R^2\sqrt{2}}{4}$$

$$= \frac{R^2}{4}(6 - \sqrt{2}).$$

$$69. \sqrt{R^2 - \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2} = \sqrt{R^2 - \left(\frac{R^2 \cdot 5}{4} - \frac{2R^2\sqrt{5}}{4} + \frac{R^2}{4}\right)}$$

$$= \sqrt{\frac{4R^2 - 6R^2 + 2R^2\sqrt{5}}{4}}$$

$$= \sqrt{\frac{R^2}{4}(2\sqrt{5} - 2)}$$

$$= \frac{R}{2}\sqrt{2\sqrt{5} - 2}.$$

$$\begin{aligned}
 70. \quad \left[R^2 - \left(\frac{R\sqrt{5} - R}{4} \right)^2 \right]^{\frac{1}{2}} &= \left[R^2 - \left(\frac{R^2 \cdot 5 - 2R^2\sqrt{5} + R^2}{16} \right) \right]^{\frac{1}{2}} \\
 &= \frac{1}{4} [16R^2 - 6R^2 + 2R^2\sqrt{5}]^{\frac{1}{2}} \\
 &= \frac{1}{4} [R^2(10 + 2\sqrt{5})]^{\frac{1}{2}} \\
 &= \frac{R}{4} \sqrt{10 + 2\sqrt{5}}.
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \left[\left(R - \frac{R}{2}\sqrt{3} \right)^2 + \left(\frac{R}{2} \right)^2 \right]^{\frac{1}{2}} &= \left[\left(R^2 - R^2\sqrt{3} + \frac{3R^2}{4} \right) + \frac{R^2}{4} \right]^{\frac{1}{2}} \\
 &= [R^2(2 - \sqrt{3})]^{\frac{1}{2}} \\
 &= R\sqrt{2 - \sqrt{3}}.
 \end{aligned}$$

$$\begin{aligned}
 72. \quad \left[R^2 - \left(\frac{R\sqrt{2 - \sqrt{3}}}{2} \right)^2 \right]^{\frac{1}{2}} &= \left[R^2 - \left(\frac{R^2(2 - \sqrt{3})}{4} \right) \right]^{\frac{1}{2}} \\
 &= \frac{1}{2} [4R^2 - 2R^2 + R^2\sqrt{3}]^{\frac{1}{2}} \\
 &= \frac{1}{2} [R^2(2 + \sqrt{3})]^{\frac{1}{2}} \\
 &= \frac{R}{2} \sqrt{2 + \sqrt{3}}.
 \end{aligned}$$

$$\begin{aligned}
 73. \quad \sqrt{R^2 - \left(\frac{R\sqrt{2 - \sqrt{2}}}{2} \right)^2} &= \left[R^2 - \frac{R^2(2 - \sqrt{2})}{4} \right]^{\frac{1}{2}} \\
 &= \left[R^2 - \frac{2R^2}{4} + \frac{R^2\sqrt{2}}{4} \right]^{\frac{1}{2}} \\
 &= \left[\frac{2R^2 + R^2\sqrt{2}}{4} \right]^{\frac{1}{2}} \\
 &= \frac{R}{2} \sqrt{2 + \sqrt{2}}.
 \end{aligned}$$

$$\begin{aligned}
 74. \quad \sqrt{\left(\frac{R\sqrt{2}}{2} \right)^2 + \left(R - \frac{R}{2}\sqrt{2} \right)^2} &= \sqrt{\frac{2R^2}{4} + \left(R^2 - R^2\sqrt{2} + \frac{2R^2}{4} \right)} \\
 &= \sqrt{2R^2 - R^2\sqrt{2}} \\
 &= R\sqrt{2 - \sqrt{2}}.
 \end{aligned}$$

$$75. \quad \left(\frac{R}{2} \sqrt{2 + \sqrt{2}} \right) \left(\frac{R\sqrt{2 - \sqrt{2}}}{2} \right) 4 = \left(\frac{R^2}{4} \sqrt{4 - 2} \right) 4 = R^2 \sqrt{2}.$$

$$\begin{aligned}
 76. \quad \text{When } x &= -2 + \sqrt{3}, \\
 x^2 + 4x + 1 &= (4 - 4\sqrt{3} + 3) + (-8 + 4\sqrt{3}) + 1 = 0.
 \end{aligned}$$

$$\begin{aligned}
 77. \quad \text{When } x &= 2 + \sqrt{5}, \\
 x^2 - 2x - 3 &= (4 + 4\sqrt{5} + 5) - (4 + 2\sqrt{5}) - 3 = 2 + 2\sqrt{5}.
 \end{aligned}$$

78. If $x = 2 + \sqrt{5}$,

$$x^2 - 4x - 1 = (4 + 4\sqrt{5} + 5) - (8 + 4\sqrt{5}) - 1 = 0.$$

If $x = 2 - \sqrt{5}$,

$$x^2 - 4x - 1 = (4 - 4\sqrt{5} + 5) - (8 - 4\sqrt{5}) - 1 = 0.$$

79. If $x = \frac{1}{2} + \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= 3\left(\frac{1}{4} + \frac{1}{2}\sqrt{13} + \frac{13}{4}\right) + \left(\frac{3}{2} + \frac{3}{2}\sqrt{13}\right) - 5 \\ &= \frac{3}{4} + \frac{3}{2}\sqrt{13} + \frac{39}{4} + \frac{3}{2} + \frac{3}{2}\sqrt{13} - 5 \\ &= 7 + 3\sqrt{13}. \end{aligned}$$

If $x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= \left(\frac{3}{4} - \frac{3}{2}\sqrt{13} + \frac{39}{4}\right) + \left(\frac{3}{2} - \frac{3}{2}\sqrt{13}\right) - 5 \\ &= 7 - 3\sqrt{13}. \end{aligned}$$

80. If $x = 2 + \sqrt{3}$, $4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + 1 = 0$. Yes.

If $x = 2 - \sqrt{3}$, $4 - 4\sqrt{3} + 3 - 8 + 4\sqrt{3} + 1 = 0$. Yes.

81. If $x = -4 + \sqrt{5}$, $16 - 8\sqrt{5} + 5 - 32 + 8\sqrt{5} + 11 = 0$. Yes.

If $x = -4 - \sqrt{5}$, $16 + 8\sqrt{5} + 5 - 32 - 8\sqrt{5} + 11 = 0$. Yes.

82. That $2 \pm \sqrt{3}$ are roots of $x^2 - 4x + 1 = 0$, and that $-4 \pm \sqrt{5}$ are roots of $x^2 + 8x + 11 = 0$.

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A rationalizing factor of:

1. $\sqrt{5}$ is $\sqrt{5}$, $\sqrt{5} \cdot \sqrt{5} = 5$.

7. $\sqrt[3]{2}$ is $\sqrt[3]{4}$, $\sqrt[3]{2} \cdot \sqrt[3]{4} = 2$.

2. $3\sqrt{6}$ is $\sqrt{6}$, $3\sqrt{6} \cdot \sqrt{6} = 18$.

8. $\sqrt[3]{3}$ is $\sqrt[3]{9}$, $\sqrt[3]{3} \cdot \sqrt[3]{9} = 3$.

3. $2\sqrt{7}$ is $\sqrt{7}$, $2\sqrt{7} \cdot \sqrt{7} = 14$.

9. $2\sqrt[3]{5}$ is $\sqrt[3]{25}$, $2\sqrt[3]{5} \cdot \sqrt[3]{25} = 10$.

4. $\sqrt{8}$ is $\sqrt{2}$, $\sqrt{8} \cdot \sqrt{2} = 4$.

10. $\sqrt[3]{16}$ is $\sqrt[3]{4}$, $\sqrt[3]{16} \cdot \sqrt[3]{4} = 4$.

5. $\sqrt{32}$ is $\sqrt{2}$, $\sqrt{32} \cdot \sqrt{2} = 8$.

11. $\sqrt[3]{25}$ is $\sqrt[3]{5}$, $\sqrt[3]{25} \cdot \sqrt[3]{5} = 5$.

6. $\sqrt{27}$ is $\sqrt{3}$, $\sqrt{27} \cdot \sqrt{3} = 9$.

12. $\sqrt[3]{36}$ is $\sqrt[3]{6}$, $\sqrt[3]{36} \cdot \sqrt[3]{6} = 6$.

13. $\sqrt[3]{49}$ is $\sqrt[3]{7}$, $\sqrt[3]{49} \cdot \sqrt[3]{7} = 7$.

14. $\sqrt{2} + 3$ is $\sqrt{2} - 3$, $(\sqrt{2} + 3)(\sqrt{2} - 3) = -7$.

15. $\sqrt{3} - \sqrt{2}$ is $\sqrt{3} + \sqrt{2}$, $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 1$.

16. $3 + \sqrt{7}$ is $3 - \sqrt{7}$, $(3 + \sqrt{7})(3 - \sqrt{7}) = 2$.

17. $3\sqrt{2} - 5$ is $3\sqrt{2} + 5$, $(3\sqrt{2} - 5)(3\sqrt{2} + 5) = -7$.

18. $4\sqrt{3} - \sqrt{2}$ is $4\sqrt{3} + \sqrt{2}$, $(4\sqrt{3} - \sqrt{2})(4\sqrt{3} + \sqrt{2}) = 46$.

19. $2\sqrt{5} + 7\sqrt{6}$ is $2\sqrt{5} - 7\sqrt{6}$,

$$(2\sqrt{5} + 7\sqrt{6})(2\sqrt{5} - 7\sqrt{6}) = -274.$$

20. $\sqrt{x} - \sqrt{a}$ is $\sqrt{x} + \sqrt{a}$, $(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a}) = x - a$.

21. $\sqrt{3a} + \sqrt{x}$ is $\sqrt{3a} - \sqrt{x}$, $(\sqrt{3a} + \sqrt{x})(\sqrt{3a} - \sqrt{x}) = 3a - x$.

22. $3\sqrt{x} - a\sqrt{2}$ is $3\sqrt{x} + a\sqrt{2}$, $(3\sqrt{x} - a\sqrt{2})(3\sqrt{x} + a\sqrt{2})$
 $= 9x - 2a^2$.
23. $\sqrt{x+a} + \sqrt{x}$ is $\sqrt{x+a} - \sqrt{x}$, $(\sqrt{x+a} + \sqrt{x})(\sqrt{x+a} - \sqrt{x}) = a$.
24. $\sqrt{x} - \sqrt{a-x}$ is $\sqrt{x} + \sqrt{a-x}$, $(\sqrt{x} - \sqrt{a-x})(\sqrt{x} + \sqrt{a-x})$
 $= 2x - a$.
25. $\sqrt{2ax - x^2} - \sqrt{ax}$ is $\sqrt{2ax - x^2} + \sqrt{ax}$,
 $(\sqrt{2ax - x^2} - \sqrt{ax})(\sqrt{2ax - x^2} + \sqrt{ax}) = ax - x^2$.

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1. $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$.
2. $\frac{(18)^{\frac{1}{2}}}{(3)^{\frac{1}{2}}} = 6^{\frac{1}{2}}$.
3. $\frac{\sqrt{6}}{\sqrt{18}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{1}{3}\sqrt{3}$.
4. $\frac{6}{2\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{3}{2}\sqrt{2}$.
5. $\frac{3\sqrt{12}}{\sqrt{6}} = 3\sqrt{2}$.
6. $\frac{8\sqrt{15}}{4\sqrt{5}} = 2\sqrt{3}$.
7. $\frac{10}{2\sqrt{5}} = \frac{5 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \sqrt{5}$.
8. $\frac{4\sqrt{10}}{8\sqrt{5}} = \frac{\sqrt{2}}{2}$.
9. $\frac{8}{4\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2}{3}\sqrt{3}$.
10. $\frac{3\sqrt{2}}{15\sqrt{8}} = \frac{1}{5\sqrt{4}} = \frac{1}{10}$.
11. $\frac{\sqrt{6} + \sqrt{18}}{3\sqrt{2}} = \frac{\sqrt{3} + \sqrt{9}}{3} = \frac{\sqrt{3} + 3}{3} = \frac{1}{3}\sqrt{3} + 1$.
12. $\frac{\sqrt{12} - \sqrt{24}}{2\sqrt{3}} = \frac{\sqrt{4} - \sqrt{8}}{2} = \frac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}$.
13. $\frac{6\sqrt{10} + 4\sqrt{15} - \sqrt{20}}{2\sqrt{5}} = 3\sqrt{2} + 2\sqrt{3} - \frac{\sqrt{4}}{2} = 3\sqrt{2} + 2\sqrt{3} - 1$.
14. $\frac{12 + \sqrt{3} + \sqrt{5}}{\sqrt{6}} = \frac{12\sqrt{6} + \sqrt{18} + \sqrt{30}}{6} = 2\sqrt{6} + \frac{1}{2}\sqrt{2} + \frac{1}{6}\sqrt{30}$.
15. $\frac{\sqrt{6} - \sqrt{9} + 18}{2\sqrt{2}} = \frac{\sqrt{12} - \sqrt{18} + 18\sqrt{2}}{2 \cdot 2} = \frac{2\sqrt{3} - 3\sqrt{2} + 18\sqrt{2}}{4}$
 $= \frac{2\sqrt{3} + 15\sqrt{2}}{4}$.
16. $\frac{\sqrt{5} + 2}{\sqrt{125}} = \frac{\sqrt{25} + 2\sqrt{5}}{\sqrt{625}} = \frac{5}{25} + \frac{2}{25}\sqrt{5} = \frac{1}{5} + \frac{2}{25}\sqrt{5}$.
17. $\frac{\sqrt{\frac{3 \cdot 2}{10}}}{\sqrt{\frac{4}{5}}} = \sqrt{\frac{3 \cdot 2}{10} \cdot \frac{5}{4}} = \sqrt{4} = 2$.

$$18. \frac{(xy)^{\frac{1}{2}}}{x^{\frac{1}{2}}} = y^{\frac{1}{2}}.$$

$$19. \frac{a \sqrt{bc}}{d \sqrt{c}} = \frac{a \sqrt{b}}{d}.$$

$$20. \frac{a^2 \sqrt{c}}{a \sqrt{bc}} = \frac{a}{\sqrt{b}} = \frac{a \sqrt{b}}{\sqrt{b^2}} = \frac{a}{b} \sqrt{b}.$$

$$21. \frac{\sqrt[3]{16}}{\sqrt[3]{4}} = \sqrt[3]{4}.$$

$$22. \frac{8 \sqrt[3]{125}}{4 \sqrt[3]{25}} = 2 \sqrt[3]{5}.$$

$$23. \frac{2 \sqrt[3]{5}}{3 \sqrt[3]{4}} = \frac{2 \sqrt[3]{5} \cdot \sqrt[3]{2}}{3 \sqrt[3]{8}} = \frac{2 \sqrt[3]{10}}{6} = \frac{1}{3} \sqrt[3]{10}.$$

$$24. \frac{a \sqrt[3]{9}}{b \sqrt[3]{8}} = \frac{a \sqrt[3]{9}}{2b}.$$

$$25. \frac{\sqrt[3]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{4}}} = \sqrt[3]{\frac{1}{8} \cdot \frac{4}{1}} = \sqrt[3]{\frac{1}{2}} = \frac{1}{2} \sqrt[3]{4}.$$

$$26. \frac{\sqrt[3]{4}}{\sqrt{2}} = \frac{\sqrt[6]{16}}{\sqrt[6]{8}} = \sqrt[6]{2}.$$

$$27. \frac{\sqrt{2}}{\sqrt[3]{4}} = \frac{\sqrt{2} \cdot \sqrt[3]{2}}{\sqrt[3]{8}} = \frac{2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}}}{2} = \frac{1}{2} \sqrt[6]{2^5} = \frac{1}{2} \sqrt[6]{32}.$$

$$28. \frac{4^{\frac{1}{3}}}{6^{\frac{1}{2}}} = \frac{\sqrt[3]{4}}{\sqrt{6}} = \frac{\sqrt[3]{4} \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{2^{\frac{2}{3}} \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{6} = \frac{2^{\frac{7}{6}} \cdot 3^{\frac{3}{6}}}{6} = \frac{2}{6} \sqrt[6]{2 \cdot 3^3} = \frac{1}{3} \sqrt[6]{54}.$$

$$29. \frac{6^{\frac{1}{2}}}{4^{\frac{1}{3}}} = \frac{\sqrt{6}}{\sqrt[3]{4}} = \frac{\sqrt{6} \cdot \sqrt[3]{2}}{\sqrt[3]{4} \cdot \sqrt[3]{2}} = \frac{2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 2^{\frac{1}{3}}}{2} = \frac{2^{\frac{5}{6}} \cdot 3^{\frac{3}{6}}}{2} = \frac{1}{2} \sqrt[6]{2^5 \cdot 3^3} = \frac{1}{2} \sqrt[6]{864}.$$

$$30. \frac{\sqrt[5]{27}}{\sqrt{27}} = \frac{\sqrt[5]{27} \cdot \sqrt{3}}{\sqrt{27} \cdot \sqrt{3}} = \frac{3^{\frac{3}{5}} \cdot 3^{\frac{1}{2}}}{9} = \frac{3^{\frac{11}{10}}}{9} = \frac{1}{3} \sqrt[10]{3}.$$

$$31. \frac{\sqrt[4]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{2}}} = \frac{\sqrt[4]{\frac{1}{8}} \cdot \sqrt[3]{\frac{1}{4}}}{\sqrt[3]{\frac{1}{2}} \cdot \sqrt[3]{\frac{1}{4}}} = \frac{(\frac{1}{2})^{\frac{3}{4}} (\frac{1}{2})^{\frac{2}{3}}}{\frac{1}{2}} = \frac{(\frac{1}{2})^{\frac{17}{12}}}{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{5}{12}} = \sqrt[12]{\frac{1}{32}} = \sqrt[12]{\frac{128}{2^{12}}} = \frac{1}{2} \sqrt[12]{128}.$$

$$32. \frac{\sqrt[6]{\frac{a^2}{72a}}}{\sqrt[3]{\frac{a}{81}}} = \frac{\sqrt[6]{\frac{a^2}{72a}}}{\sqrt[6]{\frac{a^2}{81 \cdot 81}}} = \sqrt[6]{\frac{81 \cdot 81}{72a}} = \sqrt[6]{\frac{729}{8a}} = 3 \sqrt[6]{\frac{8a^5}{64a^6}} = \frac{3}{2a} \sqrt[6]{8a^5}.$$

$$33. \frac{5}{\sqrt{5} + 2} = \frac{5(\sqrt{5} - 2)}{(\sqrt{5} + 2)(\sqrt{5} - 2)} = \frac{5\sqrt{5} - 10}{5 - 4} = 5\sqrt{5} - 10.$$

$$34. \frac{2}{2 - \sqrt{3}} = \frac{2(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{4 + 2\sqrt{3}}{4 - 3} = 4 + 2\sqrt{3}.$$

$$35. \frac{4}{\sqrt{3} - \sqrt{2}} = \frac{4(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = \frac{4\sqrt{3} + 4\sqrt{2}}{3 - 2} = 4\sqrt{3} + 4\sqrt{2}.$$

$$36. \frac{\sqrt{5}}{3\sqrt{2} - \sqrt{5}} = \frac{\sqrt{5}(3\sqrt{2} + \sqrt{5})}{(3\sqrt{2} - \sqrt{5})(3\sqrt{2} + \sqrt{5})} = \frac{3\sqrt{10} + 5}{18 - 5} = \frac{3}{13} \sqrt{10} + \frac{5}{13}.$$

37. $\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} = \frac{5 - 2\sqrt{15} + 3}{5 - 3} = 4 - \sqrt{15}.$
38. $\frac{4\sqrt{3} + 2\sqrt{2}}{4\sqrt{3} - 2\sqrt{2}} = \frac{(2\sqrt{3} + \sqrt{2})(2\sqrt{3} + \sqrt{2})}{(2\sqrt{3} - \sqrt{2})(2\sqrt{3} + \sqrt{2})} = \frac{12 + 4\sqrt{6} + 2}{12 - 2} = \frac{7}{5} + \frac{2}{5}\sqrt{6}.$
39. $\frac{2\sqrt{5} - 3\sqrt{3}}{5\sqrt{5} - 5} = \frac{(2\sqrt{5} - 3\sqrt{3})(\sqrt{5} + 1)}{5(\sqrt{5} - 1)(\sqrt{5} + 1)} = \frac{2 \cdot 5 - 3\sqrt{15} + 2\sqrt{5} - 3\sqrt{3}}{5(5 - 1)}$
 $= \frac{1}{2} - \frac{3}{20}\sqrt{15} + \frac{1}{10}\sqrt{5} - \frac{3}{20}\sqrt{3}.$
40. $3 + \sqrt{2} = 3 + 1.414 = 4.414.$
41. $14 - 5\sqrt{7} = 14 - 13.2285 = .771 +.$
42. $6 + 2\sqrt{5} = 6 + 2 \cdot 2.236 = 6 + 4.472 = 10.472.$
 $6 - 2\sqrt{5} = 6 - 4.472 + = 1.527.$
43. $\frac{7 \pm \sqrt{6}}{3} = \frac{7 \pm 2.449}{3} = \frac{9.449}{3}$ or $\frac{4.550}{3}$ which equal 3.149 or 1.516 respectively.
44. $\frac{3\sqrt{6}}{2\sqrt{5}} = \frac{3\sqrt{30}}{10} = \frac{16.431}{10} = 1.643.$
45. $\frac{2\sqrt{5} + 1}{3\sqrt{5} - \sqrt{3}} = \frac{(2\sqrt{5} + 1)(3\sqrt{5} + \sqrt{3})}{(3\sqrt{5} - \sqrt{3})(3\sqrt{5} + \sqrt{3})} = \frac{6 \cdot 5 + 3\sqrt{5} + 2\sqrt{15} + \sqrt{3}}{9 \cdot 5 - 3}$
 $= \frac{30 + 3\sqrt{5} + 2\sqrt{15} + \sqrt{3}}{42} = \frac{30 + 6.708 + 7.745 + 1.732}{42} = 1.099.$
46. $\sqrt{2 - \sqrt{3}} = \sqrt{2 - 1.732} = \sqrt{.267} = .517.$
47. $\frac{\sqrt{3}}{\sqrt{3} + \sqrt[3]{4} + \sqrt[4]{5}} = \frac{1.732}{1.732 + 1.587 + 1.495} = \frac{1.732}{4.814} = .359.$
48. $\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b})}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{a + 2\sqrt{ab} + b}{a - b}.$
49. $\frac{2\sqrt{x} - \sqrt{a}}{\sqrt{x} + 3\sqrt{a}} = \frac{(2\sqrt{x} - \sqrt{a})(\sqrt{x} - 3\sqrt{a})}{(\sqrt{x} + 3\sqrt{a})(\sqrt{x} - 3\sqrt{a})} = \frac{2x - 7\sqrt{ax} + 3a}{x - 9a}.$
50. $\frac{r\sqrt{3} + \sqrt{r}}{\sqrt{3} - \sqrt{r}} = \frac{(r\sqrt{3} + \sqrt{r})(\sqrt{3} + \sqrt{r})}{(\sqrt{3} - \sqrt{r})(\sqrt{3} + \sqrt{r})} = \frac{3r + \sqrt{3}r + r\sqrt{3}r + r}{3 - r}$
 $= \frac{4r + \sqrt{3}r + r\sqrt{3}r}{3 - r}.$
51. $\frac{m\sqrt{n} + a\sqrt{b}}{m\sqrt{n} - a\sqrt{b}} = \frac{(m\sqrt{n} + a\sqrt{b})(m\sqrt{n} + a\sqrt{b})}{(m\sqrt{n} - a\sqrt{b})(m\sqrt{n} + a\sqrt{b})}$
 $= \frac{m^2n + 2am\sqrt{bn} + a^2b}{m^2n - a^2b}.$

$$52. \frac{\sqrt{a-2}-2}{\sqrt{a-2}+2} = \frac{(\sqrt{a-2}-2)(\sqrt{a-2}-2)}{(\sqrt{a-2}+2)(\sqrt{a-2}-2)} = \frac{a-2-4\sqrt{a-2}+4}{a-2-4}$$

$$= \frac{a-4\sqrt{a-2}+2}{a-6}.$$

$$53. \frac{2}{\sqrt[4]{2+\sqrt{2}}} = \frac{2\sqrt[4]{2-\sqrt{2}}}{\sqrt[4]{2}} = \frac{2\sqrt[4]{16-8\sqrt{2}}}{2} = \sqrt[4]{16-8\sqrt{2}}.$$

$$54. \frac{x-\sqrt{a+b}}{x+\sqrt{a+b}} = \frac{(x-\sqrt{a+b})(x-\sqrt{a+b})}{(x+\sqrt{a+b})(x-\sqrt{a+b})} = \frac{x^2-2x\sqrt{a+b}+a+b}{x^2-a-b}.$$

$$55. \frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}+\sqrt{d}} = \frac{(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d})}{(\sqrt{c}+\sqrt{d})(\sqrt{c}-\sqrt{d})} = \frac{\sqrt{ac}+\sqrt{bc}-\sqrt{ad}-\sqrt{bd}}{c-d}.$$

56. No.

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1. $10^2 + 5^2 = h^2.$
 $h^2 = 125.$
 $h = \sqrt{125} = 5\sqrt{5},$
the hypotenuse.

2. $10^2 - 5^2 = l^2.$
 $l^2 = 75.$
 $l = \sqrt{75} = 5\sqrt{3},$
the leg.

$$\text{Area} = \frac{5 \cdot 5 \sqrt{3}}{2} = 12\frac{1}{2} \sqrt{3}.$$

3. $R^2 - \frac{R^2}{4} = l^2.$
 $l^2 = \frac{3R^2}{4}.$
 $l = \sqrt{\frac{3R^2}{4}} = \frac{R}{2} \sqrt{3},$
the leg.

$$\text{Area} = \frac{\frac{R}{2} \cdot \frac{R}{2} \sqrt{3}}{2} = \frac{R^2}{8} \sqrt{3}.$$

4. $10^2 + 10^2 = D^2.$
 $D^2 = 200.$
 $D = \sqrt{200} = 10\sqrt{2},$
the diagonal.

5. $10^2 = 2S^2.$
 $S^2 = 50.$
 $S = \sqrt{50} = 5\sqrt{2},$
the side.

$$\text{Area} = S^2 = 50.$$

6. $4R^2 = 2S^2.$
 $S^2 = 2R^2.$
 $S = \sqrt{2R^2} = R\sqrt{2},$
the side.

$$\text{Area} = S^2 = 2R^2.$$

7. $A^2 = 12^2 - 6^2.$
 $A^2 = 108.$
 $A = \sqrt{108} = 6\sqrt{3},$
the altitude.

$$\text{Area} = \frac{12 \cdot 6 \sqrt{3}}{2} = 36 \sqrt{3}.$$

8. $A^2 = S^2 - \frac{S^2}{4}.$
 $A^2 = \frac{3S^2}{4}.$
 $A = \sqrt{\frac{3S^2}{4}} = \frac{S}{2} \sqrt{3},$
the altitude.

$$\text{Area} = \frac{S \cdot S \sqrt{3}}{2 \cdot 2} = \frac{S^2}{4} \sqrt{3}.$$

$$9. \quad S^2 - \frac{S^2}{4} = 10^2.$$

$$\frac{3 S^2}{4} = 100.$$

$$S^2 = \frac{400}{3}.$$

$$S = \sqrt{\frac{400}{3}} = 20 \sqrt{\frac{3}{9}}$$

$$= 6\frac{2}{3} \sqrt{3}, \text{ the side.}$$

$$\text{Area} = \frac{\left(\frac{20}{3} \sqrt{3}\right)^2}{4} \sqrt{3}$$

$$= \frac{100}{3} \sqrt{3}.$$

$$10. \quad 2 l^2 = 20^2.$$

$$l^2 = 200.$$

$$l = \sqrt{200} = 10 \sqrt{2},$$

the leg.

$$\text{Area} = \frac{10 \sqrt{2} \cdot 10 \sqrt{2}}{2}$$

$$= 100.$$

$$11. \quad \frac{l^2}{2} = 32.$$

$$l^2 = 64.$$

$$l = 8, \text{ the leg.}$$

$$H^2 = 8^2 + 8^2 = 128.$$

$$H = \sqrt{128} = 8 \sqrt{2},$$

the hypotenuse.

$$12. \quad H^2 = \frac{R^2}{4} + \frac{9 R^2}{4}.$$

$$H = \sqrt{\frac{10 R^2}{4}} = \frac{R}{2} \sqrt{10},$$

the hypotenuse.

$$13. \quad l^2 = R^2 - \frac{R^2}{25}.$$

$$l^2 = \frac{24 R^2}{25}.$$

$$l = \sqrt{\frac{24 R^2}{25}} = \frac{2 R}{5} \sqrt{6}, \text{ the leg.}$$

$$14. \quad H^2 = R^2 + \frac{R^2}{4} (5 - 2 \sqrt{5} + 1)$$

$$= \frac{4 R^2 + 6 R^2 - 2 R^2 \sqrt{5}}{4}$$

$$= \frac{R^2}{4} (10 - 2 \sqrt{5}).$$

$$H = \frac{R}{2} \sqrt{10 - 2 \sqrt{5}},$$

the hypotenuse.

$$15. \quad H^2 = \frac{R^2}{4} + \left(R^2 - R^2 \sqrt{3} + \frac{3 R^2}{4} \right)$$

$$= 2 R^2 - R^2 \sqrt{3}.$$

$$H = R \sqrt{2 - \sqrt{3}},$$

the hypotenuse.

$$16. \quad \text{Area} = 5 \cdot \frac{2 R}{3} \sqrt{4 - \sqrt{3}} \cdot 3 \cdot \frac{9 R \sqrt{4 + \sqrt{3}}}{10} = 9 R^2 \sqrt{16 - 3} = 9 R^2 \sqrt{13}.$$

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$$1. \quad x^2 - 6 = (x + \sqrt{6})(x - \sqrt{6}).$$

$$2. \quad 3 x^2 - 4 = (x \sqrt{3} + 2)(x \sqrt{3} - 2).$$

$$3. \quad 5 x^2 - 1 = (x \sqrt{5} + 1)(x \sqrt{5} - 1).$$

$$4. \quad x^4 - 4 = (x^2 + 2)(x^2 - 2) = (x^2 + 2)(x + \sqrt{2})(x - \sqrt{2}).$$

$$5. \quad 4 x^4 - 1 = (2 x^2 + 1)(2 x^2 - 1) = (2 x^2 + 1)(x \sqrt{2} + 1)(x \sqrt{2} - 1).$$

$$6. \quad x^3 - 2 = (x - \sqrt[3]{2})(x^2 + x \sqrt[3]{2} + \sqrt[3]{4}).$$

$$7. \quad x^3 + 6 = (x + \sqrt[3]{6})(x^2 - x \sqrt[3]{6} + \sqrt[3]{36}).$$

$$8. \quad 3 x^3 - 1 = (x \sqrt[3]{3} - 1)(x^2 \sqrt[3]{9} + x \sqrt[3]{3} + 1).$$

$$9. \quad x^3 + 4 = (x + \sqrt[3]{4})(x^2 - x \sqrt[3]{4} + \sqrt[3]{16}) = (x + \sqrt[3]{4})(x^2 - x \sqrt[3]{4} + 2 \sqrt[3]{2}).$$

$$10. \quad 2x^3 + 8 = 2(x^3 + 4) = 2(x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + \sqrt[3]{16}) \\ = 2(x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + 2\sqrt[3]{2}).$$

$$11. \quad 2x^3 - 8 = 2(x^3 - 4) = 2(x - \sqrt[3]{4})(x^2 + x\sqrt[3]{4} + \sqrt[3]{16}) \\ = 2(x - \sqrt[3]{4})(x^2 + x\sqrt[3]{4} + 2\sqrt[3]{2}).$$

$$12. \quad 6x^3 + 24 = 6(x^3 + 4) = 6(x + \sqrt[3]{4})(x^2 - \sqrt[3]{4}x + 2\sqrt[3]{2}).$$

$$13. \quad x^2 - 2 = 0.$$

$$(x + \sqrt{2})(x - \sqrt{2}) = 0.$$

$$\text{Whence } x = \pm \sqrt{2}.$$

$$14. \quad x^2 - 6 = 0.$$

$$(x + \sqrt{6})(x - \sqrt{6}) = 0.$$

$$\text{Whence } x = \pm \sqrt{6}.$$

$$15. \quad 2x^2 - 1 = 0.$$

$$(\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0.$$

$$\text{Whence } x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}.$$

$$16. \quad x^4 + 6 = 5x^2.$$

$$(x^2 - 2)(x^2 - 3) = 0.$$

$$\text{Whence } x = \pm \sqrt{2} \text{ or } \pm \sqrt{3}.$$

$$21. \quad ax^4 - x^2 + 3a = 3a^2x^2.$$

$$(ax^2 - 1)(x^2 - 3a) = 0.$$

$$\text{Whence } x = \pm \frac{\sqrt{a}}{a} \text{ or } \pm \sqrt{3a}.$$

$$22. \quad 4x^4 + a = x^2 + 4ax^2.$$

$$(4x^2 - 1)(x^2 - a) = 0.$$

$$\text{Whence } x = \pm \frac{1}{2} \text{ or } \pm \sqrt{a}.$$

$$17. \quad 4x^4 + 5 = 12x^2.$$

$$(2x^2 - 5)(2x^2 - 1) = 0.$$

$$\text{Whence } x = \pm \sqrt{\frac{5}{2}} \text{ or } \pm \sqrt{\frac{1}{2}} \\ = \pm \frac{\sqrt{10}}{2} \text{ or } \pm \frac{\sqrt{2}}{2}.$$

$$18. \quad 3x^4 + 8 = 14x^2.$$

$$(3x^2 - 2)(x^2 - 4) = 0.$$

$$\text{Whence } x = \pm 2 \text{ or } \pm \frac{1}{3}\sqrt{6}.$$

$$19. \quad 5x^4 - 16x^2 + 3 = 0.$$

$$(5x^2 - 1)(x^2 - 3) = 0.$$

$$\text{Whence } x = \pm \frac{1}{5}\sqrt{5} \text{ or } \pm \sqrt{3}.$$

$$20. \quad x^4 + 8a = 4x^2 + 2ax^2.$$

$$(x^2 - 4)(x^2 - 2a) = 0.$$

$$\text{Whence } x = \pm 2 \text{ or } \pm \sqrt{2a}.$$

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$$1. \text{ When } f(x) = 0, x = -\frac{3}{2}.$$

$$2. \text{ If } 2x + 3 = 0, x = -\frac{3}{2}.$$

$$3. \text{ They are equal.}$$

$$4. \text{ When } f(x) = 4, x = \frac{1}{2}.$$

$$5. \text{ If } 2x + 3 = 4, x = \frac{1}{2}.$$

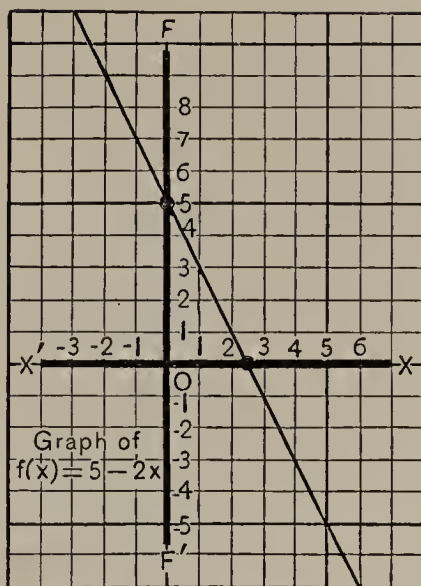
$$6. \text{ They are equal.}$$

$$7. \text{ Yes. When } f(x) = 6, x = \frac{3}{2}.$$

$$8. \text{ When } f(x), 2x + 3 = -2, x = -\frac{5}{2}.$$

9.

x	0	$\frac{5}{2}$
$f(x)$	5	0

10. Yes. When $f(x)$, $5 - 2x = 0$, $x = \frac{5}{2}$.11. (a) When $f(x)$, $5 - 2x = 9$, $x = -2$.(b) When $f(x)$, $5 - 2x = 5$, $x = 0$.(c) When $f(x)$, $5 - 2x = -3$, $x = 4$.(d) When $f(x)$, $5 - 2x = -7$, $x = 6$.

12. The roots obtained from the graph check in the equation.

13. The graph of any linear function of x should give a straight line.

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1. When $f(x)$,

$$4x^2 - 4x - 15 = 0,$$

$$x = -\frac{3}{2} \text{ or } \frac{5}{2}.$$

2. If $4x^2 - 4x - 15 = 0$,

$$\text{then } (2x + 3)(2x - 5) = 0.$$

$$\text{Whence } x = -\frac{3}{2} \text{ or } \frac{5}{2}.$$

3. They are the same.

4. When $f(x)$,

$$4x^2 - 4x - 15 = 20,$$

$$x = -\frac{5}{2} \text{ or } \frac{7}{2}.$$

5. The roots are $-\frac{5}{2}$ and $\frac{7}{2}$.6. If $4x^2 - 4x - 15 = 20$,

$$\text{then } (2x + 5)(2x - 7) = 0.$$

$$\text{Whence } x = -\frac{5}{2} \text{ or } \frac{7}{2}.$$

7. If $f(x)$,

$$4x^2 - 4x - 15 = 9,$$

$$\text{then } x = 3 \text{ or } -2.$$

If $f(x)$,

$$4x^2 - 4x - 15 = -7,$$

$$\text{then } x = 2 \text{ or } -1.$$

8. If $4x^2 - 4x - 15 = 9$,

$$\text{then } (2x - 6)(2x + 4) = 0.$$

$$\text{Whence } x = 3 \text{ or } -2.$$

If $4x^2 - 4x - 15 = -7$,

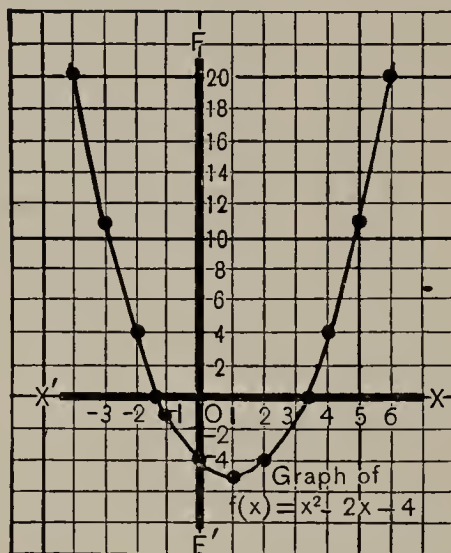
$$\text{then } (2x - 4)(2x + 2) = 0.$$

$$\text{Whence } x = 2 \text{ or } -1.$$

9. Values of x for $f(x) = -25$ or -20 cannot be read from the graph, for the graph does not reach these values.

10.

When $x =$	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x), x^2 - 2x - 4 =$	20	11	4	-1	-4	-5	-4	-1	4	11	20



11. Yes.

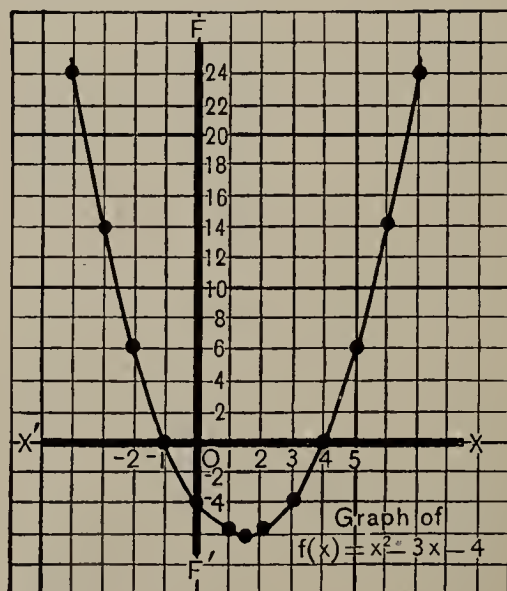
12. When $f(x), x^2 - 2x - 4 = 0, x = -\frac{5}{4}$ or $\frac{13}{4}$, nearly.13. The roots are $-\frac{5}{4}$ or $\frac{13}{4}$, nearly.

14. No. It can be obtained graphically.

15. Yes. Read values of x for $f(x) = 0$.

16.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	24	14	6	0	-4	-6	-6	-4	0	6	14	24

When $f(x) = 0, x = 4$ or -1 .If $x^2 - 3x - 4 = 0,$ then $(x - 4)(x + 1) = 0.$ Whence $x = 4$ or -1 .

17.

x	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	25	16	9	4	1	0	1	4	9	16	25

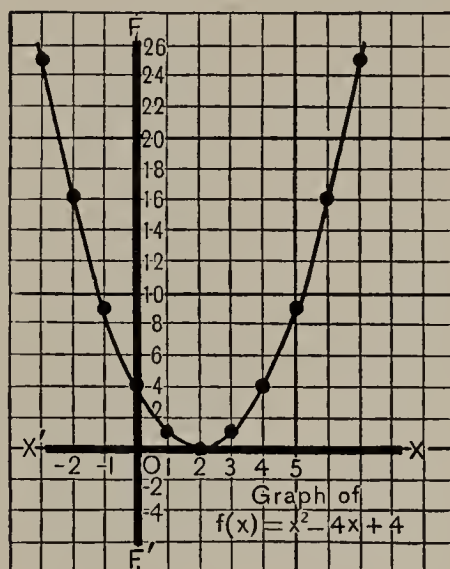
When $f(x)$,

$$x^2 - 4x + 4 = 0,$$

$$x = +2.$$

If $x^2 - 4x + 4 = 0$,
 then $(x - 2)(x - 2) = 0$.

Whence $x = +2$.



18. It touches the x -axis. The equation $x^2 - 4x + 4 = 0$ has one root. The only value of x which satisfies the equation $x^2 - 4x + 4 = 0$ is $+2$.

If $f(x)$, $x^2 - 4x + 4 = 0$, $x = 2$.

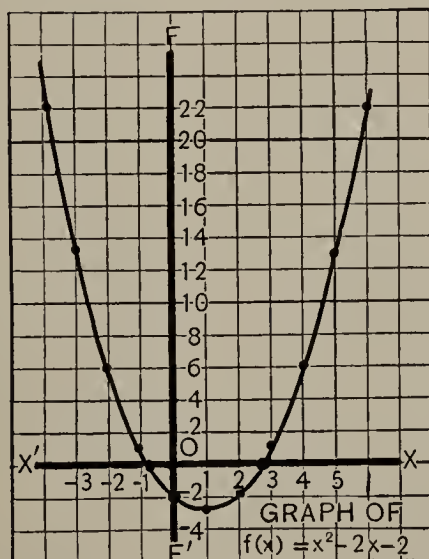
If $f(x)$, $x^2 - 4x + 4 = 4$, $x = 0$ or 4 .

If $f(x)$, $x^2 - 4x + 4 = -1$, x has no real value.

If $f(x)$, $x^2 - 4x + 4 = -10$, x has no real value.

19.

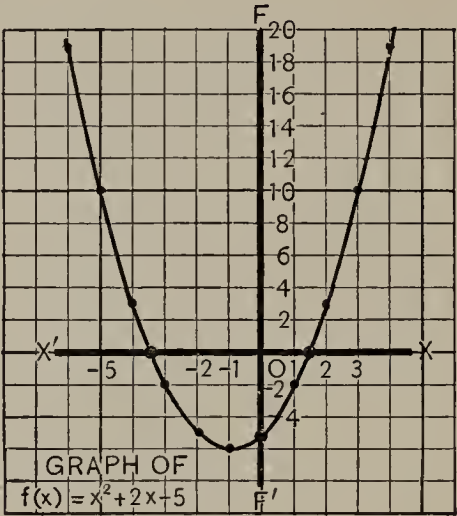
x	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	22	13	6	1	-2	-3	-2	1	6	13	22



When $f(x)$, $x^2 - 2x - 2 = 0$, $x = 2.7$ or $-.7$.

20.

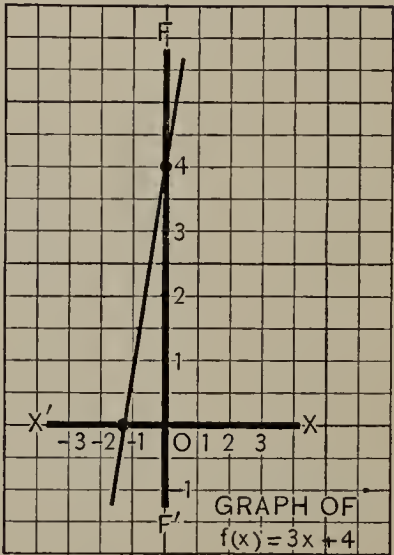
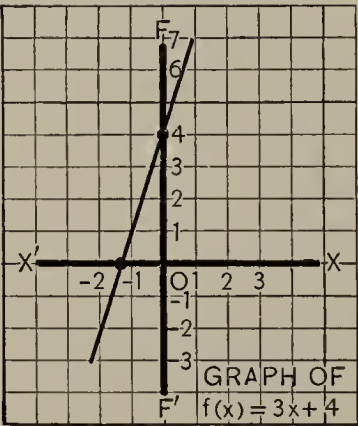
x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	19	10	3	-2	-5	-6	-5	-2	3	10	19



When $f(x)$, $x + 2x - 5 = 0$, $x = 1.4$ or -3.4 .

21.

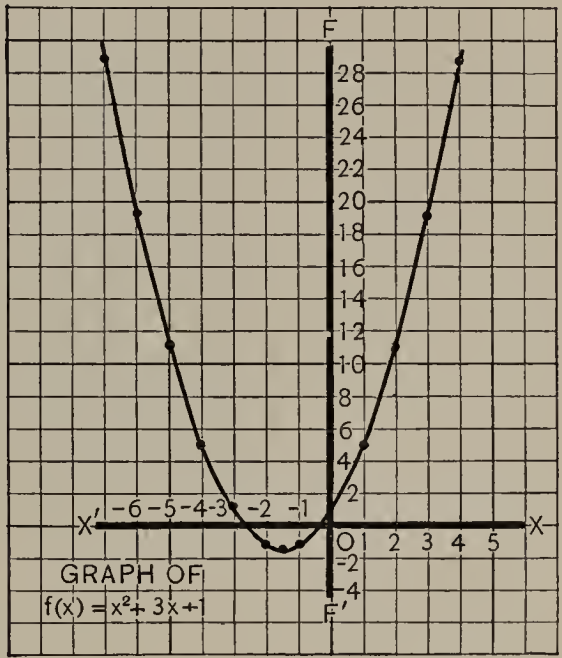
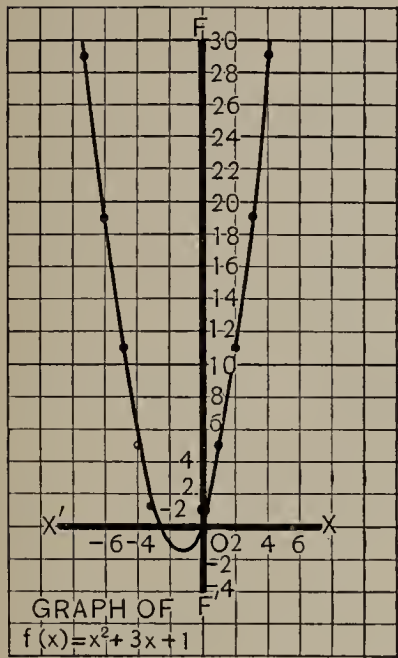
x	0	$-\frac{4}{3}$
$f(x)$	4	0



The lines cross the axes at points which have the same values respectively.

22.

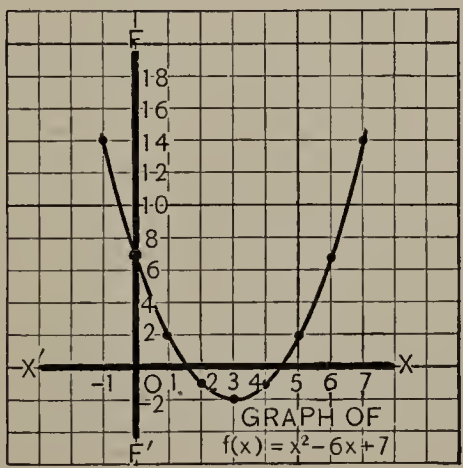
x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	29	19	11	5	1	-1	-1	1	5	11	19	29



The lines cross the axes at points which have the same values respectively.

1.

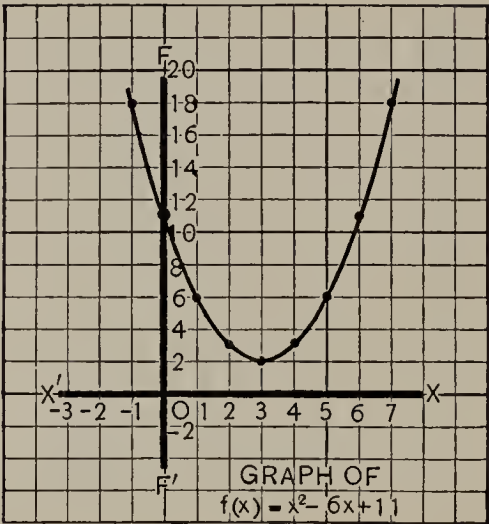
x	-1	0	1	2	3	4	5	6	7
$f(x)$	14	7	2	-1	-2	-1	2	7	14



When $f(x) = 0$, $x = 1.6$ or 4.4 .

2.

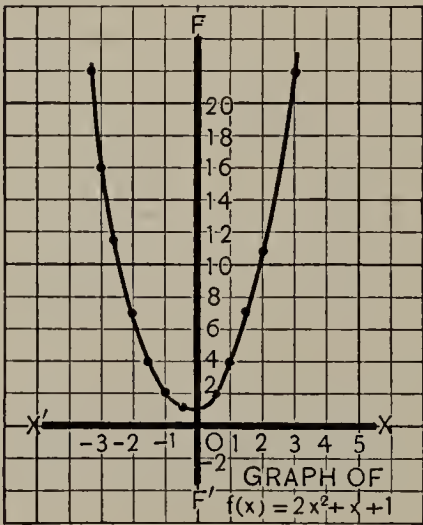
x	-1	0	1	2	3	4	5	6	7
$f(x)$	18	11	6	3	2	3	6	11	18



The equation $x^2 - 6x + 11 = 0$ has no graphical solution.

3.

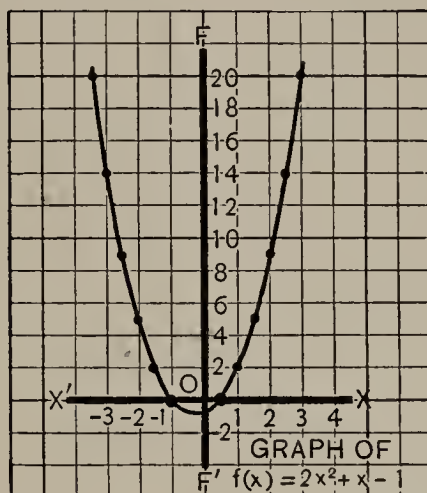
x	$-\frac{7}{2}$	-3	2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	22	16	7	4	2	1	2	4	11	22



The equation $2x^2 + x + 1 = 0$ has no graphical solution.

4.

x	$-\frac{7}{2}$	-3	-2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	20	14	5	2	0	-1	0	2	9	20



When $f(x) = 0$, $x = -1$ or $\frac{1}{2}$.

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1. $x^2 - 8x - 48 = 0$,
or $x^2 - 8x = 48$.
 $(x - 4)^2 = 64$.

Whence $x = +12$ or -4 .

2. $x^2 - 5x - 14 = 0$,
or $x^2 - 5x = 14$.
 $(x - \frac{5}{2})^2 = \frac{81}{4}$.

Whence $x = +7$ or -2 .

3. $x(x + 2) - 5(x + 2) = 0$,
or $x^2 - 3x = 10$.
 $(x - \frac{3}{2})^2 = \frac{49}{4}$.

Whence $x = -2$ or $+5$.

4. $\frac{3}{4} - y = y^2$,
or $y^2 + y = \frac{3}{4}$.
 $(y + \frac{1}{2})^2 = 1$.

Whence $y = \frac{1}{2}$ or $-\frac{3}{2}$.

5. $2y^2 - 9y + 4 = 0$,
or $y^2 - \frac{9y}{2} = -2$.
 $(y - \frac{9}{4})^2 = \frac{49}{16}$.

Whence $y = 4$ or $\frac{1}{2}$.

6. $2y^2 + 5y = 0$,
or $y^2 + \frac{5y}{2} = 0$.

$(y + \frac{5}{4})^2 = \frac{25}{16}$.

Whence $y = 0$ or $-\frac{5}{2}$.

7. $t^2 - 2t - 15 = 0$,
or $t^2 - 2t = 15$.
 $(t - 1)^2 = 16$.

Whence $t = 5$ or -3 .

8. $3t^2 - 7t = 6$,
or $t^2 - \frac{7t}{3} = 2$.

$(t - \frac{7}{6})^2 = \frac{121}{36}$.

Whence $t = 3$ or $-\frac{2}{3}$.

9. $9v = 5v^2 - 2$,
or $v^2 - \frac{9v}{5} = \frac{2}{5}$.

$(v - \frac{9}{10})^2 = \frac{121}{100}$.

Whence $v = 2$ or $-\frac{1}{5}$.

$$\begin{aligned}
 10. \quad & x^2 - 2x - 4 = 0, \\
 \text{or} \quad & x^2 - 2x = 4. \\
 & (x - 1)^2 = 5. \\
 \text{Whence} \quad & x = 1 \pm \sqrt{5} \\
 & = 1 \pm 2.236 \\
 & = 3.236 \text{ or } -1.236.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & v^2 - \frac{7}{12}v - 1 = 0, \\
 \text{or} \quad & v^2 - \frac{7v}{12} = 1. \\
 & (v - \frac{7}{24})^2 = \frac{625}{576}, \\
 \text{or} \quad & v = \frac{4}{3} \text{ or } -\frac{3}{4}.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & s^2 - 2s - 3\frac{1}{2} = 0, \\
 \text{or} \quad & s^2 - 2s = \frac{7}{2}. \\
 & (s - 1)^2 = \frac{9}{2}. \\
 \text{Whence} \quad & s = 1 \pm \frac{3}{2}\sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & h^2 + 10h + 13 = 0, \\
 \text{or} \quad & h^2 + 10h = -13. \\
 & (h + 5)^2 = 12. \\
 \text{Whence} \quad & h = -5 \pm 2\sqrt{3} \\
 & = -1.535 \text{ or } -8.464.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & 12t^2 - 25t + 12 = 0, \\
 \text{or} \quad & t^2 - \frac{25t}{12} = -1. \\
 & (t - \frac{25}{24})^2 = \frac{49}{576}. \\
 \text{Whence} \quad & t = \frac{4}{3} \text{ or } \frac{3}{4}.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & 42 + 2z^2 = -19z, \\
 \text{or} \quad & z^2 + \frac{19z}{2} = -21. \\
 & (z + \frac{19}{4})^2 = \frac{25}{16}. \\
 \text{Whence} \quad & z = -\frac{7}{2} \text{ or } -6.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & 15x^2 + 4x = 4, \\
 \text{or} \quad & x^2 + \frac{4x}{15} = \frac{4}{15}. \\
 & (x + \frac{2}{15})^2 = \frac{64}{225}. \\
 \text{Whence} \quad & x = \frac{2}{3} \text{ or } -\frac{2}{3}.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 1 - 6v^2 = 2v, \\
 \text{or} \quad & v^2 + \frac{v}{3} = \frac{1}{6}.
 \end{aligned}$$

$$\begin{aligned}
 & (v + \frac{1}{6})^2 = \frac{7}{36}. \\
 \text{Whence} \quad & v = \frac{-1 \pm \sqrt{7}}{6}.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 20s^2 + s = 1, \\
 \text{or} \quad & s^2 + \frac{s}{20} = \frac{1}{20}.
 \end{aligned}$$

$$\begin{aligned}
 & (s + \frac{1}{40})^2 = (\frac{9}{40})^2. \\
 \text{Whence} \quad & s = \frac{1}{5} \text{ or } -\frac{1}{4}.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & 9x + 4 = 9x^2, \\
 \text{or} \quad & x^2 - x = \frac{4}{9}. \\
 & (x - \frac{1}{2})^2 = \frac{25}{36}. \\
 \text{Whence} \quad & x = \frac{4}{3} \text{ or } -\frac{1}{3}.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & 25x^2 - 20x - 12 = 0, \\
 \text{or} \quad & x^2 - \frac{4x}{5} = \frac{12}{25}. \\
 & (x - \frac{2}{5})^2 = \frac{16}{25}. \\
 \text{Whence} \quad & x = \frac{6}{5} \text{ or } -\frac{2}{5}.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & 4x^2 = 1 - 4x, \\
 \text{or} \quad & x^2 + x = \frac{1}{4}. \\
 & (x + \frac{1}{2})^2 = \frac{2}{4}. \\
 \text{Whence} \quad & x = \frac{-1 \mp \sqrt{2}}{2} \\
 & = -1.207 \text{ or } .207.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & x^2 + 4\sqrt{5}x = 25, \\
 & (x + 2\sqrt{5})^2 = 45. \\
 \text{Whence} \quad & x = \sqrt{5} \text{ or } -5\sqrt{5}.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & x^2 - 3\sqrt{2}x + 4 = 0, \\
 \text{or} \quad & x^2 - 3\sqrt{2}x = -4. \\
 & (x - \frac{3}{2}\sqrt{2})^2 = \frac{2}{4}. \\
 \text{Whence} \quad & x = 2\sqrt{2} \text{ or } \sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & 1 - \frac{3\sqrt{3}}{2R} + \frac{3}{2R^2} = 0, \\
 \text{or} \quad & R^2 - \frac{3\sqrt{3}R}{2} = -\frac{3}{2}.
 \end{aligned}$$

$$\begin{aligned}
 & (R - \frac{3}{4}\sqrt{3})^2 = \frac{3}{16}. \\
 \text{Whence} \quad & R = \sqrt{3} \text{ or } \frac{1}{2}\sqrt{3}.
 \end{aligned}$$

25. $\frac{2x}{9} + \frac{2}{3} = \frac{7}{18x},$
 or $x^2 + 3x = \frac{7}{4}.$
 $(x + \frac{3}{2})^2 = 4.$
 Whence $x = \frac{1}{2}$ or $-\frac{7}{2}.$

26. $\frac{1}{3} + \frac{x}{9} - \frac{2}{x} = 0,$
 or $x^2 + 3x = 18.$
 $(x + \frac{3}{2})^2 = \frac{81}{4}.$
 Whence $x = 3$ or $-6.$

27. $\frac{2}{5v^2} + \frac{1}{4} - \frac{11}{10v} = 0,$
 or $v^2 - \frac{22v}{5} = -\frac{8}{5}.$
 $(v - \frac{11}{5})^2 = \frac{81}{25}.$
 Whence $v = 4$ or $\frac{2}{5}.$

28. $\frac{3a}{2} + \frac{1}{2} - \frac{1}{3a} = 0,$
 or $a^2 + \frac{a}{3} = \frac{2}{9}.$
 $(a + \frac{1}{6})^2 = \frac{1}{4}.$
 Whence $a = \frac{1}{3}$ or $-\frac{2}{3}.$

29. $\frac{25c}{2} - \frac{40}{c} - 40 = 0,$
 or $c^2 - \frac{16c}{5} = \frac{16}{5}.$
 $(c - \frac{8}{5})^2 = \frac{144}{25}.$
 Whence $c = 4$ or $-\frac{4}{5}.$

30. $(x-4)^2 - 3(x-9) = 15,$
 or $x^2 - 11x = -28.$
 $(x - \frac{11}{2})^2 = \frac{9}{4}.$
 Whence $x = 7$ or $4.$

31. $(x-2)(x+3) = x(5x-9) - 2,$
 or $x^2 - \frac{5x}{2} = -1.$
 $(x - \frac{5}{4})^2 = \frac{9}{16}.$
 Whence $x = 2$ or $\frac{1}{2}.$

32. $x - \frac{3}{x+2} = 0,$
 or $x^2 + 2x = 3.$
 $(x+1)^2 = 4.$
 Whence $x = 1$ or $-3.$

33. $\frac{v^2}{v-5} + \frac{5}{2} = 0,$
 or $v^2 + \frac{5v}{2} = \frac{25}{2}.$
 $(v + \frac{5}{4})^2 = \frac{225}{16}.$
 Whence $v = \frac{5}{2}$ or $-5.$

34. $\frac{3}{t-7} + \frac{t}{4} = 0,$
 or $t^2 - 7t = -12.$
 $(t - \frac{7}{2})^2 = \frac{1}{4}.$
 Whence $t = 4$ or $3.$

35. $\frac{s}{s-2} + \frac{s-2}{s} = \frac{5}{2},$
 or $s^2 - 2s = 8.$
 $(s-1)^2 = 9.$
 Whence $s = 4$ or $-2.$

36. $\frac{7}{2y-3} - \frac{5}{1-y} = 12,$
 or $y^2 - \frac{77y}{24} = -\frac{58}{24}.$
 $(y - \frac{77}{48})^2 = \frac{361}{2304}.$
 Whence $y = 2$ or $\frac{29}{24}.$

37. $\frac{3+x}{4+x} - \frac{x-5}{x-6} = \frac{1}{12},$
 or $x^2 + 22x = 48.$
 $(x+11)^2 = 169.$
 $x = 2$ or $-24.$

38. If $y = 2,$
 $x^2 - xy - 3y^2 = -12$ becomes
 $x^2 - 2x = 0.$
 Whence $(x-1)^2 = 1,$
 and $x = 2$ or $0.$

39. If $x = -3$, $x^2 - 4xy + x^3 + y^2 + 5 = 0$ becomes $y^2 + 12y = 13$.

Whence

$$(y + 6)^2 = 49,$$

and

$$y = 1 \text{ or } -13.$$

41. $x^4 - 13x^2 + 36 = 0,$

or $x^4 - 13x^2 = -36.$

$$(x^2 - \frac{13}{2})^2 = \frac{25}{4}.$$

Whence $x^2 = 9 \text{ or } 4,$

and $x = \pm 3 \text{ or } \pm 2.$

42. $4x^4 - 5x^2 + 1 = 0,$

or $x^4 - \frac{5x^2}{4} = -\frac{1}{4}.$

$$(x^2 - \frac{5}{8})^2 = \frac{9}{64}.$$

Whence $x^2 = 1 \text{ or } \frac{1}{4},$

and $x = \pm 1 \text{ or } \pm \frac{1}{2}.$

43. $9x^4 - 37x^2 + 4 = 0,$

or $x^4 - \frac{37x^2}{9} = -\frac{4}{9}.$

$$(x^2 - \frac{37}{18})^2 = \frac{1225}{324}.$$

Whence $x^2 = \frac{1}{9} \text{ or } 4,$

and $x = \pm \frac{1}{3} \text{ or } \pm 2.$

44. $4k^4 = 9k^2 - 2,$

or $k^4 - \frac{9k^2}{4} = -\frac{1}{2}.$

$$(k^2 - \frac{9}{8})^2 = \frac{49}{64}.$$

Whence $k^2 = 2 \text{ or } \frac{1}{4},$

and $k = \pm \sqrt{2} \text{ or } \pm \frac{1}{2}.$

45. $9s^4 + 12 = 31s^2,$

or $s^4 - \frac{31s^2}{9} = -\frac{12}{9}.$

$$(s^2 - \frac{31}{18})^2 = (\frac{23}{18})^2.$$

Whence $s^2 = 3 \text{ or } \frac{4}{9},$

and $s = \pm \sqrt{3} \text{ or } \pm \frac{2}{3}.$

46. $4v^4 + 5 = 21v^2,$

or $v^4 - \frac{21v^2}{4} = -\frac{5}{4}.$

$$(v^2 - \frac{21}{8})^2 = \frac{361}{64}.$$

Whence $v^2 = 5 \text{ or } \frac{1}{4},$

and $v = \pm \sqrt{5} \text{ or } \pm \frac{1}{2}.$

47. $9 - 30 = 49 - 70. \quad (1)$

$$9 - 30 + 25 = 49 - 70 + 25. \quad (2)$$

$$(3 - 5)^2 = (7 - 5)^2. \quad (3)$$

$$3 - 5 = 7 - 5. \quad (4)$$

$$3 = 7. \quad (5)$$

The error occurred in passing from (3) to (4). Square roots of numerically equal expressions are equal only when the signs of the roots are properly chosen. Line (4) could have read

$$+ (3 - 5) = - (7 - 5) \text{ or } - (3 - 5) = + (7 - 5),$$

but not $+ (3 - 5) = + (7 - 5) \text{ or } - (3 - 5) = - (7 - 5).$

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2. $x^2 - ax - 2a^2 = 0,$

or $x^2 - ax = 2a^2.$

$$\left(x - \frac{a}{2}\right)^2 = \frac{9a^2}{4}.$$

Whence $x = 2a \text{ or } -a.$

3. $x^2 + 2ax + a^2 - 4 = 0,$

or $x^2 + 2ax = 4 - a^2.$

$$(x + a)^2 = 4.$$

Whence $x = -a \pm 2.$

4. $x^2 + 1 = a + 2x$,
 or $x^2 - 2x = a - 1$.
 $(x - 1)^2 = a$.

Whence $x = 1 \pm \sqrt{a}$.

5. $3x^2 - ax = 10a^2$,
 or $x^2 - \frac{ax}{3} = \frac{10a^2}{3}$.
 $\left(x - \frac{a}{6}\right)^2 = \frac{121a^2}{36}$.

Whence $x = 2a$ or $-\frac{5a}{3}$.

6. $3mx + 2m^2 = 2x^2$,
 or $x^2 - \frac{3mx}{2} = m^2$.
 $\left(x - \frac{3m}{4}\right)^2 = \frac{25m^2}{16}$.

Whence $x = 2m$ or $-\frac{m}{2}$.

7. $a^2x^2 - 7ax + 10 = 0$,
 or $x^2 - \frac{7x}{a} = -\frac{10}{a^2}$.
 $\left(x - \frac{7}{2a}\right)^2 = \frac{9}{4a^2}$.

Whence $x = \frac{5}{a}$ or $\frac{2}{a}$.

8. $4x^2 + 4ax - 3a^2 = 0$,
 or $x^2 + ax = \frac{3a^2}{4}$.

$\left(x + \frac{a}{2}\right)^2 = a^2$.

Whence $x = \frac{a}{2}$ or $-\frac{3a}{2}$.

9. $x^2 + 4\sqrt{a}x - 5a = 0$,
 or $x^2 + 4a\sqrt{x} = 5a$.
 $(x + 2\sqrt{a})^2 = 9a$.

Whence $x = \sqrt{a}$ or $-5\sqrt{a}$.

10. $2x^2 + 9x\sqrt{h} = 5h$,
 or $x^2 + \frac{9\sqrt{h}x}{2} = \frac{5h}{2}$.

$\left(x + \frac{9\sqrt{h}}{4}\right)^2 = \frac{121h}{16}$.

Whence $x = \frac{\sqrt{h}}{2}$ or $-5\sqrt{h}$.

11. $a^2x^2 + 2ab = a^2 + b^2$,
 or $x^2 = \frac{a^2 - 2ab + b^2}{a^2}$.
 $x^2 = \frac{(a - b)^2}{a^2}$.

Whence $x = \pm \frac{a - b}{a}$.

12. $5x^2 + ax = x$,
 or $x^2 + \frac{(a - 1)x}{5} = 0$.

$\left(x + \frac{a - 1}{10}\right)^2 = \left(\frac{a - 1}{10}\right)^2$.

Whence $x = 0$ or $\frac{1 - a}{5}$.

13. $x(x - b) = a(a + b)$,
 or $x^2 - bx = a^2 + ab$.
 $\left(x - \frac{b}{2}\right)^2 = \left(a + \frac{b}{2}\right)^2$.

Whence $x = a + b$ or $-a$.

14. $1 - \frac{4}{x} + \frac{4}{x^2} = \frac{b}{4x^2}$,
 or $x^2 - 4x + 4 = \frac{b}{4}$.

$(x - 2)^2 = \frac{b}{4}$.

Whence $x = 2 \pm \frac{\sqrt{b}}{2}$.

15. $x^2 + 2x = ax + 2a$,
 or $x^2 + (2 - a)x = 2a$.
 Whence $x = a$ or -2 .

16. $x^2 - (a+1)x + a = 0$,
or $x^2 - (a+1)x = -a$.

$$\left(x - \frac{a+1}{2}\right)^2 = \left(\frac{a-1}{2}\right)^2.$$

Whence $x = a$ or 1 .

17. $x^2 + bx + cx + bc = 0$,
or $x^2 + (b+c)x = -bc$.

$$\left(x + \frac{b+c}{2}\right)^2 = \left(\frac{b-c}{2}\right)^2.$$

Whence $x = -c$ or $-b$.

18. $x^2 - ax + 4x - 4a = 0$,
or $x^2 - (a-4)x = 4a$.

$$\left(x - \frac{a-4}{2}\right)^2 = \left(\frac{a+4}{2}\right)^2.$$

Whence $x = a$ or -4 .

19. $x^2 + 2a^2b^2 = a^2x + 2b^2x$,
or $x^2 - (a^2 + 2b^2)x = -2a^2b^2$.

$$\left(x - \frac{a^2 + 2b^2}{2}\right)^2 = \left(\frac{a^2 - 2b^2}{2}\right)^2.$$

Whence $x = a^2$ or $2b^2$.

20. $\frac{1}{a} - \frac{1}{5x} = 1 - \frac{5x}{a}$,

or $x^2 + \frac{(1-a)x}{5} = \frac{a}{25}$.

$$\left(x + \frac{1-a}{10}\right)^2 = \left(\frac{1+a}{10}\right)^2.$$

Whence $x = \frac{a}{5}$ or $-\frac{1}{5}$.

21. $x^2 + bx + c = 0$,
or $x^2 + bx = -c$.

$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2 - 4c}{4}.$$

Whence $x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$.

22. $ax^2 + bx + c = 0$,
or $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Whence $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Page 274 (First set)

1. $2x^2 + 5x + 2 = 0$. $\therefore x = \frac{-5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{4} = -\frac{1}{2}$ or -2 .

2. $3x^2 + 5x = 2$, or $3x^2 + 5x - 2 = 0$.

$\therefore x = \frac{-5 \pm \sqrt{25 + 24}}{6} = \frac{1}{3}$ or -2 .

3. $x^2 - 3x - 10 = 0$. $\therefore x = \frac{3 \pm \sqrt{9 + 40}}{2} = 5$ or -2 .

4. $2x + 2 = x^2$, or $x^2 - 2x - 2 = 0$. $\therefore x = \frac{2 \pm \sqrt{4 + 8}}{2} = 1 \pm \sqrt{3}$.

5. $x^2 - x = 1$. $\therefore x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$.

6. $2x^2 - \frac{11x}{2} - \frac{15}{2} = 0$, or $4x^2 - 11x - 15 = 0$.

$\therefore x = \frac{11 \pm \sqrt{121 + 240}}{8} = \frac{15}{4}$ or -1 .

$$7. 2x^2 - 3x = 1. \therefore x = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$$

$$8. 4x + 5 = x^2, \text{ or } x^2 - 4x - 5 = 0. \therefore x = \frac{4 \pm \sqrt{16+20}}{2} = 5 \text{ or } -1.$$

$$9. x^2 + x\sqrt{5} = 10, \text{ or } x^2 + x\sqrt{5} - 10 = 0.$$

$$\therefore x = \frac{-\sqrt{5} \pm \sqrt{5+40}}{2} = \sqrt{5} \text{ or } -2\sqrt{5}.$$

$$10. 12x = 1 - 72x^2, \text{ or } 72x^2 + 12x - 1 = 0.$$

$$\therefore x = \frac{-12 \pm \sqrt{144+288}}{144} = \frac{-1 \pm \sqrt{3}}{12}.$$

$$11. x^2 + 2hx - 3h^2 = 0. \therefore x = \frac{-2h \pm \sqrt{4h^2+12h^2}}{2} = h \text{ or } -3h.$$

$$12. 2m^2 = 9mx + 5x^2, \text{ or } 5x^2 + 9mx - 2m^2 = 0.$$

$$\therefore x = \frac{-9m \pm \sqrt{81m^2+40m^2}}{10} = \frac{m}{5} \text{ or } -2m.$$

$$13. 2x^2 + kx - 3k^2 = 0. \quad x = \frac{-k \pm \sqrt{k^2+24k^2}}{4} = k \text{ or } -\frac{3k}{2}.$$

$$14. x^2 + 2x\sqrt{a} - 3a = 0. \therefore x = \frac{-2\sqrt{a} \pm \sqrt{4a+12a}}{2} = \sqrt{a} \text{ or } -3\sqrt{a}.$$

$$15. mx = -m^2 + 6x^2, \text{ or } 6x^2 - mx - m^2 = 0,$$

$$\therefore x = \frac{m \pm \sqrt{m^2+24m^2}}{12} = \frac{m}{2} \text{ or } -\frac{m}{3}.$$

$$16. x^2 + \frac{kx}{2}\sqrt{2} - k^2 = 0. \therefore x = \frac{\frac{-k\sqrt{2}}{2} \pm \sqrt{\frac{2k^2}{4} + 4k^2}}{2} = \frac{k}{2}\sqrt{2} \text{ or } -k\sqrt{2}.$$

$$17. n^2x^2 - 3knx - 10k^2 = 0. \therefore x = \frac{3kn \pm \sqrt{9k^2n^2+40k^2n^2}}{2n^2} = \frac{5k}{n} \text{ or } -\frac{2k}{n}.$$

$$18. 6m^2x^2 + 19mnx = 7n^2, \text{ or } 6m^2x^2 + 19mnx - 7n^2 = 0.$$

$$\therefore x = \frac{-19mn \pm \sqrt{361m^2n^2+168m^2n^2}}{12m^2} = \frac{n}{3m} \text{ or } -\frac{7n}{2m}.$$

$$19. x^2 + 2x = hx + 2h, \text{ or } x^2 + (2-h)x - 2h = 0. \therefore x = h \text{ or } -2.$$

$$20. x^2 + rx - sx - rs = 0, \text{ or } x^2 + (r-s)x - rs = 0,$$

$$\therefore x = \frac{s-r \pm \sqrt{(r-s)^2+4rs}}{2} = s \text{ or } -r.$$

$$21. 2x^2 + rs = rx + 2sx \text{ or } 2x^2 - (r+2s)x + rs = 0.$$

$$\therefore x = \frac{r+2s \pm \sqrt{(r+2s)^2-8rs}}{4} = s \text{ or } \frac{r}{2}.$$

22. $mnx^2 + nx = 3mx + 3$, or $mnx^2 + (n - 3m)x - 3 = 0$.

$$\therefore x = \frac{3m - n \pm \sqrt{(n - 3m)^2 + 12mn}}{2mn} = \frac{3}{n} \text{ or } -\frac{1}{m}.$$

23. $mhx^2 + 4hx = 3mx + 12$, or $mhx^2 + (4h - 3m)x - 12 = 0$.

$$\therefore x = \frac{3m - 4h \pm \sqrt{(4h - 3m)^2 + 48hm}}{2hm} = \frac{3}{h} \text{ or } \frac{-4}{m}.$$

Page 274 (Second set)

1. Let $x =$ the number.

Then $x^2 + 2x = 15$.

Whence $x = -5$ or 3 .

2. Let $x =$ the less number.

Then $x + 11 =$ the greater number.

Hence

$$x(x + 11) = 42,$$

or $x^2 + 11x = 42$.

Whence $x = -14$ or $+3$,

and $x + 11 = -3$ or $+14$.

3. Let $n =$ the number.

Then $2n^2 - n = 45$.

Whence $n = 5$ or $-\frac{9}{2}$.

4. Let $x =$ the less number.

Then $x + 1 =$ the greater number.

Therefore

$$x(x + 1) = 462,$$

or $x^2 + x = 462$.

Whence $x = 21$ or -22 ,

and $x + 1 = 22$ or -21 .

5. Let n and $n + 2$ represent the odd numbers.

Then $n(n + 2) = 255$,

or $n^2 + 2n = 255$.

Whence $n = 15$ or -17 ,

and $n + 2 = 17$ or -15 .

6. Let n , $n + 2$, and $n + 4$ represent the even numbers.

Then

$$n + n + 2 + n + 4 = \frac{n(n + 2)}{6},$$

or $n^2 - 16n - 36 = 0$.

Whence $n = 18$ or -2 ,

$$n + 2 = 20 \text{ or } 0,$$

and $n + 4 = 22$ or 2 .

7. Let $n =$ the width in rods.

Then $n + 16 =$ the length in rods.

Therefore

$$n(n + 16) = 32 \cdot 160,$$

or $n^2 + 16n = 5120$.

Whence $n = 64$ or -80 ,

and $n + 16 = 80$ or -64 .

The negative roots are rejected.

8. Let x and $\frac{1}{x}$ represent the

number and its reciprocal.

$$\text{Then } x + \frac{1}{x} = \frac{41}{20},$$

or $x^2 - \frac{41x}{20} = -1$.

Whence $x = \frac{5}{4}$ or $\frac{4}{5}$.

Then $\frac{1}{x} = \frac{4}{5}$ or $\frac{5}{4}$.

9. Let $n =$ the altitude in rods.

Then $n + 51 =$ the base in rods.

Therefore

$$\frac{n(n + 51)}{2} = \frac{45}{8} \cdot 160,$$

or $n^2 + 51n = 1800.$

Whence $n = 24$ or $-75,$
and $n + 51 = 75$ or $-24.$

The negative values are rejected.

10. Let $s =$ the side of one in rods.

11. Let $s =$ the side of the square and the breadth of the rectangle in inches.

Then $s + 7 =$ the side of the rectangle.

Therefore $s(s + 7) + 18 = 2s^2,$

or $s^2 - 7s = 18.$

Whence $s = 9$ or $-2.$

The negative value is rejected.

12. Let $l =$ the shorter leg in feet.

Then $l + 31 =$ the longer leg.

Therefore

$$l^2 + (l + 31)^2 = (41)^2,$$

or $l^2 + 31l = 360.$

Whence $l = 9$ or $-40,$

and $l + 31 = 40$ or $-9.$

The negative roots are rejected.

13. Let $3x$ and $4x$ represent the legs.

Then

$$9x^2 + 16x^2 = 400,$$

or $x^2 = 16.$

Whence $x = 4$ or $-4.$

Then $3x = 12,$

and $4x = 16.$

14. Let x and $2x$ represent the other leg and the hypotenuse.

Then $x^2 + 81 = 4x^2,$

or $27 = x^2.$

Whence $x = \pm 3\sqrt{3}.$

But $3\sqrt{3} = 5.19+.$

Then $x = 5.19+,$

and $2x = 10.39+.$

Then $s + 20 =$ the side of the other in rods.

Therefore

$$s^2 + (s + 20)^2 = 62.5 \cdot 160,$$

or $s^2 + 20s = 4800.$

Whence $s = 60$ or $-80,$

and $s + 20 = 80$ or $-60.$

The negative values are rejected.

15. Let $l =$ the other leg.

Then

$$l^2 + (5\sqrt{3})^2 = 100,$$

or $l^2 = 25.$

Whence $l = \pm 5.$

Since one leg is $\frac{1}{2}$ the hypotenuse, one angle is 30° , one 60° , and the third 90° .

16. Let $x =$ the side of the square in feet.

Then $x^2 = 4x.$

Whence $x = 4$ or $0.$

17. Let $x =$ the side of the square in inches.

Then $\left(\frac{x}{12}\right)^2 = 4x,$

or $x^2 - 576x = 0.$

Whence $x = 576$ or $0,$

and $\frac{576}{12} = 48$ feet.

18. Let $s =$ the side of the square in inches.

Then $s^2 = 4\left(\frac{s}{12}\right),$

or $3s^2 = s.$

Whence $s = \frac{1}{3}$ or $0.$

19. Let x , $x + 2$, and $x + 4$ represent the dimensions and the longest straight line respectively in inches.

Then

$$x^2 + (x + 2)^2 = (x + 4)^2,$$

or $x^2 - 4x = 12.$

Whence $x = 6$ or $-2.$

Whence $x = 6,$

$$x + 2 = 8,$$

and $x + 4 = 10.$

20. Let x , $2x$, and $3x$ represent the dimensions in inches.

Then $2 \cdot 2x^2 + 2 \cdot 3x^2 + 2 \cdot 6x^2$
 $= 792,$

or $22x^2 = 792.$

Whence $x = \pm 6.$

Then $x = 6,$

$$2x = 12,$$

and $3x = 18.$

21. Let x and $x + 1$ represent the edges in yards.

Then

$$(x + 1)^3 - x^3 = 61,$$

or $x^2 + x - 20 = 0.$

Whence $x = -5$ or $+4.$

Therefore $x = 4,$

and $x + 1 = 5.$

22. Let r and $r + 5$ represent the rates in miles per hour.

Then

$$\frac{280}{r} - \frac{280}{r + 5} = 1,$$

or $r^2 + 5r = 1400.$

Whence $r = 35$ or $-40.$

Therefore $r = 35,$

and $r + 5 = 40.$

23. Let r and $r + 4$ represent the rates in miles per hour.

Then

$$\frac{80}{r} + \frac{80}{r + 4} = 9,$$

or $9r^2 - 124r = 320.$

Whence $r = 16$ or $-2\frac{2}{9}.$

Therefore $r = 16,$

and $r + 4 = 20.$

24. Let x and $x + 2$ represent the width and the length in inches respectively.

Then

$$2(x - 2)(x) = x(x + 2),$$

or $x^2 - 6x = 0.$

Whence $x = 6$ or $0.$

Then $x = 6,$

and $x + 2 = 8.$

25. Let x and $x - 40$ represent the number of acres bought and sold respectively.

Then $\frac{16,000}{x} + 20 = \frac{16,000}{x - 40},$

or $x^2 - 40x - 32,000 = 0.$

Whence $x = 200$ or $-160,$

and $x - 40 = 160.$

26. Let $x =$ the original cost
of an orange
in cents.

Then $\frac{50}{x} - 5 = \frac{50}{x + \frac{1}{2}},$

or $x^2 + \frac{5x}{6} - \frac{25}{3} = 0.$

Whence $x = 2\frac{1}{2}$ or $-3\frac{1}{3}.$

27. Let x and $x + 2$ represent the time in hours required by each respectively.

Then

$$\frac{1}{x} + \frac{1}{x + 2} = \frac{4}{3},$$

or $2x^2 + x = 3.$

Whence $x = 1$ or $-\frac{3}{2}$.
 Therefore $x = 1$,
 and $x + 2 = 3$.

28. Let r and $r + 7$ represent the rates in miles per hour respectively.

Then

$$\frac{200}{r} - \frac{200}{r + 7} = \frac{7}{4},$$

or $r^2 + 7r = 800$.

Whence $r = 25$ or -32 .

Therefore $r = 25$,

and $r + 7 = 32$.

29. Let $x =$ the number of miles B goes,
 and $x + 17 =$ the number of miles A goes.

Then

$$x^2 + (x + 17)^2 = (53)^2,$$

or $x^2 + 17x = 1260$.

Whence $x = 28$ or -45 .

Therefore $\frac{x}{5} = \frac{28}{5}$,

B's rate in miles per hour ;

and $\frac{x + 17}{5} = \frac{45}{5} = 9$,

A's rate in miles per hour.

30. Let $x =$ the distance between them at the end of 5 hours.

Then $x - 9 =$ A's distance at the end of 5 hours,
 and $x - 8 =$ B's distance at the end of 5 hours.

Then $(x - 8)^2 + (x - 9)^2 = x^2$,
 or $x^2 - 34x + 145 = 0$.

Whence $x = 5$ or 29 .

The value 5 for x is rejected.

Hence A's rate $= \frac{29 - 9}{5} = 4$ miles per hour,

and B's rate $= \frac{29 - 8}{5} = 4\frac{1}{5}$ miles per hour.

31. Let $s =$ the height in feet.

Then $\frac{32 \cdot (15)^2}{2} = s$,

or $3600 = s$.

32. Let $x =$ the time in seconds for falling,
 and $6\frac{1}{2} - x =$ the time in seconds for the sound to return.

Then $16x^2 =$ the distance fallen,
 and $(6\frac{1}{2} - x) 1152 =$ the distance the sound returns.

Hence $16x^2 = (\frac{13}{2} - x) 1152$,

or $x^2 + 72x - 468 = 0$.

Whence $x = 6$ or -78 .

Then height $= \frac{32 \cdot 6^2}{2} = 576$ feet.

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1. $55 \cdot 3 \div 11 = 15$, the number of revolutions.

2. $132 \cdot 3 \div (4 \cdot \frac{2}{7}) = 31\frac{1}{2}$, the number of revolutions.

3. Let x = the circumference of the fore wheel
in feet.

Then $x + 2$ = the circumference of the rear wheel
in feet.

Therefore $\frac{420}{x} - \frac{420}{x+2} = 5$,

or $x^2 + 2x = 168$.

Whence $x = 12$ or -14 ,

and $x + 2 = 14$.

4. Let x = the circumference of the fore wheel
in feet.

Then $x + 2$ = the circumference of the rear wheel
in feet.

Therefore $\frac{300}{x} - \frac{300}{x+2} = 5$,

or $x^2 + 2x = 120$.

Whence $x = 10$ or -12 ,

and $x + 2 = 12$.

Again $10 \div \frac{2 \cdot 22}{7} = 1\frac{10}{11}$, radius of the fore wheel.

$12 \div \frac{2 \cdot 22}{7} = 1\frac{3}{2}$, radius of the rear wheel.

5. Let x and $x - 2$ represent the circumference of the rear and the fore wheels respectively in feet.

Then $\frac{60}{x} + \frac{60}{x-2} = 11$,

or $11x^2 - 142x + 120 = 0$.

Whence $x = 12$ or $\frac{10}{11}$,

and $x - 2 = 10$ or $-\frac{12}{11}$.

Using the first values, $12 \div \frac{2}{7} = 3\frac{9}{11}$, diameter of rear wheel in feet.

and $10 \div \frac{2}{7} = 3\frac{2}{11}$, diameter of fore wheel in feet.

6. Let r = the radius of the larger circle in inches.

Then $r - 7$ = the radius of the smaller circle in inches.

Therefore $\frac{2}{7}r^2 - \frac{2}{7}(r-7)^2 = 770$,

or $44r = 924$.

Whence $r = 21$,

and $r - 7 = 14$.

7. $AB(18 - AB) = 4 \cdot 20,$
 or $\overline{AB}^2 - 18 \overline{AB} + 80 = 0.$
 Whence $AB = 8$ or $10.$

8. $AB(5 - AB) = 4 \cdot \frac{3}{2},$
 or $\overline{AB}^2 - 5 \overline{AB} + 6 = 0.$
 Whence $AB = 2$ or 3 feet,
 and $BC = 3$ or 2 feet.

9. $81 = BC(30 - BC),$
 or $\overline{BC}^2 - 30 \overline{BC} + 81 = 0.$
 Whence $BC = 27$ or $3,$
 and $BD = 3$ or $27.$

10. $400 = 2 BD \cdot BD,$
 or $\overline{BD}^2 = 200.$
 Whence $BD = 10 \sqrt{2}.$

11. $\frac{12}{AP} = \frac{AP}{12 - AP},$
 or $\overline{AP}^2 - 12 \overline{AP} - 144 = 0.$
 $AP = -6 \pm 6 \sqrt{5}.$

The negative answer has a meaning in the dividing of a line externally by a point.

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1. $3 - 5 + 6 - 8 = -4.$

2. $6 \div 2 + 1 - 4 = 3 + 1 - 4 = 0.$

3. $24 \div 8 \cdot 4 - 4 + 6 = 3 \cdot 4 - 4 + 6 = 12 - 4 + 6 = 14.$

4. $(7 - 6)(18 - 2 \cdot 4) \div (20 \div 4) = 1 \cdot 10 \div 5 = 2.$

5. $42 - 2(18 - 2 \cdot 3) \div 4 + 3 \cdot 5 = 42 - 24 \div 4 + 15 = 42 - 6 + 15 = 51.$

6. $16 + 4 \div 8 - 10 + 51 \div 16 - 4 - 6 \cdot 3 \cdot 0 \cdot 2 + 18 \cdot 8 \div 48 - 2 \cdot 18 \div 12$
 $= 16 + \frac{1}{2} - 10 + 3\frac{3}{8} - 4 + 3 - 3 = 5\frac{1}{8}.$

7. $(16 \div 32 \times 48 \div 8 - 4 - 8 + 3)[12 \div 4 \div 3 - 1] + (42 \div 6 \cdot 7 - 42 - 6) \cdot 6$
 $= -6[0] + 1 \cdot 6 = 6.$

8. No ; No ; Yes.

9. In a^4 , 4 is an exponent ; in $4a$, 4 is a coefficient.

See pages 7-8. Also see pages 352-353.

If a number is the product of two factors, either of these factors is called the coefficient of the other in that product.

10. The power of a number is the result obtained when the operation indicated by the exponent has been performed.

An exponent indicates an operation of which the power is the result.

11. $x^2 - 5x + 6 = 5^2 - 5 \cdot 5 + 6 = 6,$ when $x = 5.$

12. $x^3 - 3x^2 + 3x - 1 = 3^3 - 3 \cdot 3^2 + 3 \cdot 3 - 1 = 8$, when $x = 3$.

13. $x^3 - 3x^2y + 3xy^2 - y^3 = 4^3 - 3 \cdot 4^2 \cdot 2 + 3 \cdot 4 \cdot 2^2 - 2^3 = 8$, when $x = 4$ and $y = 2$.

14. $\frac{x^4 + x^2y^2 + y^4}{x^2 - xy + y^2} - \frac{x^3 + y^3}{x + y} = \frac{3^4 + 3^2 \cdot 2^2 + 2^4}{3^2 - 3 \cdot 2 + 2^2} - \frac{3^3 + 2^3}{3 + 2} = 19 - 7 = 12$,

when $x = 3$ and $y = 2$.

15. The absolute value of a number is its value without regard to sign.

The absolute values of -4 , $+7$, $-9a$ are 4 , 7 , and $9a$ respectively.

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3. $16 - 3 + 2 - 8 - 7 + 4 = 4$.

4. $4a - 6a - 10a + 2a + 18a = 8a$.

5 and 7.
$$\begin{array}{rclcl} 7x - 4y - z & = & 7 - 8 - 3 & = & -4 \\ 3x - 8y + z & = & 3 - 16 + 3 & = & -10 \\ \hline -17x + 18y - 14z & = & -17 + 36 - 42 & = & -23 \\ -7x + 6y - 14z & = & -7 + 12 - 42 & = & -37 \end{array}$$

6.
$$\begin{array}{rcl} 4a^2 - 3a^2c - 4ac^2 \\ - 8a^2 + 3a^2c - 8ac^2 \\ \hline 3a^2 - 6a^2c \\ - a^2 - 6a^2c - 12ac^2 \end{array}$$

The sum of the three numerical values in Exercise 7 equals the numerical value of the result.

8. Substitute for each letter in the several expressions and in their sum a convenient numerical value. The sum of the numerical values of the several expressions should equal the numerical value of their sum.

9. $ay + by + cy = (a + b + c)y$.

10. $3ax - 4bx + 6x = (3a - 4b + 6)x$.

11. $4x - abx - x = (3 - ab)x$.

12. $3(a + b) - c(a + b) = (3 - c)(a + b)$.

13. $4b(3x - 2) - 8c(3x - 2) = (4b - 8c)(3x - 2)$.

14. $4m(5a - 3c) - 6n(-3c + 5a) = (4m - 6n)(5a - 3c)$.

15. $6a - 4a = 2a$.

16. $5a^3 - 8a^3 = -3a^3$.

17. $(8x + 6) - (4x + 3) = 4x + 3$.

18. $(5x^2 + 20) - (7x^2 - 10) = -2x^2 + 30$.

19.
$$\begin{aligned} (4x - y^2 + 8 - 5ax + 9ac) - (x - 3y^2 + z - 4ac + 7ax) \\ = 3x + 2y^2 - z + 8 - 12ax + 13ac \end{aligned}$$

20.
$$\begin{aligned} (4a^3 + m - 8x - 10ac + 4a^2m) - (a^3 - c + 3x - a^2m - 8ac) \\ = 3a^3 + m - 11x - 2ac + 5a^2m + c \end{aligned}$$

21. $(3x^2 - 5x + 2) - (x^2 - 5x + 6) = 2x^2 - 4$.

$$22. (8c^2 + 7cx - 10x^2 + 8) - (4x^2 - 3cx + c^2) \\ = -14x^2 + 10cx + 7c^2 + 8.$$

$$23. (4a^2 - 2ab + b^2) - (7a^2 - 10ab + 6b^2) = -3a^2 + 8ab - 5b^2.$$

$$24. (c^2 - 10cx + 8x^2) - (9x^2 - 10cx + 4 + c^2) = -x^2 - 4.$$

25. The sum of the remainder and the subtrahend should equal the minuend.

$$\begin{array}{llll} 26. & ax - ac - 3c^2 & (1) & 4c^2 - 8ax + a^2 & (4) \\ & \underline{-3ac + 4c^2} & (2) & \underline{-5c^2 + 3ax} & + 4ac & (5) \\ (1) + (2), & ax - 4ac + c^2 & (3) & \underline{-c^2 - 5ax + a^2 + 4ac} & (6) \\ & \underline{-5ax + 4ac - c^2 + a^2} & (6) & & \\ (3) - (6), & 6ax - 8ac + 2c^2 - a^2 & & & \end{array}$$

$$27. \quad 4x - 3 - (a - 2x) + (3x - a) \\ = 4x - 3 - a + 2x + 3x - a \\ = 9x - 3 - 2a.$$

$$28. \quad 6x + (3c - 8x + 2) - (c - x - 2) \\ = 6x + 3c - 8x + 2 - c + x + 2 \\ = -x + 2c + 4.$$

$$29. \quad 6x - [- (a - c) + (3c - 4a)] \\ = 6x - [-a + c + 3c - 4a] \\ = 6x + a - c - 3c + 4a \\ = 6x + 5a - 4c.$$

$$30. \quad 7c - [(3c - 4) - 6 - (4x - 3a - c)] \\ = 7c - [3c - 4 - 6 - 4x + 3a + c] \\ = 7c - 3c + 10 + 4x - 3a - c \\ = 3c + 10 + 4x - 3a.$$

$$31. \quad 6x - 4(3 - 5x) - 4[2(x - 4) + 3(2x - 1) - (x - 7)] \\ = 6x - 12 + 20x - 4[2x - 8 + 6x - 3 - x + 7] \\ = 6x - 12 + 20x - 28x + 16 \\ = -2x + 4.$$

$$32. \quad 3x - 2[1 - 3(2x - 3 - a) - 5\{a - (3x - 2a) - 4\}] \\ = 3x - 2[1 - 6x + 9 + 3a - 5\{a - 3x + 2a - 4\}] \\ = 3x - 2[10 - 6x + 3a - 15a + 15x + 20] \\ = 3x - 60 - 18x + 24a \\ = -15x - 60 + 24a.$$

33. See pages 55-56.

$$35. + (x^2) - (49a^2 - 14ab + b^2).$$

$$34. + (x^2 + 2xy + y^2) - (a^2). \quad 36. + (y^2 + 6xy + 9x^2) - (m^2 + 10m + 25).$$

$$37. + (x^4 + 10x^2y^3 + 25y^6) - (c^8 - 12c^4d + 36d^2).$$

38. See page 58.

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$$2. (4x^2 - 3x)(2x) = 8x^3 - 6x^2.$$

$$3. (2x + 3)(5x - 6) = 10x^2 + 3x - 18.$$

$$4. \text{Substituting in Exercise 2, } (4 \cdot 2^2 - 3 \cdot 2)(2 \cdot 2) = 8 \cdot 2^3 - 6 \cdot 2^2 = 40.$$

See page 63.

$$5. (3x^2 - 1)^2 = 9x^4 - 6x^2 + 1.$$

$$6. (7x^{2a} - 8x^a + 3)^2 = 49x^{4a} + 106x^{2a} + 9 - 112x^{3a} - 48x^a.$$

$$7. (x^{\frac{1}{2}} + x^{\frac{1}{3}})^2 = x + 2x^{\frac{5}{6}} + x^{\frac{2}{3}}.$$

$$8. (e^x + 2e^{-x})^2 = e^{2x} + 4 + 4e^{-2x}.$$

$$9. (e^x - e^{-x})^3 = (e^x - e^{-x})^2(e^x - e^{-x}) = e^{3x} - 3e^x + 3e^{-x} - e^{-3x}.$$

$$10. (e^{2x} - 3e^{-x})^4 = [(e^{2x} - 3e^{-x})^2]^2 = e^{8x} - 12e^{5x} + 54e^{2x} - 108e^{-x} + 81e^{-4x}.$$

$$11. (x^{\frac{1}{2}} + x^{\frac{1}{4}} + 1)(x^{\frac{1}{2}} - x^{\frac{1}{4}} + 1) = x + x^{\frac{1}{2}} + 1.$$

$$12. \quad \begin{array}{r} 4x^{3e} - 6x^e + 3 \\ 7x^{3e} - x^{2e} + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 28x^{6e} \qquad - 42x^{4e} + 21x^{3e} \\ - 4x^{5e} \qquad + 6x^{3e} - 3x^{2e} \\ \hline \end{array}$$

$$\begin{array}{r} + 16x^{3e} \qquad - 24x^e + 12 \\ \hline \end{array}$$

$$28x^{6e} - 4x^{5e} - 42x^{4e} + 43x^{3e} - 3x^{2e} - 24x^e + 12$$

$$13. (x^2 - 2xy^2 + y^4)(x^2 + 2xy^2 + y^4) = x^4 - 2x^2y^4 + y^8.$$

$$14. (x^{-1} - 3x - 2x^{-2})^2 = x^{-2} + 9x^2 + 4x^{-4} - 6 - 4x^{-3} + 12x^{-1}.$$

$$15. (x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} - 3x^{\frac{3}{2}})^2 = x^{-1} - 2x + 9x^3 + 4 - 12x^2.$$

$$16. \quad \begin{array}{r} \frac{2a^2}{3} - \frac{a}{5} + \frac{2}{7} \\ \frac{2a^3}{3} + \frac{a^2}{5} - \frac{2a}{7} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{4a^5}{9} - \frac{2a^4}{15} + \frac{4a^3}{21} \\ + \frac{2a^4}{15} - \frac{a^3}{25} + \frac{2a^2}{35} \\ \hline \end{array}$$

$$\begin{array}{r} - \frac{4a^3}{21} + \frac{2a^2}{35} - \frac{4a}{49} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{4a^5}{9} \qquad - \frac{a^3}{25} + \frac{4a^2}{35} - \frac{4a}{49} \\ \hline \end{array}$$

$$17. (5x^{2a} - 3x^{-2a} - 6x^{-a} + 3x^a)^2 = 25x^{4a} + 9x^{-4a} + 36x^{-2a} + 9x^{2a} - 60x^a + 30x^{3a} + 36x^{-3a} - 18x^{-a} - 66.$$

$$18. x^2 - x - 90 = 81 + 9 - 90 = 0, \text{ if } x = -9.$$

19. $2^3 - 3 \cdot 2^2(-3) + 3 \cdot 2(-3)^2 - (-3)^3 = 125$, if $x = 2$ and $y = -3$.

20. $(-4)^3 + 3(-4)^2(-2) + 3(-4)(-2)^2 + (-2)^3 = -216$, if $x = -4$ and $y = -2$.

21. See page 59.

22. See page 61.

24. $(8x^4 - 6x^2 - 4x) \div (-2x) = -4x^3 + 3x + 2$.

25.
$$\begin{array}{r} x^2 - 7x + 12 \\ x^2 - 3x \\ \hline -4x + 12 \\ -4x + 12 \\ \hline \end{array} \quad \begin{array}{r} x - 3 \\ x - 4 \\ \hline \end{array}$$

26. $(e^1 - e^{-1})^2 = (2 - \frac{1}{2})^2 = (\frac{3}{2})^2 = \frac{9}{4}$, if $e = 2$.

$(e^1 - e^{-1})^2 = (-3 + \frac{1}{3})^2 = (-\frac{8}{3})^2 = \frac{64}{9}$, if $e = -3$.

27. $e^{2x} - 2e^0 + e^{-2x} = 2^4 - 2 \cdot 2^0 + 2^{-4} = 16 - 2 + \frac{1}{16} = 14\frac{1}{16}$, if $e = 2$ and $x = 2$.

28.
$$\begin{array}{r} x^3 - 64 \\ x^3 - 4x^2 \\ \hline 4x^2 \end{array} \quad \begin{array}{r} x - 4 \\ x^2 + 4x + 16 \\ \hline \end{array} \quad (1)$$

$$\begin{array}{r} 4x^2 \\ 4x^2 - 16x \\ \hline 16x - 64 \\ 16x - 64 \\ \hline \end{array} \quad \begin{array}{r} x^2 - 4x + 16 \\ x^4 + 4x^3 + 16x^2 \\ -4x^3 - 16x^2 - 64x \\ +16x^2 + 64x + 256 \\ \hline \end{array} \quad (2)$$

$(1) \cdot (2) = x^4 + 16x^2 + 256$

29.
$$\begin{array}{r} x^4 - 8x^2 + 33x - 30 \\ x^4 + 3x^3 - 5x^2 \\ \hline -3x^3 - 3x^2 \\ -3x^3 - 9x^2 + 15x \\ \hline 6x^2 + 18x - 30 \\ 6x^2 + 18x - 30 \\ \hline \end{array} \quad \begin{array}{r} x^2 + 3x - 5 \\ x^2 - 3x + 6 \\ \hline \end{array}$$

30. See pages 79-80.

31.
$$\begin{array}{r} 8x^3 - x^2 - 5 \\ 8x^3 - 12x^2 \\ \hline 11x^2 \\ 11x^2 - \frac{33}{2}x \\ \hline \frac{33}{2}x - 5 \\ \frac{33}{2}x - \frac{99}{4} \\ \hline \frac{79}{4}, \text{ remainder} \end{array} \quad \begin{array}{r} 2x - 3 \\ 4x^2 + \frac{11}{2}x + \frac{33}{4} \\ \hline \end{array}$$

$$\begin{array}{r}
 32. \quad 4x^4 \qquad \qquad - x^2 - 3 \bigg| 2x^2 - x - 1 \\
 \underline{4x^4 - 2x^3 - 2x^2} \qquad \qquad \underline{2x^2 + x + 1} \\
 2x^3 + x^2 \\
 \underline{2x^3 - x^2 - x} \\
 2x^2 + x - 3 \\
 \underline{2x^2 - x - 1} \\
 2x - 2, \text{ remainder}
 \end{array}$$

$$\begin{array}{r}
 33. \quad x^3 \qquad \qquad - 30xy + 125 + 8y^3 \bigg| x + 2y + 5 \\
 \underline{x^3 + 2x^2y + 5x^2} \qquad \qquad \underline{x^2 - 5x - 2xy + 25 - 10y + 4y^2} \\
 - 5x^2 - 2x^2y - 30xy \\
 - 5x^2 \qquad \qquad - 10xy - 25x \\
 \underline{\qquad \qquad - 2x^2y + 25x - 20xy} \\
 - 2x^2y \qquad \qquad - 10xy - 4xy^2 \\
 \underline{\qquad \qquad \qquad 25x - 10xy + 4xy^2 \qquad \qquad + 125} \\
 25x \qquad \qquad \qquad + 50y + 125 \\
 \underline{\qquad \qquad \qquad - 10xy + 4xy^2 - 50y} \\
 - 10xy \qquad \qquad - 50y - 20y^2 \\
 \underline{\qquad \qquad \qquad \qquad \qquad 4xy^2 \qquad \qquad + 20y^2 + 8y^3} \\
 4xy^2 \qquad \qquad + 20y^2 + 8y^3
 \end{array}$$

$$\begin{array}{r}
 34. \quad x^3 \qquad \qquad - 3xyz + y^3 + z^3 \bigg| x + y + z \\
 \underline{x^3 + x^2y + x^2z} \qquad \qquad \underline{x^2 - xy - xz + y^2 - yz + z^2} \\
 - x^2y - x^2z \\
 - x^2y \qquad - xy^2 - xyz \\
 \underline{\qquad \qquad - x^2z + xy^2 - 2xyz} \\
 - x^2z \qquad - xyz - xz^2 \\
 \underline{\qquad \qquad \qquad xy^2 - xyz + xz^2} \\
 xy^2 \qquad \qquad + y^3 + y^2z \\
 \underline{\qquad \qquad - xyz + xz^2 - y^2z} \\
 - xyz \qquad - y^2z - yz^2 \\
 \underline{\qquad \qquad \qquad xz^2 \qquad + yz^2 + z^3} \\
 xz^2 \qquad + yz^2 + z^3
 \end{array}$$

$$\begin{array}{r}
 35. \quad x^6 \qquad \qquad - 4x^{-6} + 3x^{-10} \bigg| x^2 + 2x^{-2} + 3x^{-6} \\
 \underline{x^6 + 2x^2 + 3x^{-2}} \qquad \qquad \underline{x^4 - 2 + x^{-4}} \\
 - 2x^2 - 3x^{-2} - 4x^{-6} \\
 - 2x^2 - 4x^{-2} - 6x^{-6} \\
 \underline{\qquad \qquad \qquad x^{-2} + 2x^{-6} + 3x^{-10}} \\
 x^{-2} + 2x^{-6} + 3x^{-10}
 \end{array}$$

$$\begin{aligned}
 36. \quad & (x^{\frac{3}{5}} - y^{\frac{3}{5}}) \div (x^{\frac{1}{10}} - y^{\frac{1}{10}}) \\
 &= x^{\frac{5}{10}} + x^{\frac{4}{10}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{2}{10}} + x^{\frac{2}{10}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{4}{10}} + y^{\frac{5}{10}} \\
 &= x^{\frac{1}{2}} + x^{\frac{2}{5}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{1}{5}} + x^{\frac{1}{5}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{2}{5}} + y^{\frac{1}{2}}.
 \end{aligned}$$

$$\begin{array}{r}
 37. \quad 9m \qquad \qquad -13 \qquad \qquad + 4m^{-1} \left| \begin{array}{l} 3m^{\frac{1}{2}} - 5 + 2m^{-\frac{1}{2}} \\ \hline 3m^{\frac{1}{2}} + 5 + 2m^{-\frac{1}{2}} \end{array} \right. \\
 \hline
 9m - 15m^{\frac{1}{2}} + 6 \\
 \hline
 15m^{\frac{1}{2}} - 19 \\
 15m^{\frac{1}{2}} - 25 + 10m^{-\frac{1}{2}} \\
 \hline
 6 - 10m^{-\frac{1}{2}} + 4m^{-1} \\
 \hline
 6 - 10m^{-\frac{1}{2}} + 4m^{-1}
 \end{array}$$

$$\begin{array}{r}
 38. \quad x^{2a} \qquad \qquad -29 \qquad \qquad + 4x^{-2a} \left| \begin{array}{l} x^a - 5 - 2x^{-a} \\ \hline x^a + 5 - 2x^{-a} \end{array} \right. \\
 \hline
 x^{2a} - 5x^a - 2 \\
 \hline
 5x^a - 27 \\
 5x^a - 25 - 10x^{-a} \\
 \hline
 -2 + 10x^{-a} + 4x^{-2a} \\
 \hline
 -2 + 10x^{-a} + 4x^{-2a}
 \end{array}$$

$$\begin{array}{r}
 39. \quad 9x^{2a} \qquad \qquad -19x^{-a} \qquad \qquad + 25x^{-4a} \left| \begin{array}{l} 3x^a - 7x^{-\frac{a}{2}} + 5x^{-2a} \\ \hline 3x^a + 7x^{-\frac{a}{2}} + 5x^{-2a} \end{array} \right. \\
 \hline
 9x^{2a} - 21x^{\frac{a}{2}} + 15x^{-a} \\
 \hline
 21x^{\frac{a}{2}} - 34x^{-a} \\
 21x^{\frac{a}{2}} - 49x^{-a} + 35x^{-\frac{5a}{2}} \\
 \hline
 15x^{-a} - 35x^{-\frac{5a}{2}} + 25x^{-4a} \\
 \hline
 15x^{-a} - 35x^{-\frac{5a}{2}} + 25x^{-4a}
 \end{array}$$

$$\begin{array}{r}
 40. \quad 6a^3 + \frac{35}{3}a^2x + \frac{35ax^2}{2} + 6x^3 \left| \begin{array}{l} \frac{3a}{2} + \frac{2x}{3} \\ \hline 4a^2 + 6ax + 9x^2 \end{array} \right. \\
 \hline
 6a^3 + \frac{8a^2x}{3} \\
 \hline
 9a^2x \\
 9a^2x + 4ax^2 \\
 \hline
 \frac{27}{2}ax^2 + 6x^3 \\
 \hline
 \frac{27}{2}ax^2 + 6x^3
 \end{array}$$

$$\begin{array}{r}
 41. \quad \frac{9a^5}{5} - \frac{43a^4}{8} + \frac{243a^3}{20} - \frac{443a^2}{30} + \frac{59a}{4} - 12 \quad \left| \begin{array}{l} \frac{3a^3}{4} - \frac{5a^2}{6} + a - \frac{3}{2} \\ \hline \frac{12a^2}{5} - \frac{9a}{2} + 8 \end{array} \right. \\
 \hline
 \frac{9a^5}{5} - 2a^4 + \frac{12a^3}{5} - \frac{18a^2}{5} \\
 \hline
 - \frac{27a^4}{8} + \frac{195a^3}{20} - \frac{67a^2}{6} \\
 \hline
 - \frac{27a^4}{8} + \frac{15a^3}{4} - \frac{9a^2}{2} + \frac{27a}{4} \\
 \hline
 6a^3 - \frac{20a^2}{3} + 8a - 12 \\
 \hline
 6a^3 - \frac{20a^2}{3} + 8a - 12 \\
 \hline
 \end{array}$$

Page 287 (First set)

$$\begin{array}{r}
 1. \quad 1 - 8 + 16 \\
 \quad 2 - 3 \\
 \hline
 2 - 16 + 32 \\
 \quad - 3 + 24 - 48 \\
 \hline
 2 - 19 + 56 - 48 \\
 \hline
 \therefore \text{the product is } 2x^3 - 19x^2 + \\
 56x - 48.
 \end{array}$$

$$\begin{array}{r}
 2. \quad 1 - 4 + 4 \\
 \quad 1 + 4 + 4 \\
 \hline
 1 - 4 + 4 \\
 \quad + 4 - 16 + 16 \\
 \quad \quad + 4 - 16 + 16 \\
 \hline
 1 + 0 - 8 + 0 + 16 \\
 \hline
 \therefore \text{the product is } x^4 - 8x^2 + 16.
 \end{array}$$

$$\begin{array}{r}
 3. \quad 1 - 1 + 1 \\
 \quad 1 + 1 + 1 \\
 \hline
 1 - 1 + 1 \\
 \quad + 1 - 1 + 1 \\
 \quad \quad + 1 - 1 + 1 \\
 \hline
 1 + 0 + 1 + 0 + 1 \\
 \hline
 \therefore \text{the product is } a^4 + a^2b^2 + b^4.
 \end{array}$$

$$\begin{array}{r}
 4. \quad 2 + 5 + 2 \quad \left| \begin{array}{l} 2 + 1 \\ \hline 1 + 2 \end{array} \right. \\
 \quad 2 + 1 \\
 \hline
 4 + 2 \\
 \hline
 4 + 2 \quad x + 2, \text{ the quotient.}
 \end{array}$$

$$\begin{array}{r}
 5. \quad 1 + 0 + 4 - 16 \quad \left| \begin{array}{l} 1 - 2 \\ \hline 1 + 2 + 8 \end{array} \right. \\
 \quad 1 - 2 \\
 \hline
 2 + 4 \\
 \quad 2 - 4 \\
 \hline
 8 - 16 \\
 \quad 8 - 16 \\
 \hline
 \end{array}$$

$x^2 + 2x + 8$, the quotient.

$$\begin{array}{r}
 6. \quad - 2 + 3 - 6 \\
 \quad 8 - 5 - 6 \\
 \hline
 - 16 + 24 - 48 \\
 \quad + 10 - 15 + 30 \\
 \quad \quad + 12 - 18 + 36 \\
 \hline
 - 16 + 34 - 51 + 12 + 36
 \end{array}$$

$\therefore$ the product is $-16x^4 + 34x^3y - 51x^2y^2 + 12xy^3 + 36y^4$.

$$\begin{array}{r}
 7. \quad 9 - 6 + 13 - 4 + 4 \quad \left| \begin{array}{l} 3 - 1 + 2 \\ \hline 3 - 1 + 2 \end{array} \right. \\
 \quad 9 - 3 + 6 \\
 \hline
 - 3 + 7 \\
 \quad - 3 + 1 - 2 \\
 \hline
 6 - 2 + 4 \\
 \quad 6 - 2 + 4 \\
 \hline
 \end{array}$$

$3x^2 - x + 2$, the quotient.

$$\begin{array}{r}
 8. \quad 1+0+0+0+4 \overline{) 1-2+2} \\
 \underline{1-2+2} \\
 2-2 \\
 \underline{2-4+4} \\
 2-4+4 \\
 \underline{2-4+4}
 \end{array}$$

$$\begin{array}{r}
 10. \quad 4-2+2+0-5-3 \overline{) 2+0-2-1} \\
 \underline{4+0-4-2} \\
 -2+6+2 \\
 \underline{-2+0+2+1} \\
 6+0-6-3 \\
 \underline{6+0-6-3}
 \end{array}$$

$x^2 + 2xy + 2y^2$, the quotient.

$2a - 1 + 3a^{-1}$, the quotient.

$$\begin{array}{r}
 9. \quad 81+0-171+0+25 \overline{) 9+9-5} \\
 \underline{81+81-45} \\
 -81-126 \\
 \underline{-81-81+45} \\
 -45-45+25 \\
 \underline{-45-45+25}
 \end{array}$$

$$\begin{array}{r}
 11. \quad 8-12+6-1 \overline{) 2-1} \\
 \underline{8-4} \\
 -8+6 \\
 \underline{-8+4} \\
 2-1 \\
 \underline{2-1}
 \end{array}$$

$9a^2 - 9ab - 5b^2$, the quotient.

$4x^{\frac{2}{3}} - 4x^{\frac{1}{3}}y^{-1} + y^{-2}$, the quotient.

12. (a) The expressions in Exercises 10 and 11 are not integral.

(b) Neither expression in Exercise 11 is rational.

Page 287 (Second set)

1. See pages 93-100.
2. $(x+3)^2 = x^2 + 6x + 9$.
3. $(x-5)^2 = x^2 - 10x + 25$.
4. $(2x+4)^2 = 4x^2 + 16x + 16$.
5. $(4x-3)^2 = 16x^2 - 24x + 9$.
6. $(x^2-x)^2 = x^4 - 2x^3 + x^2$.
7. $(x-3c^2)^2 = x^2 - 6c^2x + 9c^4$.
8. $(3x^2-4xc)^2 = 9x^4 - 24x^3c + 16x^2c^2$.
9. $(x^2-x^{-2})^2 = x^4 - 2 + x^{-4}$.
10. $(x^4-3x^{-4})^2 = x^8 - 6 + 9x^{-8}$.
11. $(16x^2+8x^2c+x^2c^2) \div (4x+xc) = 4x+xc$. Formula I.
12. $(2a^x-a^{-x})(2a^x-a^{-x})(2a^x-a^{-x}) \div (2a^x-a^{-x}) = 4a^{2x}-4+a^{-2x}$.
13. $(x-c)(x+c) = x^2 - c^2$.
14. $(x+6)(x-6) = x^2 - 36$.
15. $(a-3c)(a+3c) = a^2 - 9c^2$.
16. $(m-x)(x+m) = m^2 - x^2$.
17. $(4x-3c)(3c+4x) = 16x^2 - 9c^2$.
18. $(x^3-cx)(x^3+cx) = x^6 - c^2x^2$.
19. $(4c^3-a^5)(a^5+4c^3) = 16c^6 - a^{10}$.
20. $(x+3)(x+4) = x^2 + 7x + 12$.
21. $(b-3)(b-4) = b^2 - 7b + 12$.
22. $(c-1)(c+2) = c^2 + c - 2$.
23. $(x-3)(x+5) = x^2 + 2x - 15$.
24. $(a^2-4a)(a^2+6a) = a^4 + 2a^3 - 24a^2$.
25. $(a^x-2a^{-x})(a^x+5a^{-x}) = a^{2x} + 3 - 10a^{-2x}$.
26. $(cx-4c^2)(cx+8c^2) = c^2x^2 + 4c^3x - 32c^4$.
27. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.
28. $(a+c+x)^2 = a^2 + c^2 + x^2 + 2ac + 2ax + 2cx$.
29. $(a-c+x)^2 = a^2 + c^2 + x^2 - 2ac + 2ax - 2cx$.
30. $(a-c-x)^2 = a^2 + c^2 + x^2 - 2ac - 2ax + 2cx$.

31. $(a - c + 2)^2 = a^2 + c^2 + 4 - 2ac + 4a - 4c.$
32. $(x - c - 3a)^2 = x^2 + c^2 + 9a^2 - 2cx - 6ax + 6ac.$
33. $\frac{a^2 + 9c^2 + x^2 - 6ac + 2ax - 6cx}{a - 3c + x} = a - 3c + x.$ Formula V.
34. $(3c - 5a + 2x)^2 = 9c^2 + 25a^2 + 4x^2 - 30ac + 12cx - 20ax.$
35. $(x^2a + xa - 5)^2 = x^4a + 25 + 2x^3a - 9x^2a - 10xa.$
36. $(a + a^{-1} - 3)^2 = a^2 + a^{-2} + 11 - 6a - 6a^{-1}.$
37. $(a^x - a^{-x} + 4)^2 = a^{2x} + a^{-2x} + 14 + 8a^x - 8a^{-x}.$
38. Yes; regard $a + b$ as one term and c as the other. Then use Formula I.
39. $(x + c)^3 = x^3 + 3cx^2 + 3c^2x + c^3.$
40. $(x - c)^3 = x^3 - 3cx^2 + 3c^2x - c^3.$
41. $(x - 1)^3 = x^3 - 3x^2 + 3x - 1.$
42. $(x + 2)^3 = x^3 + 6x^2 + 12x + 8.$
43. $(x + 3)^3 = x^3 + 9x^2 + 27x + 27.$
44. $(x - 4)^3 = x^3 - 12x^2 + 48x - 64.$
45. $(x^2 - 2)^3 = x^6 - 6x^4 + 12x^2 - 8.$
46. $(5a - 4c)^3 = 125a^3 - 300a^2c + 240ac^2 - 64c^3.$
47. $(2x^5 - 7x^2)^3 = 8x^{15} - 84x^{12} + 294x^9 - 343x^6.$
48. $(x^3 - 6x^2 + 12x - 8) \div (x - 2) = (x - 2)^2.$ By § 46.
49. $(8 - 12x + 6x^2 - x^3) \div (4 - 4x + x^2) = 2 - x.$ By § 46.
50. $25 - 70 + 49 = (-2)^2 = 4.$
51. $x^2 + 18x + 81.$ Results are the same. Even powers of both positive and negative numbers are positive.
52. $81 - 9 - 12 = 60,$ which equals $5 \cdot 12.$
53. (a) $(42)^2 = (40 + 2)^2 = 1600 + 160 + 4 = 1764.$
 (b) $(59)^2 = (60 - 1)^2 = 3600 - 120 + 1 = 3481.$
 (c) $(73)^2 = (70 + 3)^2 = 4900 + 420 + 9 = 5329.$
 (d) $(105)^2 = (100 + 5)^2 = 10,000 + 1000 + 25 = 11,025.$
 (e) $(97)^2 = (100 - 3)^2 = 10,000 - 600 + 9 = 9409.$
 (f) $(1005)^2 = (1000 + 5)^2 = 1,000,000 + 10,000 + 25 = 1,010,025.$
54. $16 + 81 + 25 + 72 - 40 - 90 = 64,$ which equals $8 \cdot 8.$
55. $27 - 54 + 36 - 8 = 1,$ which equals $1^3.$
56. $(a - 2b)^3 = a^3 - 6a^2b + 12ab^2 - 8b^3.$ $(2b - a)^3 = 8b^3 - 12ab^2 + 6a^2b - a^3.$ Results have opposite signs. Odd powers of any number have the same sign as the number. Here one expression is the negative of the other.
57. $+1.$
58. 24a. When arranged the middle term of a trinomial square is twice the product of the square roots of the other two.
59. $x^2 - 6x + 9.$
60. $4x^2 - 12ax + 9a^2.$
61. $x^2 + 4x + 4.$
62. $9x^2 \pm 24x + 16.$
63. $4x^2 \pm 12x + 9.$
64. $9x^2 - 4ax + \frac{4a^2}{9}.$
65. $a^2 + 2 + a^{-2}.$

66. $a^2x - 2 + a^{-2x}$.

67. $a^{2x} - 8 + 16a^{-2x}$.

68. $a^{4x} + 10 + 25a^{-4x}$.

69. $a^{10} - 14a^2 + 49a^{-6}$.

70. $a^6 - 6a^2 + 9a^{-2}$.

71. $a^{4x} - 12a^x + 36a^{-2x}$.

72. $4a^{6x} + 20a^{2x} + 25a^{-2x}$.

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1. $4x + 8 = 4(x + 2)$.
2. $ax - 7ay = a(x - 7y)$.
3. $a^2c - ac^2 - 4ac = ac(a - c - 4)$.
4. $3xy + 21y^5 - 15y^3 = 3y(x + 7y^4 - 5y^2)$.
5. $x^{2a} - 3x^a + 12x = x(x^{2a-1} - 3x^{a-1} + 12)$.
6. $y^{2a} - 6y^a + 2y^{a-1} = y^a(y^a - 6 + 2y^{-1})$.
7. $5xy + 30y(x^2 + xy) = 5xy(1 + 6x + 6y)$.
8. $(7a^2 - 21ab + 7a) - 14ax = 7a(a - 3b + 1 - 2x)$.
9. $(3c^2 - 3cd) - a(45c^2 - 15c^3x) = 3c(c - d - 15ac + 5ac^2x)$.
10. $2r^{2x+3} + 12r^{x-7} - 16sr^{x+2} + 8sr^{x+4} = 2r^x(r^{x+3} + 6r^{-7} - 8sr^2 + 4sr^4)$.
11. $3(x + y) + a(x + y) = (x + y)(3 + a)$.
12. $a(x - 3) - b(x - 3) = (x - 3)(a - b)$.
13. $4x - 4y + bx - by = (x - y)(4 + b)$.
14. $3cx + 6ac + 8ax + 4x^2 = 3c(x + 2a) + 4x(2a + x) = (x + 2a)(3c + 4x)$.
15. $-6x^2 + 10x + 21xm - 35m = 2x(-3x + 5) - 7m(-3x + 5) = (5 - 3x)(2x - 7m)$.
16. $rs - 2s + 3r - 6 - 5rx + 10x = s(r - 2) + 3(r - 2) - 5x(r - 2) = (r - 2)(s + 3 - 5x)$.
17. $x^3a - 3x^2a - x^a + 3 - 6x^4a + 2x^5a = x^2a(x^a - 3) - 1(x^a - 3) + 2x^4a(x^a - 3) = (x^a - 3)(x^{2a} - 1 + 2x^4a)$.
18. $x^3a^{-2} + 2xa^{+1} - 15x^{2a-3} - 10 + 10x^{2a-3} = x^{a+1}(x^{2a-3} + 2) - 5(x^{2a-3} + 2) = (x^{2a-3} + 2)(x^{a+1} - 5)$.
19. $x^2 + 6x + 9 = (x + 3)^2$.
20. $x^2 + x + \frac{1}{4} = (x + \frac{1}{2})^2$.
21. $9b^2 - 12b + 4 = (3b - 2)^2$.
22. $9r^2 + 49 - 42r = (3r - 7)^2$.
23. $a^2 - 2 + a^{-2} = (a - a^{-1})^2$.
24. $a^4 - 6 + 9a^{-4} = (a^2 - 3a^{-2})^2$.
25. $\frac{x^2}{9a^2} - \frac{8x}{a} + 144 = \left(\frac{x}{3a} - 12\right)^2$.
26. $x^4a - 10ax^2a + 25a^2 = (x^{2a} - 5a)^2$.
27. $r^4x^{+2} + 4 - 4r^{2x+1} = (r^{2x+1} - 2)^2$.
28. $a^{2x} + 4a^{-2x} - 4 = (a^x - 2a^{-x})^2$.
29. $a^{4x} - 2a^x + a^{-2x} = (a^{2x} - a^{-x})^2$.
30. $4(a + 5)^2 - 12b(a + 5) + 9b^2 = [2(a + 5) - 3b]^2$.
31. $(a - b)^{2x} - 18x(a - b)^x + 81x^2 = [(a - b)^x - 9x]^2$.
32. $x^2 + 2x - 24 = (x + 6)(x - 4)$.
33. $x^2 + x - \frac{3}{4} = (x + \frac{3}{2})(x - \frac{1}{2})$.
34. $a^2 + .3a - .1 = (a + \frac{1}{2})(a - \frac{1}{5})$.
35. $c^2 - ac - 90a^2 = (c - 10a)(c + 9a)$.
36. $r^2s^2 + 6rs - 40 = (rs + 10)(rs - 4)$.

37. $a^2 + 5 + 6a^{-2} = (a + 3a^{-1})(a + 2a^{-1})$.
 38. $15m^2 - 14mx - x^2 = (m - x)(x + 15m)$.
 39. $90 + x - x^2 = (10 - x)(9 + x)$.
 40. $12a^4 - a^2x - x^2 = (3a^2 - x)(x + 4a^2)$.
 41. $a^{2x} - 20 + 19a^x = (a^x + 20)(a^x - 1)$.
 42. $a^2 + 12a^{-2} + 7 = (a + 4a^{-1})(a + 3a^{-1})$.
 43. $a^{2x} - 8a^{-2x} - 2 = (a^x - 4a^{-x})(a^x + 2a^{-x})$.
 44. $120 + 7m^n - m^{2n} = (8 + m^n)(15 - m^n)$.
 45. $a^{4x} - a^x - 6a^{-2x} = (a^{2x} - 3a^{-x})(a^{2x} + 2a^{-x})$.
 46. $(m + n)^{4e} - 9(m + n)^{2e} - 22 = [(m + n)^{2e} - 11][(m + n)^{2e} + 2]$.

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2. $2a^2 - 3a - 2 = 2a^2 - 4a + a - 2$
 $= 2a(a - 2) + 1(a - 2)$
 $= (a - 2)(2a + 1)$.
 3. $3a^2 + 8a - 3 = 3a^2 + 9a - a - 3$
 $= 3a(a + 3) - 1(a + 3)$
 $= (a + 3)(3a - 1)$.
 4. $4a^2 + a - 5 = 4a^2 + 5a - 4a - 5$
 $= a(4a + 5) - 1(4a + 5)$
 $= (4a + 5)(a - 1)$.
 5. $9c^2 - 71c - 8 = 9c^2 - 72c + c - 8$
 $= 9c(c - 8) + 1(c - 8)$
 $= (c - 8)(9c + 1)$.
 6. $5r^2 - 22r + 8 = 5r^2 - 20r - 2r + 8$
 $= 5r(r - 4) - 2(r - 4)$
 $= (r - 4)(5r - 2)$.
 7. $7x^2 + 62x - 9 = 7x^2 + 63x - x - 9$
 $= 7x(x + 9) - 1(x + 9)$
 $= (x + 9)(7x - 1)$.
 8. $6x^2 + 19x - 7 = 6x^2 + 21x - 2x - 7$
 $= 3x(2x + 7) - 1(2x + 7)$
 $= (2x + 7)(3x - 1)$.
 9. $6x^2 + 13x - 5 = 6x^2 + 15x - 2x - 5$
 $= 3x(2x + 5) - 1(2x + 5)$
 $= (2x + 5)(3x - 1)$.
 10. $2x^2 + 7x - 15 = 2x^2 + 10x - 3x - 15$
 $= 2x(x + 5) - 3(x + 5)$
 $= (x + 5)(2x - 3)$.
 11. $3x^2 - ax - 2a^2 = 3x^2 - 3ax + 2ax - 2a^2$
 $= 3x(x - a) + 2a(x - a)$
 $= (x - a)(3x + 2a)$.

$$12. 4a^4 - 12a^2 + 9 = (2a^2 - 3)^2.$$

$$13. 25 + 4c^2d^2 - 20cd = (2cd - 5)^2.$$

$$14. -8n^4 + 3n^8 - 3 = 3n^8 - 9n^4 + n^4 - 3 \\ = 3n^4(n^4 - 3) + 1(n^4 - 3) \\ = (n^4 - 3)(3n^4 + 1).$$

$$15. 6x^{2y} - 13xy + 6 = 6x^{2y} - 9xy - 4xy + 6 \\ = 3xy(2x^y - 3) - 2(2xy - 3) \\ = (2x^y - 3)(3xy - 2).$$

$$16. 20x^2 - 9xy^3 - 20y^6 = 20x^2 + 16xy^3 - 25xy^3 - 20y^6 \\ = 4x(5x + 4y^3) - 5y^3(5x + 4y^3) \\ = (5x + 4y^3)(4x - 5y^3).$$

$$17. c^2x^a + x^{2a} - 12c^4 = x^{2a} - 3c^2x^a + 4c^2x^a - 12c^4 \\ = x^a(x^a - 3c^2) + 4c^2(x^a - 3c^2) \\ = (x^a + 4c^2)(x^a - 3c^2).$$

$$18. 2x^2 - (a + 2b)x + ab = 2x^2 - ax - 2bx + ab \\ = x(2x - a) - b(2x - a) \\ = (2x - a)(x - b).$$

$$19. 5m^{2n-4} + 9am^{n-2} - 2a^2 = 5m^{2n-4} + 10am^{n-2} - am^{n-2} - 2a^2 \\ = 5m^{n-2}(m^{n-2} + 2a) - a(m^{n-2} + 2a) \\ = (m^{n-2} + 2a)(5m^{n-2} - a).$$

$$20. 6x^{4a} + (3 - 2y^b)x^{2a} - y^b = 6x^{4a} - 2x^{2a}y^b + 3x^{2a} - y^b \\ = 2x^{2a}(3x^{2a} - y^b) + 1(3x^{2a} - y^b) \\ = (3x^{2a} - y^b)(2x^{2a} + 1).$$

$$21. 20a^2b^{4-2x} - 9a - 20b^{2x-4} = 20a^2b^{4-2x} + 16a - 25a - 20b^{2x-4} \\ = 4ab^{2-x}(5ab^{2-x} + 4b^{x-2}) - 5b^{x-2}(5ab^{2-x} + 4b^{x-2}) \\ = (5ab^{2-x} + 4b^{x-2})(4ab^{2-x} - 5b^{x-2}).$$

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1. $m^2 - n^2 = (m + n)(m - n).$
2. $x^2 - 4 = (x + 2)(x - 2).$
3. $x^2 - \frac{1}{9} = (x + \frac{1}{3})(x - \frac{1}{3}).$
4. $25x^2 - 49b^2 = (5x + 7b)(5x - 7b).$
5. $a^2 - a^{-2} = (a + a^{-1})(a - a^{-1}).$
6. $a^4 - \frac{4c^2}{25} = \left(a^2 + \frac{2c}{5}\right)\left(a^2 - \frac{2c}{5}\right).$
7. $16a^6 - 25c^4d^{10} = (4a^3 + 5c^2d^5)(4a^3 - 5c^2d^5).$
8. $a^{2x} - a^{-2x} = (a^x + a^{-x})(a^x - a^{-x}).$
9. $a^4 - a^{-4} = (a^2 + a^{-2})(a + a^{-1})(a - a^{-1}).$
10. $(a + c)^2 - 1 = (a + c + 1)(a + c - 1).$
11. $(a - x)^4 - 4 = (a^2 - 2ax + x^2 + 2)(a^2 - 2ax + x^2 - 2).$
12. $9 - (2 + x)^2 = (5 + x)(1 - x).$
13. $16 - (x - a)^{6a} = [4 + (x - a)^{3a}][4 - (x - a)^{3a}].$
14. $5^{2a} - (m - 1)^{10a} = [5^a + (m - 1)^{5a}][5^a - (m - 1)^{5a}].$

15. $(a + c)^2 - (m - n)^2 = (a + c + m - n)(a + c - m + n).$
16. $(a - x)^{2r} - (c - 5)^2 = [(a - x)^r + c - 5][(a - x)^r - c + 5].$
17. $a^2 + 2ax + x^2 - 9 = (a + x + 3)(a + x - 3).$
18. $25 - 10x + x^2 - 16m^2 = (5 - x)^2 - 16m^2$
 $= (5 - x + 4m)(5 - x - 4m).$
19. $9 - 12a + 4a^2 - b^2m = (3 - 2a)^2 - b^2m$
 $= (3 - 2a + bm)(3 - 2a - bm).$
20. $m^2 - a^2 - 6a - 9 = (m + a + 3)(m - a - 3).$
21. $x^2 - 4y^2 + 20y - 25 = x^2 - (2y - 5)^2$
 $= (x + 2y - 5)(x - 2y + 5).$
22. $-28c^3d^2 - 49c^6 + 1 - 4d^4 = 1 - (7c^3 + 2d^2)^2$
 $= (1 + 7c^3 + 2d^2)(1 - 7c^3 - 2d^2).$
23. $12rs - 36 + 5^2a - r^2s = 5^2a - (rs - 6)^2$
 $= (5^a + rs - 6)(5^a - rs + 6).$
24. $(m - 2)^2 - 4n^2 + 28n - 49 = (m - 2)^2 - (2n - 7)^2$
 $= (m - 2 + 2n - 7)(m - 2 - 2n + 7)$
 $= (m + 2n - 9)(m - 2n + 5).$
25. $x^2 - 6x + 9 - y^2 + 8ay - 16a^2 = (x - 3)^2 - (y - 4a)^2$
 $= (x - 3 + y - 4a)(x - 3 - y + 4a).$
26. $4bd + 4c^2 - 4d^2 - 4c - b^2 + 1 = (2c - 1)^2 - (2d - b)^2$
 $= (2c - 1 + 2d - b)(2c - 1 - 2d + b).$
27. $6c + h^2 - 1 - 9c^2 - 4hk + 4k^2 = (h - 2k)^2 - (1 - 3c)^2$
 $= (h - 2k + 1 - 3c)(h - 2k - 1 + 3c).$
28. $h^2 - 4y^2 - 10h + 8xy + 25 - 4x^2 = (h - 5)^2 - (2y - 2x)^2$
 $= (h - 5 + 2y - 2x)(h - 5 - 2y + 2x).$
29. $25x^2 - 20x + 4 - 4y^2 - 9a^2 - 12ay = (5x - 2)^2 - (2y + 3a)^2$
 $= (5x - 2 + 2y + 3a)(5x - 2 - 2y - 3a).$

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1. $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 = (x^2 + 1)^2 - x^2 = (x^2 + 1 + x)(x^2 + 1 - x).$
2. $x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$
 $= (x^2 + y^2)^2 - x^2y^2$
 $= (x^2 + xy + y^2)(x^2 - xy + y^2).$
3. $x^4 + 4x^2 + 16 = x^4 + 8x^2 + 16 - 4x^2$
 $= (x^2 + 4)^2 - 4x^2$
 $= (x^2 + 2x + 4)(x^2 - 2x + 4).$
4. $16y^4 + 4y^2 + 1 = 16y^4 + 8y^2 + 1 - 4y^2$
 $= (4y^2 + 1)^2 - 4y^2$
 $= (4y^2 + 2y + 1)(4y^2 - 2y + 1).$
5. $c^4 + c^2d^2 + 25d^4 = c^4 + 10c^2d^2 + 25d^4 - 9c^2d^2$
 $= (c^2 + 5d^2)^2 - 9c^2d^2$
 $= (c^2 + 5d^2 + 3cd)(c^2 + 5d^2 - 3cd).$

6. $1 - 19y^6 + 25y^{12} = 1 - 10y^6 + 25y^{12} - 9y^6$
 $= (1 - 5y^6)^2 - 9y^6$
 $= (1 - 5y^6 + 3y^3)(1 - 5y^6 - 3y^3).$
7. $4x^4 + 3x^2y^2 + 9y^4 = 4x^4 + 12x^2y^2 + 9y^4 - 9x^2y^2$
 $= (2x^2 + 3y^2)^2 - 9x^2y^2$
 $= (2x^2 + 3y^2 + 3xy)(2x^2 + 3y^2 - 3xy).$
8. $4x^4 - 28x^2y^6 + 9y^{12} = 4x^4 - 12x^2y^6 + 9y^{12} - 16x^2y^6$
 $= (2x^2 - 3y^6)^2 - 16x^2y^6$
 $= (2x^2 - 3y^6 + 4xy^3)(2x^2 - 3y^6 - 4xy^3).$
9. $9a^8 - 19a^4b^2 + 25b^4 = 9a^8 + 30a^4b^2 + 25b^4 - 49a^4b^2$
 $= (3a^4 + 5b^2)^2 - 49a^4b^2$
 $= (3a^4 + 5b^2 + 7a^2b)(3a^4 + 5b^2 - 7a^2b).$
10. $49h^4 - 44h^2k^4 + 4k^8 = 49h^4 - 28h^2k^4 + 4k^8 - 16h^2k^4$
 $= (7h^2 - 2k^4)^2 - 16h^2k^4$
 $= (7h^2 - 2k^4 + 4hk^2)(7h^2 - 2k^4 - 4hk^2).$
11. $x^8 + x^4 + 1 = x^8 + 2x^4 + 1 - x^4$
 $= (x^4 + x^2 + 1)(x^4 - x^2 + 1)$
 $= (x^4 - x^2 + 1)(x^2 + x + 1)(x^2 - x + 1).$
12. $c^8 - 6c^4 + 1 = c^8 - 2c^4 + 1 - 4c^4$
 $= (c^4 - 1)^2 - 4c^4$
 $= (c^4 + 2c^2 - 1)(c^4 - 2c^2 - 1).$
13. $16 + 4x^4 + x^8 = 16 + 8x^4 + x^8 - 4x^4$
 $= (4 + x^4)^2 - 4x^4$
 $= (4 + x^4 + 2x^2)(4 + x^4 - 2x^2).$
14. $y^8 + 16y^4 + 256 = y^8 + 32y^4 + 256 - 16y^4$
 $= (y^4 + 16 + 4y^2)(y^4 + 16 - 4y^2)$
 $= (y^2 + 4 + 2y)(y^2 + 4 - 2y)(y^4 + 16 - 4y^2).$
15. $3x^9y + 3x^5y^5 + 3xy^9 = 3xy(x^8 + x^4y^4 + y^8)$
 $= 3xy(x^4 + x^2y^2 + y^4)(x^4 - x^2y^2 + y^4)$
 $= 3xy(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4).$
16. $16h^4 - 33h^2k^2 + 36k^4 = 16h^4 + 48h^2k^2 + 36k^4 - 81h^2k^2$
 $= (4h^2 + 6k^2 + 9hk)(4h^2 + 6k^2 - 9hk).$
17. $25c^4 - 51c^2d^2 + 49d^4 = 25c^4 + 70c^2d^2 + 49d^4 - 121c^2d^2$
 $= (5c^2 + 7d^2 + 11cd)(5c^2 + 7d^2 - 11cd).$
18. $49a^4 - 32a^2b^2 + 64b^4 = 49a^4 + 112a^2b^2 + 64b^4 - 144a^2b^2$
 $= (7a^2 + 8b^2 + 12ab)(7a^2 + 8b^2 - 12ab).$
19. $64x^4 + 119x^2y^2 + 81y^4 = 64x^4 + 144x^2y^2 + 81y^4 - 25x^2y^2$
 $= (8x^2 + 9y^2 + 5xy)(8x^2 + 9y^2 - 5xy).$
20. $81a^4 - 171a^2b^2 + 25b^4 = 81a^4 - 90a^2b^2 + 25b^4 - 81a^2b^2$
 $= (9a^2 - 5b^2 + 9ab)(9a^2 - 5b^2 - 9ab).$
21. $1 + 4x^4 = 1 + 4x^2 + 4x^4 - 4x^2$
 $= (1 + 2x^2 + 2x)(1 + 2x^2 - 2x).$

22. $64c^4 + 1 = 64c^4 + 16c^2 + 1 - 16c^2$
 $= (8c^2 + 1 + 4c)(8c^2 + 1 - 4c).$
23. $x^4 + 4y^4 = x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$
 $= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy).$
24. $x^8 + 4y^8 = x^8 + 4x^4y^4 + 4y^8 - 4x^4y^4$
 $= (x^4 + 2y^4 + 2x^2y^2)(x^4 + 2y^4 - 2x^2y^2).$
25. $x^8 + 64 = x^8 + 16x^4 + 64 - 16x^4$
 $= (x^4 + 8 + 4x^2)(x^4 + 8 - 4x^2).$
26. $x^{4a} + 4y^{8a} = x^{4a} + 4x^{2a}y^{4a} + 4y^{8a} - 4x^{2a}y^{4a}$
 $= (x^{2a} + 2y^{4a} + 2x^ay^{2a})(x^{2a} + 2y^{4a} - 2x^ay^{2a}).$
27. $a^4bc^{12d} + 64e^{4x+4}$
 $= a^4bc^{12d} + 16a^2bc^6de^{2x+2} + 64e^{4x+4} - 16a^2bc^6de^{2x+2}$
 $= (a^2bc^6d + 8e^{2x+2} + 4a^bc^3de^{x+1})(a^2bc^6d + 8e^{2x+2} - 4a^bc^3de^{x+1}).$

Page 294 (Second set)

1. $x^3 + 64 = x^3 + 4^3 = (x + 4)(x^2 - 4x + 16).$
2. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9).$
3. $a^3 - 64 = a^3 - 4^3 = (a - 4)(a^2 + 4a + 16).$
4. $8 + m^3 = 2^3 + m^3 = (2 + m)(4 - 2m + m^2).$
5. $27 - m^6 = 3^3 - (m^2)^3 = (3 - m^2)(9 + 3m^2 + m^4).$
6. $x^3 - \frac{a^3}{8} = x^3 - \left(\frac{a}{2}\right)^3 = \left(x - \frac{a}{2}\right)\left(x^2 + \frac{ax}{2} + \frac{a^2}{4}\right).$
7. $x^3 - y^{-3} = x^3 - (y^{-1})^3 = (x - y^{-1})(x^2 + xy^{-1} + y^{-2}).$
8. $8a^3 + 27b^3 = (2a)^3 + (3b)^3 = (2a + 3b)(4a^2 - 6ab + 9b^2).$
9. $1 - 125x^6 = 1^3 - (5x^2)^3 = (1 - 5x^2)(1 + 5x^2 + 25x^4).$
10. $\frac{x^3}{27} - y^6 = \left(\frac{x}{3}\right)^3 - (y^2)^3 = \left(\frac{x}{3} - y^2\right)\left(\frac{x^2}{9} + \frac{xy^2}{3} + y^4\right).$
11. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2).$
12. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)(x^4 - x^2y^2 + y^4).$
13. $x^9 - a^3 = (x^3)^3 - a^3 = (x^3 - a)(x^6 + x^3a + a^2).$
14. $x^6 + a^9 = (x^2)^3 + (a^3)^3 = (x^2 + a^3)(x^4 - x^2a^3 + a^6).$
15. $(x + y)^3 - 8 = (x + y)^3 - 2^3 = (x + y - 2)(x^2 + 2xy + y^2 + 2x + 2y + 4).$
16. $x^3 - 9x^2 + 27x - 28 = (x^3 - 9x^2 + 27x - 27 - 1)$
 $= (x - 3)^3 - 1^3$
 $= (x - 3 - 1)(x^2 - 6x + 9 + x - 3 + 1)$
 $= (x - 4)(x^2 - 5x + 7).$
17. $a^3b^6c^9 - 8d^{12} = (ab^2c^3)^3 - (2d^4)^3 = (ab^2c^3 - 2d^4)(a^2b^4c^6 + 2ab^2c^3d^4 + 4d^8).$
18. $x^{3e} - y^{-3e} = (x^e)^3 - (y^{-e})^3 = (x^e - y^{-e})(x^{2e} + x^ey^{-e} + y^{-2e}).$
19. $c^{6e} + 27d^{9x} = (c^{2e})^3 + (3d^{3x})^3 = (c^{2e} + 3d^{3x})(c^{4e} - 3c^{2e}d^{3x} + 9d^{6x}).$

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$$\begin{array}{r}
 1. \ x^2 + bx + c \quad | \ x - n \\
 \underline{x^2 - nx} \qquad \qquad \underline{x + (n + b)} \\
 (n + b)x + c \\
 \underline{(n + b)x - n^2 - nb} \\
 n^2 + nb + c, \text{ remainder}
 \end{array}$$

$$\begin{array}{r}
 2. \ x^3 + ax^2 + bx + c \quad | \ x - n \\
 \underline{x^3 - nx^2} \qquad \qquad \underline{x^2 + (n + a)x + (n^2 + na + b)} \\
 (n + a)x^2 \\
 \underline{(n + a)x^2 - (n^2 + na)x} \\
 + (n^2 + na + b)x \\
 \underline{+ (n^2 + na + b)x - n^3 - n^2a - nb} \\
 n^3 + n^2a + nb + c, \text{ remainder}
 \end{array}$$

$$3. \ x^3 - x - 8 = 27 - 3 - 8 = 16, \text{ if } 3 \text{ is put for } x.$$

16, remainder.

$$\begin{array}{r}
 4. \ 1 + 0 + 0 - 1 + \quad 6 \underline{| - 2} \quad \text{By synthetic division} \\
 \underline{- 2 + 4 - 8 + 18} \\
 1 - 2 + 4 - 9 + 24 \quad \quad \quad 24, \text{ remainder.}
 \end{array}$$

$$\begin{array}{r}
 5. \ 1 + 0 - 3 + 2 \underline{| + 1} \\
 \underline{+ 1 + 1 - 2} \\
 1 + 1 - 2 \quad 0,
 \end{array}$$

$$\therefore x^3 - 3x + 2 = (x - 1)(x^2 + x - 2) = (x - 1)^2(x + 2).$$

$$\begin{array}{r}
 6. \ 1 + 0 - 4 + 3 \underline{| + 1} \\
 \underline{+ 1 + 1 - 3} \\
 1 + 1 - 3 \quad 0
 \end{array}$$

$$\therefore x^3 - 4x + 3 = (x - 1)(x^2 + x - 3).$$

$$\begin{array}{r}
 7. \ 1 + 0 + 2 + 3 \underline{| - 1} \\
 \underline{- 1 + 1 - 3} \\
 1 - 1 + 3 \quad 0
 \end{array}$$

$$\therefore x^3 + 2x + 3 = (x + 1)(x^2 - x + 3).$$

$$\begin{array}{r}
 10. \ 1 + 0 - 11 + 6 \underline{| + 3} \\
 \underline{+ 3 + 9 - 6} \\
 1 + 3 - 2 \quad 0
 \end{array}$$

$$\therefore x^3 - 11x + 6 = (x - 3)(x^2 + 3x - 2).$$

$$\begin{array}{r}
 8. \ 1 + 0 - 1 - 6 \underline{| + 2} \\
 \underline{+ 2 + 4 + 6} \\
 1 + 2 + 3 \quad 0
 \end{array}$$

$$\therefore x^3 - x - 6 = (x - 2)(x^2 + 2x + 3).$$

$$\begin{array}{r}
 11. \ 1 + 0 - 11 - 6 \underline{| - 3} \\
 \underline{- 3 + 9 + 6} \\
 1 - 3 - 2 \quad 0
 \end{array}$$

$$\therefore x^3 - 11x - 6 = (x + 3)(x^2 - 3x - 2).$$

$$\begin{array}{r}
 9. \ 1 + 0 - 1 + 6 \underline{| - 2} \\
 \underline{- 2 + 4 - 6} \\
 1 - 2 + 3 \quad 0
 \end{array}$$

$$\therefore x^3 - x + 6 = (x + 2)(x^2 - 2x + 3).$$

$$\begin{array}{r}
 12. \ 1 + 0 - 14 - 8 \underline{| + 4} \\
 \underline{+ 4 + 16 + 8} \\
 1 + 4 + 2 \quad 0
 \end{array}$$

$$\therefore x^3 - 14x - 8 = (x - 4)(x^2 + 4x + 2).$$

$$\begin{array}{r} 13. \quad 1 + 0 - 27 - 10 \mid - 5 \\ \quad - 5 + 25 + 10 \\ \hline 1 - 5 - 2 \quad 0 \end{array}$$

$$\therefore x^3 - 27x - 10 = (x + 5)(x^2 - 5x - 2).$$

$$\begin{array}{r} 14. \quad 1 + 3 + 3 + 2 \mid - 2 \\ \quad - 2 - 2 - 2 \\ \hline 1 + 1 + 1 \quad 0 \end{array}$$

$$\therefore x^3 + 3x^2 + 3x + 2 = (x + 2)(x^2 + x + 1).$$

$$\begin{array}{r} 15. \quad 1 + 4 + 5 + 2 \mid - 1 \\ \quad - 1 - 3 - 2 \\ \hline 1 + 3 + 2 \quad 0 \end{array}$$

$$\therefore x^3 + 4x^2 + 5x + 2 = (x + 1)(x^2 + 3x + 2) = (x + 1)(x + 1)(x + 2).$$

$$\begin{array}{r} 16. \quad 1 - 6 + 11 - 6 \mid + 1 \\ \quad + 1 - 5 + 6 \\ \hline 1 - 5 + 6 \quad 0 \end{array}$$

$$\therefore x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 2)(x - 3).$$

$$\begin{array}{r} 17. \quad 1 + 0 - 11 + 2 + 12 \mid - 1 \\ \quad - 1 + 1 + 10 - 12 \\ \hline 1 - 1 - 10 + 12 \quad 0 \\ 1 - 1 - 10 + 12 \mid + 3 \\ \quad + 3 + 6 - 12 \\ \hline 1 + 2 - 4 \quad 0 \end{array}$$

$$\begin{aligned} \therefore x^4 - 11x^2 + 2x + 12 &= (x + 1)(x^3 - x^2 - 10x + 12) \\ &= (x + 1)(x - 3)(x^2 + 2x - 4). \end{aligned}$$

$$\begin{array}{r} 18. \quad 4 + 0 + 0 - 3 - 1 \mid + 1 \\ \quad + 4 + 4 + 4 + 1 \\ \hline 4 + 4 + 4 + 1 \quad 0 \end{array}$$

$$\therefore 4x^4 - 3x - 1 = (x - 1)(4x^3 + 4x^2 + 4x + 1).$$

$$\begin{array}{r} 19. \quad 1 + 0 - 5a^2 + 2a^3 \mid + 2a \\ \quad + 2a + 4a^2 - 2a^3 \\ \hline 1 + 2a - a^2 \quad 0 \end{array}$$

$$x^3 - 5a^2x + 2a^3 = (x - 2a)(x^2 + 2ax - a^2).$$

$$\begin{array}{r} 20. \quad 1 + 0 - 7m^2 - 6m^3 \mid - m \\ \quad - m + m^2 + 6m^3 \\ \hline 1 - m - 6m^2 \quad 0 \end{array}$$

$$\therefore x^3 - 7m^2x - 6m^3 = (x + m)(x^2 - mx - 6m^2) = (x + m)(x - 3m)(x + 2m).$$

$$\begin{array}{r} 21. \quad 1 - 2n - 5n^2 + 6n^3 \mid + n \\ \quad + n - n^2 - 6n^3 \\ \hline 1 - n - 6n^2 \quad 0 \end{array}$$

$$\therefore x^3 - 2nx^2 - 5n^2x + 6n^3 = (x - n)(x^2 - nx - 6n^2) = (x - n)(x - 3n)(x + 2n).$$

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1. $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1).$
2. $x^5 + y^5 = (x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4).$
3. $x^7 - y^7 = (x - y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6).$
4. $x^7 + 1 = (x + 1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1).$
5. $x^5 - 32 = (x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16).$
6. $3125 - c^5 = (5 - c)(625 + 125c + 25c^2 + 5c^3 + c^4).$
7. $c^7 - 128 = (c - 2)(c^6 + 2c^5 + 4c^4 + 8c^3 + 16c^2 + 32c + 64).$
8. $c^7 - x^{14} = (c - x^2)(c^6 + c^5x^2 + c^4x^4 + c^3x^6 + c^2x^8 + cx^{10} + x^{12}).$
9. $x^{10} + y^5 = (x^2 + y)(x^8 - x^6y + x^4y^2 - x^2y^3 + y^4).$
10. $1 + c^7d^{14} = (1 + cd^2)(1 - cd^2 + c^2d^4 - c^3d^6 + c^4d^8 - c^5d^{10} + c^6d^{12}).$
11. $c^{14} + 128 = (c^2 + 2)(c^{12} - 2c^{10} + 4c^8 - 8c^6 + 16c^4 - 32c^2 + 64).$
12. $x^{10} + a^{15} = (x^2 + a^3)(x^8 - x^6a^3 + x^4a^6 - x^2a^9 + a^{12}).$

13. $x^5 - 32y^{10} = (x - 2y^2)(x^4 + 2x^3y^2 + 4x^2y^4 + 8xy^6 + 16y^8)$.
 14. $128x^7 - 1 = (2x - 1)(64x^6 + 32x^5 + 16x^4 + 8x^3 + 4x^2 + 2x + 1)$.
 15. $1024 - 243x^5 = (4 - 3x)(256 + 192x + 144x^2 + 108x^3 + 81x^4)$.

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1. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1)$.
 2. $x^{10} - x^2 = x^2(x^8 - 1) = x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$.
 3. $x^8 - 2x^4 + 1 = (x^4 - 1)^2 = (x^2 + 1)^2(x + 1)^2(x - 1)^2$.
 4. $x^5 - 8x^3 + 16x = x(x^2 - 4)^2 = x(x + 2)^2(x - 2)^2$.
 5. $x^4 - 10x^2 + 9 = (x^2 - 9)(x^2 - 1) = (x + 3)(x - 3)(x + 1)(x - 1)$.
 6. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x + 3)(x - 3)(x + 2)(x - 2)$.
 7. $18a^2x^2 - 24a^2x - 10a^2 = 2a^2(9x^2 - 12x - 5) = 2a^2(3x - 5)(3x + 1)$.
 8. $x^2 - \frac{x}{6} - \frac{1}{6} = \left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$.
 9. $16x^4 + 8x^2 - 3 = (4x^2 + 3)(4x^2 - 1) = (4x^2 + 3)(2x + 1)(2x - 1)$.
 10. $a^3 - a + a^2b - b = a(a^2 - 1) + b(a^2 - 1) = (a + b)(a + 1)(a - 1)$.
 11. $3x^4 - 15x^2 + 12 = 3(x^2 - 4)(x^2 - 1)$
 $= 3(x + 2)(x - 2)(x + 1)(x - 1)$.
 12. $12a - 39ay - 51ay^2 = 3a(4 - 13y - 17y^2) = 3a(4 - 17y)(1 + y)$.
 13. $x^4 - 3x^3 + 4x^2 - 12x = x^3(x - 3) + 4x(x - 3) = x(x^2 + 4)(x - 3)$.
 14. $2a^3b + 3a^2b - 8ab - 12b = a^2b(2a + 3) - 4b(2a + 3)$
 $= b(a + 2)(a - 2)(2a + 3)$.
 15. $4a^2 - a^4 + 81 + 10a^2x - 36a - 25x^2 = (2a - 9)^2 - (a^2 - 5x)^2$
 $= (2a - 9 + a^2 - 5x)(2a - 9 - a^2 + 5x)$.
 16. $12cd^3 - 6a^3x - a^6 + 4c^2 + 9d^6 - 9x^2 = (2c + 3d^3)^2 - (a^3 + 3x)^2$
 $= (2c + 3d^3 + a^3 + 3x)(2c + 3d^3 - a^3 - 3x)$.
 17. $x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1)$.
 18. $x^5 - \frac{a^4x}{16} = x\left(x^2 + \frac{a^2}{4}\right)\left(x + \frac{a}{2}\right)\left(x - \frac{a}{2}\right)$.
 19. $x^{10} - y^{10} = (x^5 + y^5)(x^5 - y^5)$
 $= (x + y)(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.
 20. $x^8 - x^2 = x^2(x + 1)(x - 1)(x^2 + x + 1)(x^2 - x + 1)$.
 21. $x^{12} + y^{12} = (x^4 + y^4)(x^8 - x^4y^4 + y^8)$.
 22. $x^{12} - y^{12} = (x^6 + y^6)(x^6 - y^6)$
 $= (x^6 + y^6)(x^3 + y^3)(x^3 - y^3)$
 $= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.
 23. $x^{12} - 64 = (x^6 - 8)(x^6 + 8)$
 $= (x^2 - 2)(x^4 + 2x^2 + 4)(x^2 + 2)(x^4 - 2x^2 + 4)$.
 24. $x^{10} + xy^9 = x(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$.
 25. $x^{12} - 8 = (x^4 - 2)(x^8 + 2x^4 + 4)$.
 26. $x^4 - y^{16} = (x^2 + y^8)(x + y^4)(x - y^4)$.

27. $64x^{12} - 4x^4 = 4x^4(16x^8 - 1)$
 $= 4x^4(2x^2 + 1)(2x^2 - 1)(2x^2 + 2x + 1)(2x^2 - 2x + 1).$
28. $x^{12} + 64y^{12} = (x^4 + 4y^4)(x^8 - 4x^4y^4 + 16y^8)$
 $= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)(x^4 + 2x^3y + 2x^2y^2 + 4xy^3 + 4y^4)$
 $(x^4 - 2x^3y + 2x^2y^2 - 4xy^3 + 4y^4).$
29. $32x^{10} + y^{10} = (2x^2 + y^2)(16x^8 - 8x^6y^2 + 4x^4y^4 - 2x^2y^6 + y^8).$
30. $x^{16} - y^{16} = (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y).$
31. $16x^{16} - 1 = (4x^8 + 1)(2x^4 + 1)(2x^4 - 1)$
 $= (2x^4 + 2x^2 + 1)(2x^4 - 2x^2 + 1)(2x^4 + 1)(2x^4 - 1).$
32. $5^{-4x} - 1 = (5^{-2x} + 1)(5^{-2x} - 1) = (5^{-2x} + 1)(5^{-x} + 1)(5^{-x} - 1).$
33. $10 - 10c^{14}d^4 = 10(1 - c^{14}d^4) = 10(1 + c^7d^2)(1 - c^7d^2).$
34. $2cd - c^2 - d^2 = (c - d)(d - c).$
35. $x^4 - 9x^2 - x + 3 = (x - 3)(x^3 + 3x^2 - 1).$
36. $x^4 - 7x^2y^2 + 81y^4 = (x^2 + 9y^2 + 5xy)(x^2 + 9y^2 - 5xy).$
37. $4c^4 + 20c^3d - 11c^2d^2 = c^2(2c + 11d)(2c - d).$
38. $a^5 - a^3 + a - 1 = a^3(a^2 - 1) + (a - 1) = (a^4 + a^3 + 1)(a - 1).$
39. $a^5 - a^4 - a^3 + a = a^4(a - 1) - a(a + 1)(a - 1) = a(a^3 - a - 1)(a - 1).$
40. $5d^2 - 5cd - 10c^2 = 5(d - 2c)(d + c).$

$$\begin{array}{r} 41. \quad 1 - 3 + 8 - 12 \mid 2 \\ \quad \quad \quad + 2 - 2 \div 12 \\ \hline \quad \quad \quad 1 - 1 + 6 \quad \quad 0 \end{array}$$

$$\therefore x^3 - 3x^2 + 8x - 12 = (x - 2)(x^2 - x + 6).$$

42. $121x^4 - 476x^2y^2 + 100y^4 = (11x^2 - 10y^2 + 16xy)(11x^2 - 10y^2 - 16xy).$
43. $x^4 - x^2 + 12xy - 36y^2 = (x^2 + x - 6y)(x^2 - x + 6y).$
44. $y^4 - 18y^2 + 81 - 16x^4 - 24x^2y^3 - 9y^6 = (y^2 - 9)^2 - (4x^2 + 3y^3)^2$
 $= (y^2 - 9 + 4x^2 + 3y^3)(y^2 - 9 - 4x^2 - 3y^3).$
45. $h^5k^5 - 1024k^5 = k^5(h - 4)(h^4 + 4h^3 + 16h^2 + 64h + 256).$
46. $x^3 - 83x^5 + 289x^7 = x^3(1 - 17x^2 + 7x)(1 - 17x^2 - 7x).$
47. $x^2 - y^2 - x - y = (x + y)(x - y - 1).$
48. $289 - 100a^2 - b^2 - 20ab = (17 + 10a + b)(17 - 10a - b).$
49. $625a^8 - 169d^4 + 78cd^2 - 9c^2 = (25a^4 + 13d^2 - 3c)(25a^4 - 13d^2 + 3c).$
50. $x^{3n} - 125y^{6n} = (x^n - 5y^{2n})(x^{2n} + 5x^ny^{2n} + 25y^{4n}).$
51. $4x^4 - 37x^2 + 9 = (x + 3)(x - 3)(2x + 1)(2x - 1).$
52. $256 - 16k^4 + 8h^2k^2 - h^4 = (16 + 4k^2 - h^2)(16 - 4k^2 + h^2).$
53. $x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2 = (x^2 + 2 + 2x)(x^2 + 2 - 2x).$
54. $x^{12} - 729 = (x^6 - 27)(x^6 + 27)$
 $= (x^2 - 3)(x^4 + 3x^2 + 9)(x^2 + 3)(x^4 - 3x^2 + 9).$
55. $a^{4x+8} + 64 = a^{4x+8} + 16a^{2x+4} + 64 - 16a^{2x+4}$
 $= (a^{2x+4} + 8 + 4a^{x+2})(a^{2x+4} + 8 - 4a^{x+2}).$
56. $a^4 + 225a^{-4} - 39 = a^4 - 30 + 225a^{-4} - 9$
 $= (a^2 - 15a^{-2} + 3)(a^2 - 15a^{-2} - 3).$

$$57. x^3 - 6x^2 + 12x - 8 = (x - 2)^3.$$

$$58. a^2 - 9d^2 - 8ab + 6cd - c^2 + 16b^2 = (a - 4b)^2 - (3d - c)^2 \\ = (a - 4b + 3d - c)(a - 4b - 3d + c).$$

$$59. 4h^{-6} - 20h^{-3}k + 25k^2 - 6ab^{-2} - 9a^2 - b^{-4} \\ = (2h^{-3} - 5k)^2 - (3a + b^{-2})^2 \\ = (2h^{-3} - 5k + 3a + b^{-2})(2h^{-3} - 5k - 3a - b^{-2}).$$

$$60. x^2a - 2x^a - 15 = (x^a - 5)(x^a + 3).$$

$$61. a^3 + a + b^3 + b = a^3 + b^3 + a + b = (a + b)(a^2 - ab + b^2 + 1).$$

$$62. x^2a - 12x^a + 36 = (x^a - 6)^2.$$

$$63. 25x^{2c} + 50x^c - 39 = (5x^c + 13)(5x^c - 3).$$

$$64. 128 - x^{28} = (2 - x^4)(64 + 32x^4 + 16x^8 + 8x^{12} + 4x^{16} + 2x^{20} + x^{24}).$$

$$65. 3x^2 + 10x - 8 = (3x - 2)(x + 4).$$

$$66. a^5 - a^3 - a^2 + 1 = a^3(a + 1)(a - 1) - (a + 1)(a - 1) \\ = (a + 1)(a - 1)(a^3 - 1) \\ = (a + 1)(a - 1)^2(a^2 + a + 1).$$

$$67. x^3 + 3x^2 + 9x + 27 = (x + 3)(x^2 + 9).$$

$$68. x^3 - 6x^2 + 12x - 7 = x^3 - 6x^2 + 12x - 8 + 1 = [(x - 2)^3 + 1] \\ = (x - 2 + 1)[(x - 2)^2 - (x - 2) + 1] \\ = (x - 1)(x^2 - 5x + 7).$$

$$69. 4x^4 - 25y^6 + 10y^3 - 12x^2 + 8 \\ = (4x^4 - 12x^2 + 9) - (25y^6 - 10y^3 + 1) \\ = (2x^2 + 5y^3 - 4)(2x^2 - 5y^3 - 2).$$

$$70. e^{2x} - 2 + e^{-2x} = (e^x - e^{-x})^2.$$

$$71. e^{2x} - 5 + 6e^{-2x} = (e^x - 3e^{-x})(e^x - 2e^{-x}).$$

$$72. 3x^{2a} + 5x^a - 28 = (3x^a - 7)(x^a + 4).$$

$$73. e^{3x} - 2e^x - 24e^{-x} = e^{-x}(e^{2x} + 4)(e^{2x} - 6) \text{ or } (e^x + 4e^{-x})(e^{2x} - 6).$$

$$74. 6e^{2x} - 5e^{-2x} - 13 = (2e^x - 5e^{-x})(3e^x + e^{-x}).$$

$$75. \begin{array}{r} 1 + 0 - 7y^2 + 6y^3 \overline{) -3y} \\ -3y + 9y^2 - 6y^3 \\ \hline 1 - 3y + 2y^2 \quad 0 \end{array}$$

$$\therefore x^3 - 7xy^2 + 6y^3 = (x + 3y)(x^2 - 3xy + 2y^2) = (x + 3y)(x - 2y)(x - y).$$

$$76. e^{3x} + e^{2x} + e^x + 1 = (e^x + 1)(e^{2x} + 1).$$

$$77. a^{4x-2} - 10 + 25a^{2-4x} = (a^{2x-1} - 5a^{1-2x})^2.$$

$$78. e^{3x} - e^{-3x} + 3e^{-x} - 3e^x = (e^x - e^{-x})^3.$$

$$79. e^{x+3} + e^{x+2} - e^{3-x} - e^{2-x} = (e + 1)(e^{x+2} - e^{2-x}) \text{ or } \\ e^{2-x}(e + 1)(e^x + 1)(e^x - 1).$$

$$80. \text{ Let } x = -y \text{ and the expression becomes zero.}$$

$$\text{ Let } x = -z \text{ and the expression becomes zero.}$$

$$\text{ Let } y = -z \text{ and the expression becomes zero.}$$

$$\therefore (x + y)(x + z)(y + z) = xy^2 + xz^2 + x^2y + x^2z + yz^2 + y^2z + 2xyz.$$

81. Let $a = -b$ and the expression becomes zero.

Let $a = c$ and the expression becomes zero.

Let $b = -c$ and the expression becomes zero.

Let $a = b - c$ and the expression becomes zero.

$$\therefore (a + b)(c - a)(b + c)(a - b + c) = ab^3 - a^3b + ac^3 - a^3c + bc^3 - b^3c.$$

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1. $x^2 - 25 = 0.$

$$(x + 5)(x - 5) = 0.$$

$$\therefore x = 5 \text{ or } -5.$$

2. $x^2 + 10 = 7x.$

$$(x - 2)(x - 5) = 0.$$

$$\therefore x = 2 \text{ or } 5.$$

3. $y^2 - 9y = 0.$

$$y(y - 9) = 0.$$

$$\therefore y = 0 \text{ or } 9.$$

4. $r^2 - ra - 30a^2 = 0.$

$$(r - 6a)(r + 5a) = 0.$$

$$\therefore r = 6a \text{ or } -5a.$$

5. $4z^3 - 36z = 0.$

$$z(z + 3)(z - 3) = 0.$$

$$\therefore z = 0, 3, \text{ or } -3.$$

6. $x^4 - 5x^2 + 4 = 0.$

$$(x + 1)(x - 1)(x + 2)(x - 2) = 0.$$

$$\therefore x = -1, 1, -2, \text{ or } 2.$$

7. $t^6 - 13t^4 + 36t^2 = 0.$

$$t^2(t + 3)(t - 3)(t + 2)(t - 2) = 0.$$

$$\therefore t = 0, -3, 3, 2, \text{ or } -2.$$

8. $3x^2 - xb - 2b^2 = 0.$

$$(3x + 2b)(x - b) = 0.$$

$$\therefore x = -\frac{2b}{3} \text{ or } b.$$

9. $x^3 - 2x^2 - x + 2 = 0.$

$$(x - 2)(x + 1)(x - 1) = 0.$$

$$\therefore x = 2, -1, \text{ or } 1.$$

10. $x^{2x} - 2x^x + 1 = 0.$

$$(x^x - 1)^2 = 0.$$

Then $x^x = 1.$

$$\therefore x = 1.$$

11. $x^{2x} - 8x^x + 16 = 0.$

$$(x^x - 4)^2 = 0.$$

Then $x^x = 4.$

$$\therefore x = 2.$$

12. $x^3 + 5x^2 - 18x - 72 = 0.$

$$1 + 5 - 18 - 72 \mid + 4$$

$$+ 4 + 36 + 72$$

$$1 + 9 + 18 \quad 0$$

$$(x - 4)(x + 3)(x + 6) = 0.$$

$$\therefore x = 4, -3, \text{ or } -6.$$

13. $x^4 + 3x^3 - 8x^2 - 12x + 16 = 0.$

$$1 + 3 - 8 - 12 + 16 \mid - 4$$

$$- 4 + 4 + 16 - 16$$

$$1 - 1 - 4 + 4 \quad 0$$

$$1 - 1 - 4 + 4 \mid 2$$

$$+ 2 + 2 - 4$$

$$1 + 1 - 2 \quad 0$$

$$\therefore (x + 4)(x - 2)(x + 2)(x - 1) = 0.$$

$$\therefore x = -4, +2, -2, \text{ or } 1.$$

14. $x^3 - 2ax^2 - 5a^2x + 6a^3 = 0.$

$$1 - 2a - 5a^2 + 6a^3 \mid 3a$$

$$+ 3a + 3a^2 - 6a^3$$

$$1 + a - 2a^2 \quad 0$$

$$\therefore (x - 3a)(x + 2a)(x - a) = 0.$$

$$\therefore x = 3a, -2a, \text{ or } a.$$

15. $x^5 - 9x^4a - x^3 + 9ax^2 = 0.$

$$x^2(x - 9a)(x + 1)(x - 1) = 0.$$

$$\therefore x = 0, 9a, -1, \text{ or } 1.$$

16. $x^3 + 5x^2c - 16xc^2 - 80c^3 = 0.$

$$(x - 4c)(x + 4c)(x + 5c) = 0.$$

$$\therefore x = 4c, -4c, \text{ or } -5c.$$

17. Let $x =$ the less part.

Then $272 - x =$ the greater part.

Therefore $272 - x = x^2$.

Whence $(x - 16)(x + 17) = 0$,

or $x = 16, -17$. -17 is rejected.

Therefore 16 and 256 are the required parts.

18. Let $x =$ the least number.

Then the next greater even number $= x + 2$, and the greatest number $= x + 4$.

Therefore $2x^2 = x^2 + 6x + 8 + 104$.

Whence $x^2 - 6x - 112 = 0$,

or $(x - 14)(x + 8) = 0$.

Whence $x = 14, -8$. -8 is rejected.

Therefore the numbers are 14, 16, and 18.

19. Let $x =$ the number.

Therefore $x + 4\left(\frac{1}{x}\right) = 20.2$,

or $x^2 - 20.2x + 4 = 0$.

Factoring, $(x - 20)(x - .2) = 0$.

Whence $x = 20, .2$.

Therefore the number is 20 or .2.

20. Let $b =$ the number of days B requires.

Then $b + 2 =$ the number of days A requires.

Therefore $\frac{1}{b} =$ the portion of the work B does in one day,

and $\frac{1}{b + 2} =$ the portion of the work A does in one day.

Therefore $\frac{1}{b} + \frac{1}{b + 2} = \frac{8}{15}$.

Whence $4b^2 - 7b - 15 = 0$.

Factoring, $(b - 3)(4b + 5) = 0$.

Then $b = 3, -\frac{5}{4}$. $-\frac{5}{4}$ is impossible.

Therefore A and B require 5 days and 3 days respectively.

21. Let $x =$ the shorter leg.

Then $x + 8 =$ the other leg.

Therefore $x^2 + (x + 8)^2 = 1600$,

or $x^2 + 8x - 768 = 0$.

Factoring, $(x + 32)(x - 24) = 0$.

Whence $x = -32, 24$. -32 is impossible.

Therefore the sides are 40 ft., 24 ft., and 32 ft.

22. Let $x =$ the rate of the slower train.
 Then $x + 10 =$ the rate of the other train.
 Then $\frac{240}{x} = \frac{240}{x + 10} + 2$.
 Whence $x^2 + 10x - 1200 = 0$.
 Factoring, $(x + 40)(x - 30) = 0$.
 Whence $x = -40, 30$. -40 is impossible.
 Therefore the two rates are 30 mi. and 40 mi. per hour.
23. Let $a =$ the altitude in feet.
 Then the bases are $a + 2$ and $a + 8$ respectively.
 Therefore $a \frac{(a + 2) + (a + 8)}{2} = 126$.
 Whence $a^2 + 5a - 126 = 0$.
 Factoring, $(a - 9)(a + 14) = 0$.
 Whence $a = 9, -14$. -14 is impossible.
 Therefore the altitude is 9 ft. and the bases are 11 ft. and 17 ft.

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- | | | | |
|----|--|----|---|
| 1. | $28 = 2^2 \cdot 7$. | 4. | $816 = 2^4 \cdot 3 \cdot 17$. |
| | $56 = 2^3 \cdot 7$. | | $1224 = 2^3 \cdot 3^2 \cdot 17$. |
| | $84 = 2^2 \cdot 3 \cdot 7$. | | $1360 = 2^4 \cdot 5 \cdot 17$. |
| | $35 = 5 \cdot 7$. | | $4080 = 2^4 \cdot 5 \cdot 17 \cdot 3$. |
| | $\therefore \text{H.C.F.} = 7$. | | $\therefore \text{H.C.F.} = 2^3 \cdot 17 = 136$. |
| 2. | $225 = 3^2 \cdot 5^2$. | 5. | $91x^4y^3 = 13 \cdot 7x^4y^3$. |
| | $120 = 2^3 \cdot 3 \cdot 5$. | | $133x^2y^6 = 19 \cdot 7x^2y^6$. |
| | $210 = 2 \cdot 3 \cdot 5 \cdot 7$. | | $343x^5y^2 = 7^3 \cdot x^5y^2$. |
| | $135 = 3^3 \cdot 5$. | | $\therefore \text{H.C.F.} = 7x^2y^2$. |
| | $\therefore \text{H.C.F.} = 3 \cdot 5 = 15$. | | |
| 3. | $198 = 2 \cdot 3^2 \cdot 11$. | 6. | $a^2 - 9a + 14 = (a - 2)(a - 7)$. |
| | $495 = 11 \cdot 3^2 \cdot 5$. | | $a^2 - 4 = (a - 2)(a + 2)$. |
| | $693 = 11 \cdot 7 \cdot 3^2$. | | $5a^2 - 10a = 5a(a - 2)$. |
| | $1155 = 11 \cdot 3 \cdot 7 \cdot 5$. | | $\therefore \text{H.C.F.} = a - 2$. |
| | $\therefore \text{H.C.F.} = 11 \cdot 3 = 33$. | | |
| 7. | $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$. | | |
| | $2x^2 + 3x - 9 = (x + 3)(2x - 3)$. | | |
| | $5x^3 + 15x^2 = 5x^2(x + 3)$. | | |
| | $\therefore \text{H.C.F.} = x + 3$. | | |
| 8. | $2x^5 + 8x = 2x(x^2 + 2 + 2x)(x^2 + 2 - 2x)$. | | |
| | $3ax^3 + 6ax + 6ax^2 = 3ax(x^2 + 2 + 2x)$. | | |
| | $3ax^7 + 12ax^3 = 3ax^3(x^2 + 2 + 2x)(x^2 + 2 - 2x)$. | | |
| | $\therefore \text{H.C.F.} = x(x^2 + 2x + 2)$. | | |

$$9. [(x + y)(x - y)]^3 = (x + y)^3(x - y)^3.$$

$$x^4 - 2x^2y^2 + y^4 = (x + y)^2(x - y)^2.$$

$$(x^6 - y^6)^2 = (x + y)^2(x - y)^2(x^2 + xy + y^2)^2(x^2 - xy + y^2)^2.$$

$$\therefore \text{H.C.F.} = (x + y)^2(x - y)^2 = x^4 - 2x^2y^2 + y^4.$$

$$10. x^7 + a^7e = (x + ae)(x^6 - x^5ae + x^4a^2e - x^3a^3e + x^2a^4e - xa^5e + a^6e).$$

$$x^2 - a^2e = (x + ae)(x - ae).$$

$$x^2 - 3xa^e - 4a^2e = (x + ae)(x - 4ae).$$

$$x^3 + a^3e = (x + ae)(x^2 - xa^e + a^2e).$$

$$\therefore \text{H.C.F.} = x + ae.$$

$$11. a^{5e} + 4a^eb^8 = a^e(a^{2e} + 2b^4 + 2a^eb^2)(a^{2e} + 2b^4 - 2a^eb^2).$$

$$a^{2e+1}b^2 + 2ab^6 - 2a^{e+1}b^4 = ab^2(a^{2e} + 2b^4 - 2a^eb^2).$$

$$a^8e - 16b^{16} = (a^{2e} + 2b^4 + 2a^eb^2)(a^{2e} + 2b^4 - 2a^eb^2)(a^{2e} + 2b^4)(a^{2e} - 2b^4).$$

$$\therefore \text{H.C.F.} = a^{2e} + 2b^4 - 2a^eb^2.$$

$$12. 24 = 2^3 \cdot 3.$$

$$30 = 2 \cdot 3 \cdot 5.$$

$$54 = 2 \cdot 3^3.$$

$$\therefore \text{L.C.M.} = 2^3 \cdot 3^3 \cdot 5 = 1080.$$

$$13. 105 = 3 \cdot 5 \cdot 7.$$

$$140 = 2^2 \cdot 5 \cdot 7.$$

$$245 = 5 \cdot 7^2.$$

$$\therefore \text{L.C.M.} = 2^2 \cdot 3 \cdot 5 \cdot 7^2 = 2940.$$

$$14. 174 = 2 \cdot 3 \cdot 29.$$

$$485 = 5 \cdot 97.$$

$$4611 = 3 \cdot 29 \cdot 53.$$

$$5141 = 97 \cdot 53.$$

$$\therefore \text{L.C.M.} = 2 \cdot 3 \cdot 29 \cdot 5 \cdot 97 \cdot 53 = 4,472,670.$$

$$15. 30ax^2 = 2 \cdot 3 \cdot 5ax^2.$$

$$225a^5xy^2 = 3^2 \cdot 5^2 \cdot a^5xy^2.$$

$$75a^4x^3y = 3 \cdot 5^2 \cdot a^4x^3y.$$

$$\therefore \text{L.C.M.} = 2 \cdot 3^2 \cdot 5^2a^5x^3y^2 = 450a^5x^3y^2.$$

$$16. 12x^2 + 6x = 6x(2x + 1).$$

$$12x^3 - 3x = 3x(2x + 1)(2x - 1).$$

$$16x^4 + 2x = 2x(2x + 1)(4x^2 - 2x + 1).$$

$$\therefore \text{L.C.M.} = 6x(2x + 1)(2x - 1)(4x^2 - 2x + 1).$$

$$17. a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2).$$

$$4b^2 - a^2 = (2b + a)(2b - a).$$

$$a^3b + 4ab^3 + 2a^2b^2 = ab(a^2 + 4b^2 + 2ab).$$

$$\therefore \text{L.C.M.} = ab(2b + a)(2b - a)(a^2 + 4b^2 + 2ab).$$

$$18. x^3 - 2x^2 - 5x + 6 = (x + 2)(x - 1)(x - 3).$$

$$4 - x^2 = (2 + x)(2 - x).$$

$$a - ax^2 = a(1 + x)(1 - x).$$

$$\therefore \text{L.C.M.} = a(x + 1)(x + 2)(x - 3)(2 - x)(x - 1).$$

19. $a^4 + 4a^2 + 16 = (a^2 + 4 + 2a)(a^2 + 4 - 2a)$.
 $a^2 - 4 = (a + 2)(a - 2)$.
 $a^3 + 8 = (a + 2)(a^2 - 2a + 4)$.
 $a^3 - 8 = (a - 2)(a^2 + 2a + 4)$.
 $\therefore \text{L.C.M.} = (a^2 + 2a + 4)(a^2 - 2a + 4)(a + 2)(a - 2)$.
20. $x^3 - 2a^2x + ax^2 = x(x + 2a)(x - a)$.
 $2a^3 + 3a^2x + ax^2 = a(x + 2a)(x + a)$.
 $4a^4x^2 - a^2x^4 = a^2x^2(2a + x)(2a - x)$.
 $\therefore \text{L.C.M.} = a^2x^2(x - a)(x + 2a)(2a - x)(x + a)$.
21. $m^4 - 3m^2n^2 + 9n^4 = (m^2 + 3n^2 + 3mn)(m^2 + 3n^2 - 3mn)$.
 $m^3 + 3mn^2 + 3m^2n = m(m^2 + 3n^2 + 3mn)$.
 $3n^3 + m^2n - 3mn^2 = n(3n^2 + m^2 - 3mn)$.
 $\therefore \text{L.C.M.} = mn(m^2 + 3n^2 + 3mn)(m^2 + 3n^2 - 3mn)$.
22. $e^{2x} + e^{-2x} - 2 = (e^x - e^{-x})^2$.
 $e^{2x} - e^{-2x} = (e^x + e^{-x})(e^x - e^{-x})$.
 $e^{2x} - 3 + 2e^{-2x} = (e^x - e^{-x})(e^x - 2e^{-x})$.
 $\therefore \text{L.C.M.} = (e^x - e^{-x})^2(e^x + e^{-x})(e^x - 2e^{-x})$.
23. $x^3 - 2x^2 - 2x - 3 = (x - 3)(x^2 + x + 1)$.
 $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$.
 $x^2 + x - 12 = (x - 3)(x + 4)$.
 $\therefore \text{L.C.M.} = (x - 3)(x + 4)(x^2 + x + 1)(x^2 + 3x + 9)$.

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1. $\frac{18a^3c^2}{24a^2c^3} = \frac{3a}{4c}$.
2. $\frac{3x - 6}{x^2 - 4} = \frac{3(x - 2)}{(x + 2)(x - 2)} = \frac{3}{x + 2}$.
3. $\frac{7a^2 - 14b^2}{a^4 + a^2b^2 - 6b^4} = \frac{7(a^2 - 2b^2)}{(a^2 - 2b^2)(a^2 + 3b^2)} = \frac{7}{a^2 + 3b^2}$.
4. $\frac{x^3 - 8}{(x - 2)^3} = \frac{(x - 2)(x^2 + 2x + 4)}{(x - 2)^3} = \frac{x^2 + 2x + 4}{(x - 2)^2}$.
5. $\frac{x^{2a} - c^{2b}}{(x^a + c^b)^2} = \frac{(x^a + c^b)(x^a - c^b)}{(x^a + c^b)^2} = \frac{x^a - c^b}{x^a + c^b}$.
6. $\frac{(x^2 - c^2)^2}{x^4 - c^4} = \frac{(x + c)^2(x - c)^2}{(x^2 + c^2)(x + c)(x - c)} = \frac{x^2 - c^2}{x^2 + c^2}$.
7. $\frac{2x^{6e} - 128}{x^{2e} - 4} = \frac{2(x^e + 2)(x^{2e} - 2x^e + 4)(x^e - 2)(x^{2e} + 2x^e + 4)}{(x^e + 2)(x^e - 2)}$
 $= 2x^{4e} + 8x^{2e} + 32$.
8. $\frac{x^6 - 1}{x^4 + x^2 + 1} = \frac{(x^2 - 1)(x^4 + x^2 + 1)}{x^4 + x^2 + 1} = x^2 - 1$.
9. $\frac{3x^2 + 2x - 21}{27x^4 - 147x^2} = \frac{(3x - 7)(x + 3)}{3x^2(3x - 7)(3x + 7)} = \frac{x + 3}{9x^3 + 21x^2}$.

$$10. \frac{x^4 - x^2 + x - 1}{x^4 - 1} = \frac{(x-1)(x^3 + x^2 + 1)}{(x^2 + 1)(x+1)(x-1)} = \frac{x^3 + x^2 + 1}{(x^2 + 1)(x+1)}.$$

$$11. -\frac{c}{-2a} = \frac{c}{2a} = \frac{-c}{-2a} = -\frac{-c}{2a}.$$

$$12. \frac{c}{a-2c} = \frac{-c}{2c-a} = -\frac{-c}{a-2c} = -\frac{c}{2c-a}.$$

$$13. \frac{2x-3y}{3x^2-6y^2-xy} = \frac{-2x+3y}{-3x^2+6y^2+xy}$$

$$= -\frac{-2x+3y}{3x^2-6y^2-xy}$$

$$= -\frac{2x-3y}{-3x^2+6y^2+xy}.$$

$$14. \frac{-3}{(c-a)(b-a)} = \frac{-3}{(a-c)(a-b)}.$$

$$15. \frac{x-y}{(y-x)(z-x)(z-y)} = \frac{y-x}{(x-y)(x-z)(y-z)}.$$

$$16. \frac{x-3}{5-x-x^2} \neq \frac{x+3}{x^2+x-5} \text{ for } x+3 \neq -(x-3).$$

$$17. \frac{7}{24}, \frac{9}{56} \text{ equal } \frac{49}{168}, \frac{27}{168} \text{ respectively.}$$

$$18. \frac{4}{ab}, \frac{3}{a^2c} \text{ equal } \frac{4ac}{a^2bc}, \frac{3b}{a^2bc} \text{ respectively.}$$

$$19. \frac{3a+b}{6a}, \frac{a-2b}{4ab} \text{ equal } \frac{6ab+2b^2}{12ab}, \frac{3a-6b}{12ab} \text{ respectively.}$$

$$20. \frac{2}{6x-12}, \frac{3}{x^2-2x} \text{ equal } \frac{x}{3x^2-6x}, \frac{9}{3x^2-6x} \text{ respectively.}$$

$$21. \frac{x-1}{x^2-5x+6}, \frac{x}{x^2-9} \text{ equal } \frac{x^2+2x-3}{(x-3)(x+3)(x-2)},$$

$$\frac{x^2-2x}{(x-3)(x-2)(x+3)} \text{ respectively.}$$

22. No. The subtraction of equal numbers from both numerator and denominator of a fraction changes the value of the fraction. See pages 135-136.

23. No. The term may be *subtracted* from each member of the equation, but not *canceled*. Cancellation means division.

$$24. \frac{3a}{6} - \frac{a-x}{9} - \frac{3x}{4} = \frac{18a - (4a - 4x) - 27x}{36} = \frac{14a - 23x}{36}.$$

$$25. \frac{x-3}{4x^2} - \frac{3x^2-2}{14x^3} - \frac{x^3-8}{3x^4} = \frac{21x^3 - 63x^2 - (18x^3 - 12x) - (28x^3 - 224)}{84x^4}$$

$$= \frac{-25x^3 - 63x^2 + 12x + 224}{84x^4}.$$

$$26. \frac{5-x}{x-4} - \frac{5}{7} = \frac{35-7x-(5x-20)}{7(x-4)} = \frac{55-12x}{7x-28}.$$

$$27. \frac{4m^2}{x^2-9m^2} - \frac{3m-x}{x+3m} = \frac{4m^2-(3m-x)(x-3m)}{x^2-9m^2} = \frac{x^2-6mx+13m^2}{x^2-9m^2}.$$

$$28. \frac{x-3}{x^2-9x+14} - \frac{3}{x^2-4} = \frac{(x-3)(x+2)-3(x-7)}{(x-2)(x-7)(x+2)} \\ = \frac{x^2-4x+15}{(x-2)(x-7)(x+2)}.$$

$$29. \frac{x}{x^2-2x} - \frac{1}{x} - \frac{2+x}{x^2-4x+4} = \frac{x(x-2)-(x-2)^2-x(2+x)}{x(x-2)^2} \\ = \frac{-x^2-4}{x(x-2)^2}.$$

$$30. 2 - \frac{1}{v-1} + \frac{1}{1-v} = 2 + \frac{1}{1-v} + \frac{1}{1-v} = 2 + \frac{2}{1-v} = \frac{4-2v}{1-v}.$$

$$31. x^2+x+1 - \frac{x^3}{x-1} = \frac{(x^2+x+1)(x-1)-x^3}{x-1} = \frac{1}{1-x}.$$

$$32. x^2 - \frac{x^4-2x^2}{x^2-x+1} + x+1 = \frac{(x^2+x+1)(x^2-x+1)-(x^4-2x^2)}{x^2-x+1} \\ = \frac{3x^2+1}{x^2-x+1}.$$

$$33. \frac{2x^2-3a^2}{27x^3-64a^3} - \frac{2x+a}{9x^2-16a^2} \\ = \frac{(2x^2-3a^2)(3x+4a)-(2x+a)(9x^2+12ax+16a^2)}{(3x-4a)(9x^2+12ax+16a^2)(3x+4a)} \\ = \frac{-12x^3-53a^2x-25ax^2-28a^3}{(27x^3-64a^3)(3x+4a)}.$$

$$34. \frac{2}{(a-c)(x-c)} - \frac{3}{(c-a)(c-x)} = \frac{-2+3}{(a-c)(c-x)} = \frac{1}{(a-c)(c-x)}.$$

$$35. \frac{2x-4}{9-x^2} - \frac{3x+1}{x-3} - \frac{x}{9-6x+x^2} = \frac{4-2x}{x^2-9} - \frac{3x+1}{x-3} - \frac{x}{x^2-6x+9} \\ = \frac{(4-2x)(x-3)-(3x+1)(x+3)(x-3)-x(x+3)}{(x+3)(x-3)^2} \\ = \frac{-3x^3-4x^2+34x-3}{(x+3)(x-3)^2}.$$

$$36. \frac{2x+12c}{6x^2-13cx-5c^2} - \frac{3x-7c}{4x^2+4cx-35c^2} \\ = \frac{(2x+12c)(2x+7c)-(3x-7c)(3x+c)}{(2x-5c)(3x+c)(2x+7c)} \\ = \frac{-5x^2+56cx+91c^2}{(2x-5c)(3x+c)(2x+7c)}.$$

37. $\frac{(a-s)(c-s)}{s(s-m)} + \frac{ac}{ms} + \frac{(a-m)(c-m)}{m(m-s)}$
 $= \frac{(s-a)(c-s)m + ac(m-s) + s(a-m)(c-m)}{ms(m-s)}$
 $= 1.$
38. $\frac{2x^3 - x^2}{x^2 - x + 1} - \frac{x^2 - 3}{x^4 + x^2 + 1} + \frac{3x - 5}{x^3 + 1}$
 $= \frac{(2x^3 - x^2)(x+1)(x^2 + x + 1) - (x^2 - 3)(x+1) + (3x - 5)(x^2 + x + 1)}{(x^2 - x + 1)(x^2 + x + 1)(x+1)}$
 $= \frac{2x^6 + 3x^5 + 2x^4 + 2x^3 - 4x^2 + x - 2}{(x^4 + x^2 + 1)(x+1)}.$
39. $\frac{e^{3x} - e^{-3x}}{e^x - e^{-x}} = \frac{e^{3x} - \frac{1}{e^{3x}}}{e^x - \frac{1}{e^x}} = \frac{e^{6x} - 1}{e^{2x}(e^{2x} - 1)} = \frac{4^3 - 1}{4(4-1)} = 5\frac{1}{4}, \text{ when } e^{2x} = 4.$
40. $c(c-y) \div d - (y+d)d \div c = \frac{c(c-c+d)}{d} - \frac{(c-d+d)d}{c}$
 $= c-d, \text{ when } y = c-d.$
41. $\frac{a}{b} - \frac{c}{d} = \frac{x+2}{x^2+2x+4} - \frac{x-2}{x^2-2x+4}$
 $= \frac{(x+2)(x^2-2x+4) - (x-2)(x^2+2x+4)}{(x^2-2x+4)(x^2+2x+4)} = \frac{16}{x^4+4x^2+16},$
 when $a = x+2$, $b = x^2+2x+4$, $c = x-2$, and $d = x^2-2x+4$.

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1. $\frac{4x^2c^2}{9a^3} \cdot \frac{27a^2}{10x^3c} = \frac{6c}{5ax}.$
2. $\left(\frac{2a}{cx}\right)^2 \cdot \left(\frac{3c^2x}{4a^2}\right)^3 = \frac{4a^2}{c^2x^2} \cdot \frac{27c^6x^3}{64a^6} = \frac{27c^4x}{16a^4}.$
3. $\left(\frac{a}{6c^2}\right) \cdot \left(\frac{3a}{c}\right)^2 \cdot \left(\frac{2c^2}{3a^4}\right)^2 = \frac{a}{6c^2} \cdot \frac{9a^2}{c^2} \cdot \frac{4c^4}{9a^8} = \frac{2}{3a^5}.$
4. $\left(\frac{-a}{x}\right)^3 \cdot \left(\frac{-2x}{a}\right)^2 \cdot \left(\frac{-c}{2a}\right)^4 = \frac{-a^3}{x^3} \cdot \frac{4x^2}{a^2} \cdot \frac{c^4}{16a^4} = \frac{-c^4}{4a^3x}.$
5. $\left(\frac{3c^2}{2a}\right)^2 \div \left(\frac{9c^2x^3}{4a^2}\right)^2 \div \left(\frac{-2a^3c^2}{5x^3}\right)^3 \frac{(3a^2c)^4}{(10x^3)^2}$
 $= \frac{9c^4}{4a^2} \cdot \frac{16a^4}{81c^4x^6} \cdot \frac{125x^9}{(-8a^9c^6)} \cdot \frac{81a^8c^4}{100x^6} = -\frac{45a}{8c^2x^3}.$
6. $\frac{4x^e - 2y^e}{(2x^e - y^e)^2} \cdot \frac{4x^{2e} - y^{2e}}{(2x^e + y^e)^2} \cdot \frac{2x^e + y^e}{3}$
 $= \frac{2(2x^e - y^e)}{(2x^e - y^e)^2} \cdot \frac{(2x^e + y^e)(2x^e - y^e)}{(2x^e + y^e)^2} \cdot \frac{2x^e + y^e}{3} = \frac{2}{3}.$

7. $\frac{x^2-1}{x^4-1} \left(1 - \frac{2}{x^6+1}\right) \frac{x^4-x^2+1}{x^2+x+1}$
 $= \frac{x^2-1}{(x^2+1)(x^2-1)} \cdot \frac{(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{(x^2+1)(x^4-x^2+1)} \cdot \frac{x^4-x^2+1}{x^2+x+1}$
 $= \frac{x^4-x^3+x-1}{(x^2+1)^2}.$
8. $\left(x^2-2x+4 - \frac{16}{x+2}\right) \frac{x^2-4}{3} = \frac{x^3-8}{x+2} \cdot \frac{x^2-4}{3} = \frac{x^4-2x^3-8x+16}{3}.$
9. $\frac{a^2-a-90}{a^2-100} \div \frac{a^3+9a^2}{a^2+10a} \div \frac{4a+6}{2a^2x+3ax}$
 $= \frac{(a-10)(a+9)}{(a+10)(a-10)} \cdot \frac{a(a+10)}{a^2(a+9)} \cdot \frac{ax(2a+3)}{2(2a+3)} = \frac{x}{2}.$
10. $\left(a - \frac{13a}{a+6}\right) \div \frac{a^2-a-42}{a^2-36} \div \frac{2a^3-12a}{ax+6x}$
 $= \frac{a(a-7)}{a+6} \cdot \frac{(a-6)(a+6)}{(a-7)(a+6)} \cdot \frac{x(a+6)}{2a(a^2-6)} = \frac{ax-6x}{2a^2-12}.$
11. $\left(\frac{3a^4-75a^2}{3a-7}\right) \left(6a - \frac{7}{a} - 11\right) \div \left(2 + \frac{5}{a^2} + \frac{11}{a}\right)$
 $= \frac{3a^2(a+5)(a-5)}{3a-7} \cdot \frac{(3a-7)(2a+1)}{a} \cdot \frac{a^2}{(2a+1)(a+5)}$
 $= 3a^4 - 15a^3.$
12. $\left(\frac{4c}{a} - \frac{15c^2}{a^2} + 4\right) \left(3 - \frac{4a+20c}{2a+5c}\right) \div \left(4 - \frac{16c}{a} + \frac{15c^2}{a^2}\right)$
 $= \frac{(2a+5c)(2a-3c)}{a^2} \cdot \frac{2a-5c}{2a+5c} \cdot \frac{a^2}{(2a-5c)(2a-3c)} = 1.$
13. $\frac{1}{mx} \left(\frac{m}{x} + \frac{x}{m}\right) \div \left(\frac{m^6+x^6}{m^3x^3}\right) \left(m^2-x^2 + \frac{x^4}{m^2}\right) \left(\frac{am+mx}{cx-ax}\right)$
 $= \frac{1}{mx} \cdot \frac{m^2+x^2}{mx} \cdot \frac{m^3x^3}{(m^2+x^2)(m^4-m^2x^2+x^4)} \cdot \frac{m^4-m^2x^2+x^4}{m^2} \cdot \frac{m(a+x)}{x(c-a)}$
 $= \frac{a+x}{c-a}.$
14. $\frac{x^3-8}{x^4+4x^2+16} \cdot \frac{x^2-4}{(x-2)^2} \cdot \left(1 \div \frac{(x+2)^2}{x^3+8}\right)$
 $= \frac{x^3-8}{(x^2+4+2x)(x^2+4-2x)} \cdot \frac{x^2-4}{(x-2)(x-2)} \cdot \frac{x^3+8}{(x+2)^2} = 1.$
15. $\frac{2x^2+5x+3}{6x^2+x-1} \cdot \frac{3x^2+3ax-x-a}{2x^2-2cx+3x-3c} \cdot \left(\frac{c+2a+(2a+1+2c)x}{x^2+x+ax+a} - 2\right)$
 $= \frac{(2x+3)(x+1)}{(3x-1)(2x+1)} \cdot \frac{(x+a)(3x-1)}{(x-c)(2x+3)} \cdot \frac{(c-x)(2x+1)}{(x+a)(x+1)} = -1.$

$$16. \left(\frac{2x^3}{c^2}\right)^5 - 5\left(\frac{2x^3}{c^2}\right)^4 \left(\frac{c^3}{15x^4}\right) + 10\left(\frac{2x^3}{c^2}\right)^3 \left(\frac{c^3}{15x^4}\right)^2 - 10\left(\frac{2x^3}{c^2}\right)^2 \left(\frac{c^3}{15x^4}\right)^3$$

$$= \frac{32x^{15}}{c^{10}} - \frac{16x^8}{3c^5} + \frac{16x}{45} - \frac{8c^5}{675x^6}.$$

$$17. \left(\frac{a^2}{b^3}\right)^{10} - 10\left(\frac{a^2}{b^3}\right)^9 \left(\frac{b^2}{6a^4}\right) + \frac{10 \cdot 9}{2} \left(\frac{a^2}{b^3}\right)^8 \left(\frac{b^2}{6a^4}\right)^2 - \frac{10 \cdot 9 \cdot 8}{2 \cdot 3} \left(\frac{a^2}{b^3}\right)^7 \left(\frac{b^2}{6a^4}\right)^3$$

$$= \frac{a^{20}}{b^{30}} - \frac{5a^{14}}{3b^{25}} + \frac{5a^8}{4b^{20}} - \frac{5a^2}{9b^{15}}.$$

13. For $1 \div \frac{2}{5}$ we may write $\frac{5}{5} \div \frac{2}{5}$, which equals $\frac{5}{2}$. Similarly, $1 \div \frac{c}{a} = \frac{a}{a} \div \frac{c}{a} = \frac{a}{c}$. Hence the number of times a fraction is contained in the number 1 is the reciprocal of the fraction. In the number n a number will be contained n times as often as in the number 1. Therefore $n \div \frac{c}{a} = n \times \frac{a}{c} = \frac{na}{c}$.

19. For $\frac{a}{b} \div \frac{c}{d}$ we may (using the principle of § 28) write $\frac{ad}{bd} \div \frac{bc}{bd} = \frac{ad}{bc}$;

that is, the result of dividing one fraction by another is a fraction whose numerator is the product of the numerator of the dividend and the denominator of the divisor, and whose denominator is the product of the denominator of the dividend and the numerator of the divisor.

Hence $\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \times \frac{d}{b} = \frac{ad}{bc}$ gives in abbreviated form the essentials, the reduction of divisor and dividend to similar fractions being unnecessary except for the purpose of explanation.

$$20. \frac{8 - \frac{1}{27}}{2 - \frac{1}{3}} = \frac{215}{27} \cdot \frac{3}{5} = 4\frac{7}{9}. \quad 21. \frac{7\frac{3}{7}}{40\frac{5}{9}} \cdot \frac{146}{17\frac{1}{3}} = \frac{52}{7} \cdot \frac{9}{365} \cdot 146 \cdot \frac{3}{52} = \frac{54}{35}.$$

$$22. \frac{3\frac{1}{2} \cdot 3\frac{1}{2} \cdot 3\frac{1}{2} - 1}{3\frac{1}{2} \cdot 3\frac{1}{2} - 1} = \frac{335}{8} \cdot \frac{4}{45} = \frac{67}{18}.$$

$$23. \frac{\frac{a}{x} + 3}{\frac{a^2}{x^2} - 9} = \frac{a + 3x^2}{x} \cdot \frac{x^2}{a^2 - 9x^2} = \frac{x}{a - 3x}.$$

$$24. \frac{1 - \frac{m}{x}}{\frac{4m^4}{x^4} - 4} = \frac{x - m}{x} \cdot \frac{x^4}{4m^4 - 4x^4} = \frac{-x^3}{4(m^3 + m^2x + mx^2 + x^3)}.$$

$$25. \frac{2a - \frac{a^2 + c^2}{c}}{\frac{a}{c} - 1} = \frac{2ac - a^2 - c^2}{c} \cdot \frac{c}{a - c} = c - a.$$

$$26. \frac{3 - \frac{a}{c}}{\frac{(a+3c)^2}{2ac} - 6} = \frac{3c-a}{c} \cdot \frac{2ac}{a^2-6ac+9c^2} = \frac{2a}{3c-a}.$$

$$27. \frac{c - \frac{4}{c}}{\frac{1}{c^2} + \frac{2}{c^3} - \frac{8}{c^4}} = \frac{c^2-4}{c} \cdot \frac{c^4}{c^2+2c-8} \\ = \frac{(c+2)(c-2)}{c} \cdot \frac{c^4}{(c+4)(c-2)} = \frac{c^4+2c^3}{c+4}.$$

$$28. \frac{\left(\frac{4c+3a}{3a}\right)^2 - \frac{6c}{a}}{\frac{(3a+2c)^2}{3a} - 8c} = \frac{16c^2-30ac+9a^2}{9a^2} \cdot \frac{3a}{9a^2-12ac+4c^2} \\ = \frac{(2c-3a)(8c-3a)}{9a^2} \cdot \frac{3a}{(3a-2c)^2} = \frac{8c-3a}{6ac-9a^2}.$$

$$29. 1 + \frac{1}{1 + \frac{1}{1+2}} = 1 + \frac{1}{\frac{4}{3}} = 1 + \frac{3}{4} = \frac{7}{4}.$$

$$30. 1 - \frac{2}{3 + \frac{4}{5 - \frac{6}{7}}} = 1 - \frac{2}{3 + \frac{28}{29}} = 1 - \frac{2}{\frac{115}{29}} = 1 - \frac{58}{115} = \frac{57}{115}.$$

$$31. \frac{\frac{m^2}{n^2} + \frac{mn}{mn} + \frac{n^2}{m^2}}{\frac{m^2+n^2}{mn} - 1} = \frac{m^4+m^2n^2+n^4}{m^2n^2} \cdot \frac{mn}{m^2+n^2-mn} = \frac{m^2+mn+n^2}{mn}.$$

$$32. \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} = \frac{1}{1 - \frac{1}{\frac{x-1}{x}}} = \frac{1}{1 - \frac{x}{x-1}} = \frac{1}{\frac{-1}{x-1}} = 1 - x.$$

$$33. 3 - \frac{1}{1 - \frac{4}{6 + \frac{3}{7 - \frac{2}{5}}}} = 3 - \frac{1}{1 - \frac{4}{\frac{71}{11}}} = 3 - \frac{1}{\frac{27}{11}} = \frac{10}{27}.$$

$$34. \frac{6x-11 - \frac{7}{x}}{2 + \frac{11}{x} + \frac{5}{x^2}} \div \frac{\frac{5-x}{3x}}{x^2-25} = \frac{6x^2-11x-7}{x} \cdot \frac{x^2}{2x^2+11x+5} \div \frac{5-x}{3x(x^2-25)} \\ = \frac{(3x-7)(2x+1)}{x} \cdot \frac{x^2}{(2x+1)(x+5)} \cdot \frac{3x(x-5)(x+5)}{5-x} \\ = 21x^2 - 9x^3.$$

$$35. \frac{1 - \frac{a^2 b^2}{(a^2 + b^2)^2}}{\frac{ab}{a^2 + b^2} + 1} = \frac{a^4 + a^2 b^2 + b^4}{(a^2 + b^2)^2} \cdot \frac{a^2 + b^2}{a^2 + ab + b^2} = \frac{a^2 - ab + b^2}{a^2 + b^2}.$$

$$36. \frac{1 + \frac{y}{x} + \frac{x}{y}}{\frac{x^2}{y^2} + 1 + \frac{y^2}{x^2}} = \frac{xy + y^2 + x^2}{xy} \cdot \frac{x^2 y^2}{x^4 + x^2 y^2 + y^4} = \frac{xy}{x^2 - xy + y^2}.$$

$$37. \frac{\frac{1}{x} - \frac{x-a}{x^2 - a^2}}{\frac{1}{a} - \frac{a-x}{a^2 + x^2}} = \frac{x^2 - a^2 - x^2 + ax}{(x+a)(x-a)x} \div \frac{a^2 + x^2 - a^2 + ax}{a(a^2 + x^2)}$$

$$= \frac{a(x-a)}{(x+a)(x-a)x} \cdot \frac{a(a^2 + x^2)}{x(a+x)}$$

$$= \frac{a^2(a^2 + x^2)}{x^2(a+x)^2}.$$

$$38. \frac{\frac{xy+1}{y}}{x + \frac{1}{\frac{yz+1}{z}}} - \frac{1}{y(x+xyz+z)} = \frac{xy+1}{y} \cdot \frac{yz+1}{xyz+x+z} - \frac{1}{y(x+xyz+z)}$$

$$= \frac{xy^2z + xy + yz + 1 - 1}{y(xyz+x+z)} = 1.$$

$$39. \frac{1}{1 + \frac{a}{a+1 + \frac{2a^2}{1-a}}} = \frac{1}{1 + \frac{a-a^2}{1+a^2}} = \frac{1}{\frac{1+a}{1+a^2}} = \frac{a^2+1}{a+1}.$$

$$40. \frac{a^3 - b^3}{a^2 + b^2} = \frac{(e^x - e^{-x})^3 - (e^x + e^{-x})^3}{(e^x - e^{-x})^2 + (e^x + e^{-x})^2}$$

$$= \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x} - e^{3x} - 3e^x - 3e^{-x} - e^{-3x}}{e^{2x} - 2 + e^{-2x} + e^{2x} + 2 + e^{-2x}}$$

$$= \frac{-6e^x - 2e^{-3x}}{2e^{2x} + 2e^{-2x}} = -\frac{3e^x + e^{-3x}}{e^{2x} + e^{-2x}}$$

$$= \frac{-3e^{4x} - 1}{e^{5x} + e^x}, \text{ if } a = e^x - \frac{1}{e^x}, \text{ and } b = e^x + \frac{1}{e^x}.$$

41. The fraction continues thus:

$$(a) \ 2 + \frac{81}{2 + \frac{121}{2 + \frac{169}{2 + \frac{225}{2 + \dots}}}}$$

$$\begin{aligned}
 (b) \quad & \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{25}{2 + \left[2 + \frac{49}{\left[2 + \frac{8.5}{2}\right]}}}\right]}} = \frac{1}{1 + \frac{1}{2 + \frac{9}{\left[2 + \frac{25}{\left[2 + \frac{2.68}{8.5}\right]}}}\right]} \\
 & = \frac{1}{1 + \left[\frac{1}{2 + \frac{9}{\frac{2.661}{2.68}}}\right]} = \frac{1}{1 + \frac{2.661}{77.34}} = \frac{7734}{10395} \\
 & \frac{7734}{10395} \cdot 4 = 2.9760 +. \quad 3.1416 - 2.9760 + = .1655.
 \end{aligned}$$

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1. See pages 33, 43, 34, 44, 309, 310, 30.

2. (a) $3 \cdot 2 + 16 = 24 - 2$.
 (b) $(a - 2b)^2 = a^2 - 4ab + 4b^2$.
 (c) $4x - 3 = 18$.
 (d) $\frac{2}{x} - \frac{3}{x+2} = 1$, $x^2 + 3x = 4$.
 (e) $x - 5 = x + 5$. See also pages 161 and 377.

3. See pages 44-46.

- (a) The subtraction axiom.
 (b) Yes. See page 309, § 123, Principle I.

4. (a) $x = -4$. (d) $x = 0$.

(b) None. (e) None.
 (c) $x = c$. (f) $x = -2, -1$.

5. (a) $x = 2$. (d) $x = 0$.
 (b) $x = 3$. (e) $x = +2$ and -2 .
 (c) $x = -4$. (f) None.

$$6. \quad \frac{5x}{21} - \frac{a}{6} = \frac{x}{14}. \quad (1)$$

$$(1) \cdot 42, 10x - 7a = 3x.$$

$$x = a.$$

$$7. \quad \frac{3}{2x} - \frac{3}{20} = \frac{7}{5x}. \quad (1)$$

$$(1) \cdot 20x, 30 - 3x = 28.$$

$$x = \frac{2}{3}.$$

$$8. \quad \frac{m-2}{m-3} = \frac{17}{18}. \quad (1)$$

$$(1) \cdot 18(m-3), \quad 18m - 36 = 17m - 51.$$

$$m = -15.$$

$$9. \quad \frac{2x+5}{10x} - \frac{3(2x+1)}{2x} = 3\frac{1}{5}. \quad (1)$$

$$(1) \cdot 10x, \quad 2x + 5 - 30x - 15 = 32x.$$

$$x = -\frac{1}{6}.$$

$$10. \quad \frac{5x-7}{6} - \frac{5}{2} \left(\frac{4-x}{10} \right) = \frac{15x-22}{6}. \quad (1)$$

$$(1) \cdot 60, \quad 50x - 70 - 60 + 15x = 150x - 220.$$

$$-85x = -90, x = \frac{18}{17}.$$

$$11. \quad \frac{1-8x}{5} - \frac{2(1-6x)}{24x-3} = \frac{-24x+1}{15}. \quad (1)$$

$$(1) \cdot 15(8x-1), \quad -3 + 48x - 192x^2 - 10 + 60x \\ = -192x^2 + 32x - 1. \\ 76x = 12, \quad x = \frac{3}{19}.$$

$$12. \quad \frac{3x-9}{3x-5} - 2 = \frac{3x-5}{8-3x}.$$

$$(1) \cdot (3x-5)(8-3x), \quad -9x^2 + 51x - 72 + 18x^2 - 78x + 80 \\ = 9x^2 - 30x + 25. \\ 3x = 17, \quad x = \frac{17}{3}.$$

$$13. \quad \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{x}. \quad (1)$$

$$(1) \cdot abcx, \quad bcx + acx - abx = abc. \\ (bc + ac - ab)x = abc. \\ x = \frac{abc}{bc + ac - ab}.$$

$$14. \quad \frac{x}{a} + \frac{x+m}{b} = 1. \quad (1)$$

$$(1) \cdot ab, \quad bx + ax + am = ab. \\ (b+a)x = ab - am. \\ x = \frac{ab - am}{a+b}.$$

$$15. \quad \frac{cy}{2d} - c^2 = \frac{2dy - c^3}{c}. \quad (1)$$

$$(1) \cdot 2cd, \quad c^2y - 2c^3d = 4d^2y - 2c^3d. \\ (c^2 - 4d^2)y = 0. \\ y = 0.$$

$$16. \quad \frac{2x-3b}{a} - \frac{2a-3x}{b} + \frac{9b}{a} = \frac{4a}{b} - 5. \quad (1)$$

$$(1) \cdot ab, \quad 2bx - 3b^2 - 2a^2 + 3ax + 9b^2 = 4a^2 - 5ab. \\ (2b+3a)x = 6a^2 - 5ab - 6b^2. \\ x = 2a - 3b.$$

$$17. \quad \frac{3}{b(a-x)} + \frac{1}{a-x} = \frac{-3(b+3)}{2ab}. \quad (1)$$

$$(1) \cdot 2ab(a-x), \quad 6a + 2ab = -3ab + 3bx - 9a + 9x. \\ 15a + 5ab = x(3b+9). \\ \frac{5a}{3} = x.$$

$$18. \quad \frac{3}{2y+1} + \frac{1+2y}{-2y+1} = \frac{4y^2}{1-4y^2}. \quad (1)$$

$$(1) \cdot (1-4y^2), \quad 3-6y+1+4y+4y^2 = 4y^2. \\ -2y = -4, \quad y = 2.$$

$$19. \quad \left(1 - \frac{1}{c}\right) \div \left(1 + \frac{1}{z}\right) + \frac{c+z}{z+1} = 1. \quad (1)$$

$$\frac{cz-z}{c(z+1)} + \frac{c+z}{z+1} = 1. \quad (2)$$

$$(2) \cdot c(z+1), \quad cz-z+c^2+cz = cz+c. \\ (c-1)z = c-c^2. \\ z = -c.$$

$$20. \quad \frac{3b+9x}{9a+6x} - \frac{a-2x}{2x+3a} = 2.$$

$$\frac{b+3x-a+2x}{2x+3a} = 2. \quad (1)$$

$$(1) \cdot (2x+3a), \quad 5x-a+b = 4x+6a. \\ x = 7a-b.$$

$$21. \quad \frac{8}{x-7} - \frac{2-6x}{x^2-6x-7} = \frac{27}{x+1}. \quad (1)$$

$$(1) \cdot (x-7)(x+1), \quad 8x+8-2+6x = 27x-189. \\ -13x = -195, \quad x = 15.$$

$$22. \quad \frac{x}{2(a+b)} - \frac{5}{b-a} = \frac{bx}{b^2-a^2}. \quad (1)$$

$$(1) \cdot 2(b+a)(b-a), \quad bx-ax-10b-10a = 2bx. \\ (-a-b)x = 10b+10a. \\ x = -10.$$

$$23. \quad \frac{2x}{3} - \frac{d}{5} \left(\frac{6x}{c} - 10d \right) = 2cd \left(\frac{2}{3} - \frac{d}{5c} \right). \quad (1)$$

$$(1) \cdot 15c, \quad 10cx-18dx+30cd^2 = 20c^2d-6cd^2. \\ (10c-18d)x = 20c^2d-36cd^2. \\ x = 2cd.$$

$$24. \quad 82.4 - 13x = 32x - 52.6. \\ -45x = -135.0, \\ x = 3.$$

$$25. \quad .01(2x+.205) - .0125(1.5x-.5) = .01955. \quad (1) \\ (1) \cdot 10,000, \quad 100(2x+.205) - 125(1.5x-.5) = 195.5. \\ 200x+20.5-187.5x+62.5 = 195.5. \\ 12.5x = 112.5. \\ x = 9.$$

$$26. \quad \frac{1}{x - .33a} + \frac{1}{\frac{2x}{3} - .22a} = \frac{5}{4a}.$$

$$\frac{100}{100x - 33a} + \frac{300}{200x - 66a} = \frac{5}{4a}.$$

$$\frac{500}{200x - 66a} = \frac{5}{4a}.$$

$$400a = 200x - 66a.$$

$$2.33a = x.$$

$$27. \quad \frac{c^2}{ax} + \frac{1}{c} = \frac{3c - 3a}{x} + \frac{1}{a} + \frac{a^2}{cx}. \quad (1)$$

$$(1) \cdot acx,$$

$$c^3 + ax = 3ac^2 - 3a^2c + cx + a^3.$$

$$(a - c)x = a^3 - 3a^2c + 3ac^2 - c^3.$$

$$x = (a - c)^2.$$

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2. Let x equal the number of minute spaces that the minute hand travels from its position at 7 o'clock until it overtakes the hour hand. Obviously the minute hand travels 35 minute spaces more than the hour hand.

$$\text{Then} \quad x = (x - 35) 12.$$

$$\text{Whence} \quad x = 38\frac{2}{11}.$$

3. Let x equal the number of minute spaces that the minute hand travels from its position at 2 o'clock until it is in the required position. For the first of the required positions the minute hand must gain 10 minute spaces over the hour hand; for the second position 40 spaces.

$$\text{Then} \quad x = (x - 10) 12.$$

$$\text{Whence} \quad x = 10\frac{10}{11},$$

$$\text{and} \quad x = (x - 40) 12.$$

$$\text{Whence} \quad x = 43\frac{7}{11}.$$

4. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 6 o'clock to the required position. For the first it must gain 40 spaces and for the second 20 spaces.

$$\text{Then} \quad x = (x - 40) 12.$$

$$\text{Whence} \quad x = 43\frac{10}{11},$$

$$\text{and} \quad x = (x - 20) 12.$$

$$\text{Whence} \quad x = 21\frac{9}{11}.$$

5. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 5 o'clock to the required position. Now if the hands are at right angles, one is 15 minute spaces from the other. For the first position the minute hand must gain 10 spaces and for the second 40 spaces.

Then $x = (x - 10) 12.$

Whence $x = 10\frac{10}{11},$

and $x = (x - 40) 12.$

Whence $x = 43\frac{7}{11}.$

7. Let x equal the number of days from April 28, 1909, until the next conjunction.

Then, as in Exercise 6, $\frac{x}{225} = \frac{x}{365} + 1.$

Whence $x = 586 + .$

Therefore the required date is December 5, 1910.

8. Let x equal the number of days from opposition to opposition.

Then $\frac{x}{365} = \frac{x}{4332} + 1.$

Whence $x = 398 + .$

Therefore the required date is April 2, 1910.

9. Let x equal the number of hours from the start until they are again together.

Then $\frac{x}{2\frac{1}{2}} = \frac{x}{3} + 1.$

Whence $x = 15.$

10. Reducing $2\frac{3}{4}$, $3\frac{1}{3}$, and $4\frac{2}{5}$ to a common denominator gives $\frac{165}{60}$, $\frac{200}{60}$, and $\frac{264}{60}$.

The L.C.M. of the numerators is 6600.

The L.C.M. of the fractions is $\frac{6600}{60} = 110.$

Hence the three are together 110 hours after the start.

11. Yes, $15 \div 2\frac{1}{2} = 6$, and $15 \div 3 = 5$. Yes.

12. Yes; Yes.

13. $\frac{11}{4}, \frac{10}{3}, \frac{22}{5}, \frac{\text{L.C.M. of numerators}}{\text{G.C.D. of denominators}} = \frac{110}{1}.$

14. RULE. Divide the L.C.M. of the numerators by the G.C.D. of the denominators.

L.C.M. of $\frac{7}{6}$, $\frac{7}{3}$, and $\frac{28}{9}$ is $\frac{28}{3}$, or $9\frac{1}{3}.$

15. Assuming that a and c have no common integral factor and that b and d have none, the L.C.M. of $\frac{a}{b}$ and $\frac{c}{d} = \frac{ac}{1} = ac$.

With an assumption similar to the preceding one, the L.C.M. of

$$\frac{a}{b}, \frac{c}{d}, \text{ and } \frac{e}{f} = \frac{ace}{1} = ace.$$

16. Let x equal the number of ounces needed.

$$\text{Then} \quad \frac{56}{56 + x} = \frac{70}{100}.$$

$$\text{Whence} \quad x = 24.$$

$$\text{17.} \quad \frac{\frac{16 \cdot 410}{100} + x}{410 + x} = \frac{18}{100}.$$

$$6560 + 100x = 7380 + 18x.$$

$$\text{Whence} \quad x = 10.$$

18. Let m equal the number of gallons of milk and w the number of gallons of water.

$$\text{Then} \quad m + w = 30, \quad (1)$$

$$\text{and} \quad \frac{\frac{20m}{100}}{30} = \frac{16}{100}. \quad (2)$$

$$\text{Solving (1) and (2),} \quad m = 24,$$

$$\text{and} \quad w = 6.$$

19. Let x equal the number of gallons of 90% alcohol.

$$\text{Then} \quad \frac{\frac{90}{100}x + \frac{95}{100} \cdot 10}{x + 10} = \frac{92}{100}.$$

$$\text{Whence} \quad x = 15.$$

20. Let e equal the diameter of the earth in miles, and m the diameter of the moon in miles.

$$\text{Then} \quad e = \frac{11m}{3},$$

$$\text{and} \quad e - m = 5760.$$

$$\text{Whence} \quad e = 7920,$$

$$\text{and} \quad m = 2160.$$

21. Let s equal the diameter of the sun in miles, and e the diameter of the earth in miles.

$$\text{Then} \quad s = 3220 + 109e,$$

$$\text{and} \quad s + e = 874,420.$$

$$\text{Whence} \quad s = 866,500,$$

$$\text{and} \quad e = 7920.$$

22. Let s equal the sun's distance from the earth in miles, and m the moon's distance from the earth in miles.

Then
$$s = 387\frac{1}{2} m,$$

and
$$\frac{s}{186000} = \frac{m}{186000} + 498\frac{2}{3}\frac{2}{1}.$$

Whence
$$s = 93,000,000,$$

and
$$m = 240,000.$$

23. Let j equal the diameter of Jupiter in miles, and e equal the diameter of the earth in miles.

Then
$$j = \frac{120 e}{11},$$

and
$$j + e = 94,320.$$

Whence
$$j = 86,400,$$

and
$$e = 7920.$$

24. Let x be the number of days B requires to finish the work.

Then
$$3 \cdot \frac{1}{15} + 3 \cdot \frac{1}{25} + x \cdot \frac{1}{25} = 1.$$

Whence
$$x = 17.$$

25. Let x be the number of days required to do the job, when all work together.

Then
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{a+b} = \frac{1}{x}.$$

Whence
$$x = \frac{a^2b + ab^2}{a^2 + 3ab + b^2}.$$

26. Let h be the number of hours required to fill the cistern when both pipes are open.

Then
$$\frac{1}{2m} - \frac{1}{\frac{n+1}{3}} = \frac{1}{h}.$$

Whence
$$h = \frac{2mn + 2m}{n+1-6m}.$$

27. By the conditions of the problem the numerator $2mn + 2m$ is a positive number. Therefore, if $6m$ is less than $n+1$, h in Problem 26 is positive. This would mean that the water comes in faster than it goes out.

If $6m$ equals $n+1$, h equals a fraction whose denominator is zero. This would mean that the water runs out as fast as it runs in.

If $6m$ is greater than $n+1$, h equals a fraction whose denominator is negative. This would mean that more water could run out than runs in.

28. Here
$$\frac{\frac{1}{2m} + \frac{1}{n+1}}{3} = \frac{1}{h} \cdot \frac{1}{x}.$$

Whence
$$h = \frac{2mn + 2m}{nx + x + 6mx}.$$

29. Let r equal the length of the radius in inches.

Then
$$\frac{2}{7} r^2 + 440 = \frac{2}{7} (r + 7)^2.$$

Whence
$$r = 6\frac{1}{2}.$$

30. Let R be the radius of the earth and x be the radius of the hoop, both in feet.

Then
$$2\pi R + 1 = 2\pi x.$$

Whence
$$x = \frac{2\pi R + 1}{2\pi} = R + \frac{1}{2\pi}.$$

The distance from the earth to the hoop $= R + \frac{1}{2\pi} - R = \frac{1}{\frac{4}{7}}$ feet, or $1\frac{10}{11}$ inches.

31. Here if x equals radius of the hoop in inches, $2\pi \cdot 3 + 12 = 2\pi x.$

Whence
$$x = 3 + \frac{6}{\pi}.$$

The distance of the hoop from the pole is $3 + \frac{6}{\pi} - 3 = 1\frac{10}{11}$ inches.

32. Let x equal the rate of the freight in miles per hour.

Then
$$42\left(b + \frac{a}{60}\right) = x\left(a + \frac{b}{60} + b + \frac{a}{60}\right).$$

Whence
$$x = \frac{2520b + 42a}{61a + 61b}.$$

33. Let x be the required weight in pounds. Then by the law of the lever (see page 168),

$$3x = 4 \cdot 100.$$

Whence
$$x = 133\frac{1}{3}.$$

Therefore the weight is $133\frac{1}{3}$ pounds.

34. Let AB be the beam and C the point on which the load rests, AC being 7 feet. Then A may be regarded as the fulcrum, and the force downward at C equal to the upward force at B . Then, if x is the pressure in tons on B ,

$$AC \cdot 3 = AB \cdot x,$$

or
$$7 \cdot 3 = 12 \cdot x.$$

Whence
$$x = \frac{7}{4}, \text{ the pressure on } B.$$

Since the pressure on A and B together is 3 tons, the pressure on A alone equals 3 tons minus $\frac{7}{4}$ tons, or $\frac{5}{4}$ tons.

35. Let the load on the shorter and on the longer arms be $3l$ and $2l$ respectively, and let x be the required distance.

Then $x \cdot 8 + (2l - 4) 12 = 8 \cdot 3l$.

Solving, $x = 6$.

36. Let x equal the pressure in pounds on the support. Then, as in the solution of Problem 35, we have

$$8 \cdot x = 12 \cdot 130.$$

Whence $x = 195$.

37. (a) 100 pounds on each support.

(b) The support 3 feet from the end does exactly what it did at the end, — support the beam. In one case it does it at the end of an arm 14 feet long, in the other at the end of an arm 11 feet long.

Let x equal the pressure in pounds on the support.

Then $11x = 14 \cdot 100$.

$$\therefore x = 127\frac{3}{11}.$$

38. Solution like (b) in Problem 37. Let x equal the weight in pounds on the support which is 4 feet from one end.

Then $12 \cdot x = 16 \cdot 150$.

$$x = 200, \text{ weight on one support.}$$

$$300 - 200 = 100, \text{ weight on the other support.}$$

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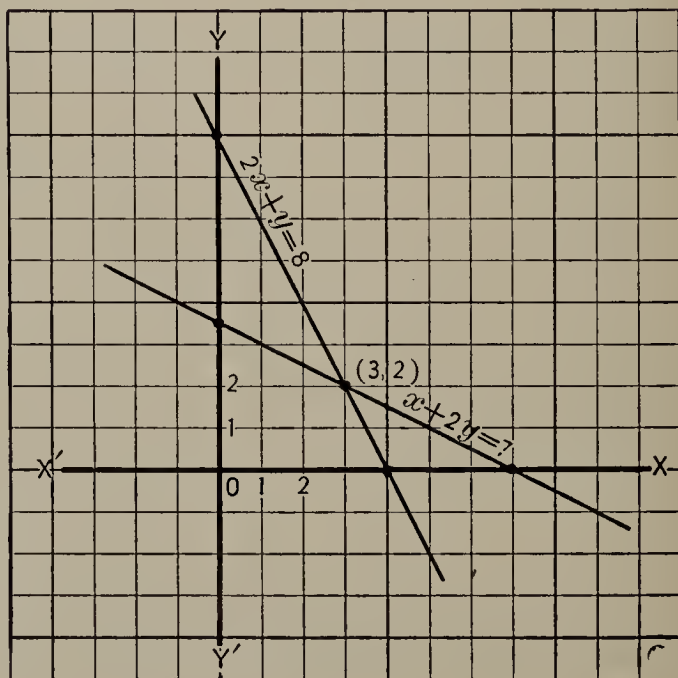
1. $2x + y = 8$

x	0	4	2
y	8	0	4

$$x + 2y = 7$$

x	0	7	1
y	$\frac{7}{2}$	0	3

$$x = 3, y = 2.$$



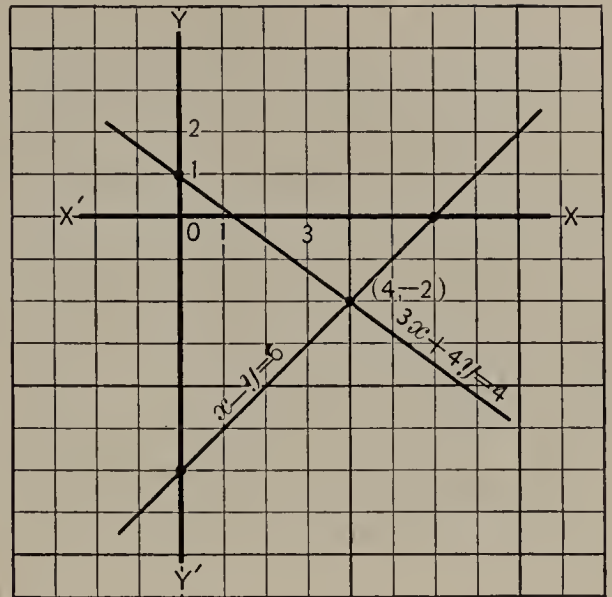
2. $x - y = 6$

x	0	6	2
y	-6	0	-4

$$3x + 4y = 4$$

x	0	$\frac{4}{3}$	4
y	1	0	-2

$$x = 4, y = -2.$$



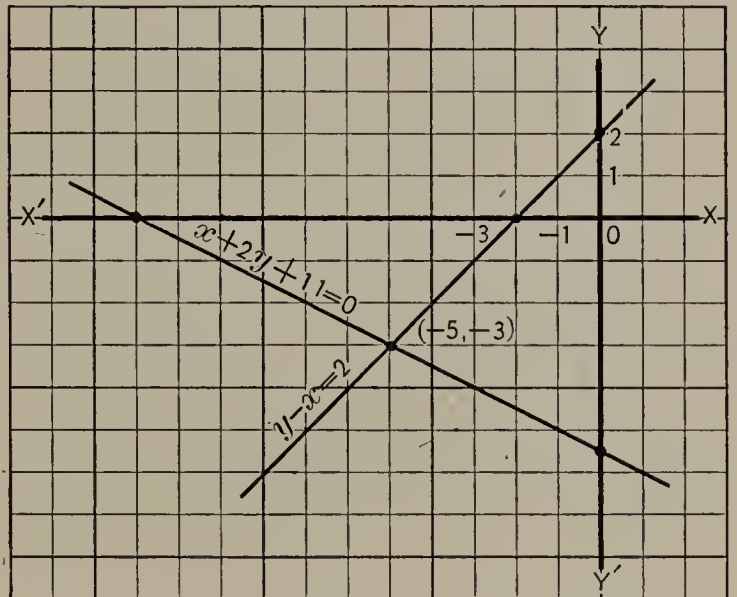
3. $x + 2y + 11 = 0$

x	0	-11	1
y	$-\frac{11}{2}$	0	-6

$$y - x = 2$$

x	0	-2	-4
y	2	0	-2

$$x = -5, y = -3.$$



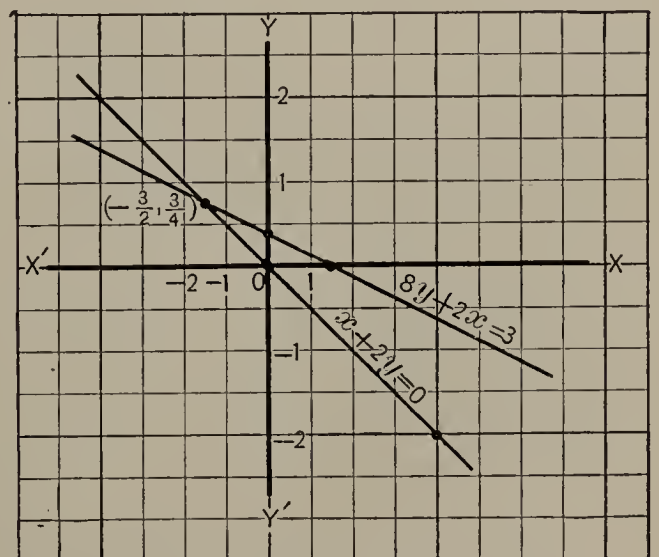
4. $x + 2y = 0$

x	0	4	-2
y	0	-2	1

$$8y + 2x = 3$$

x	0	$\frac{3}{2}$	8
y	$\frac{3}{8}$	0	$-\frac{13}{8}$

$$x = -\frac{3}{2}, y = \frac{3}{4}.$$



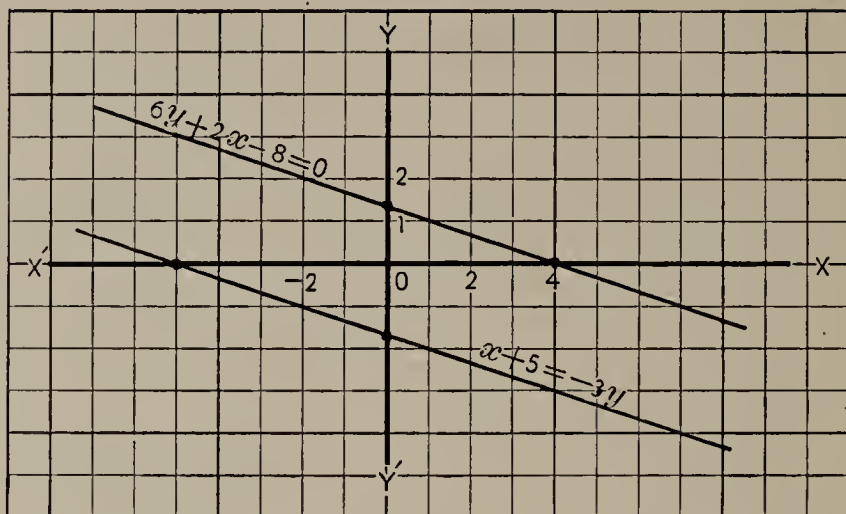
5. $x + 5 = -3y$

x	0	-5	7
y	$-\frac{5}{3}$	0	-4

$6y + 2x - 8 = 0$

x	0	4	-2
y	$\frac{4}{3}$	0	2

Inconsistent.



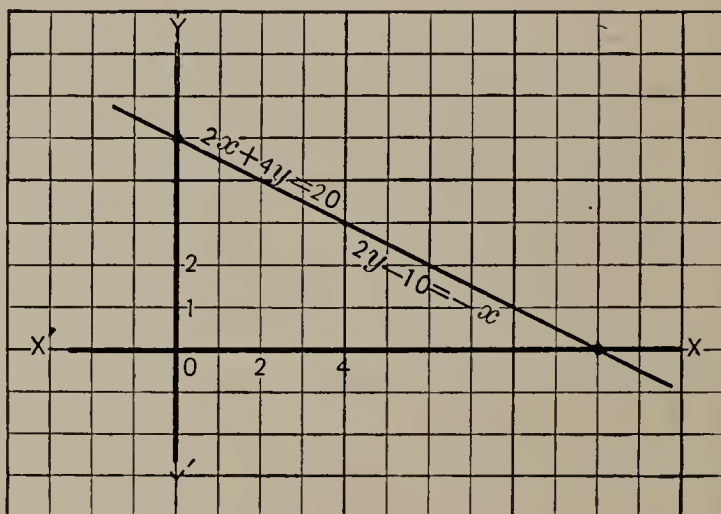
6. $2x + 4y = 20$

x	0	10	8
y	5	0	1

$2y - 10 = -x$

x	0	10	6
y	5	0	2

Infinite number of sets of roots.



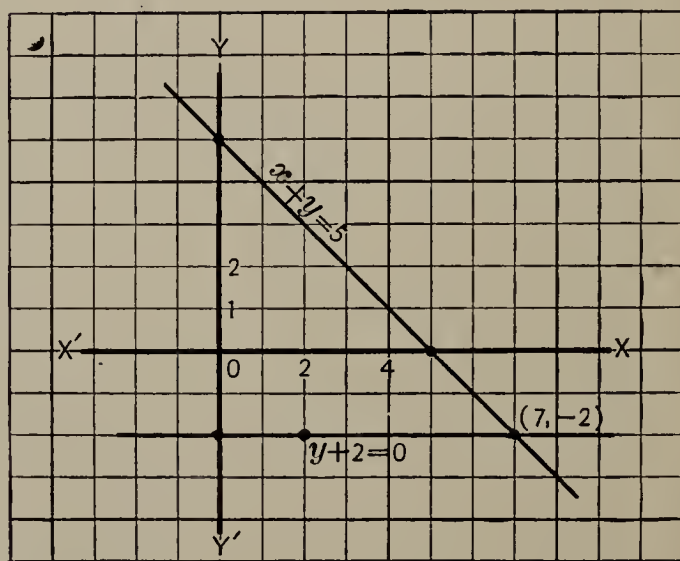
7. $x + y = 5$

x	0	5	2
y	5	0	3

$0x + y + 2 = 0$

x	0	2	4
y	-2	-2	-2

$x = 7, y = -2$



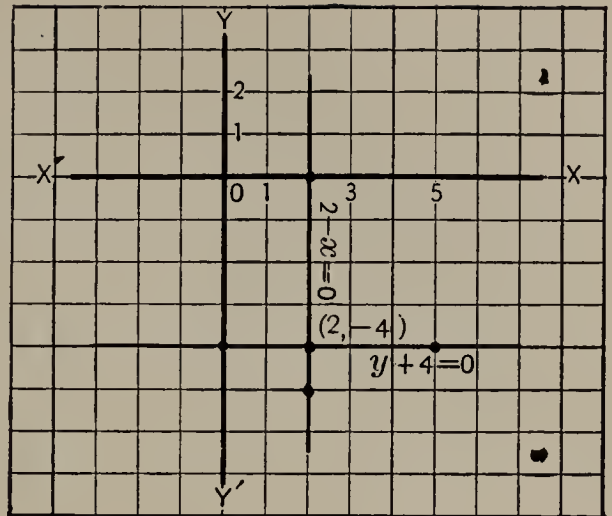
8. $0x + y + 4 = 0$

x	0	5	-2
y	-4	-4	-4

$0y + 2 - x = 0$

x	2	2	2
y	0	-5	-3

$x = 2, y = -4.$



9. $x + y = 4$

x	0	4	2
y	4	0	2

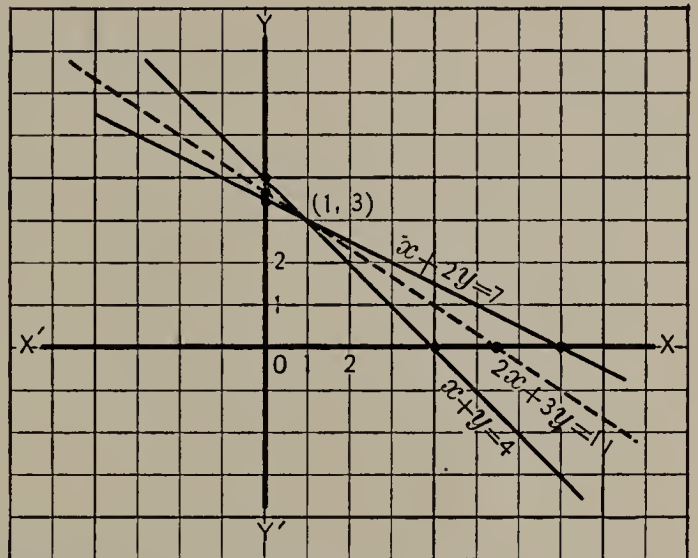
$x + 2y = 7$

x	0	7	1
y	$\frac{7}{2}$	0	3

$2x + 3y = 11$

x	0	$\frac{11}{2}$	4
y	$\frac{11}{3}$	0	1

$x = 1, y = 3.$



The third graph passes through the point of intersection of the other two.

10. $x - y = 5$

x	0	5	7
y	-5	0	2

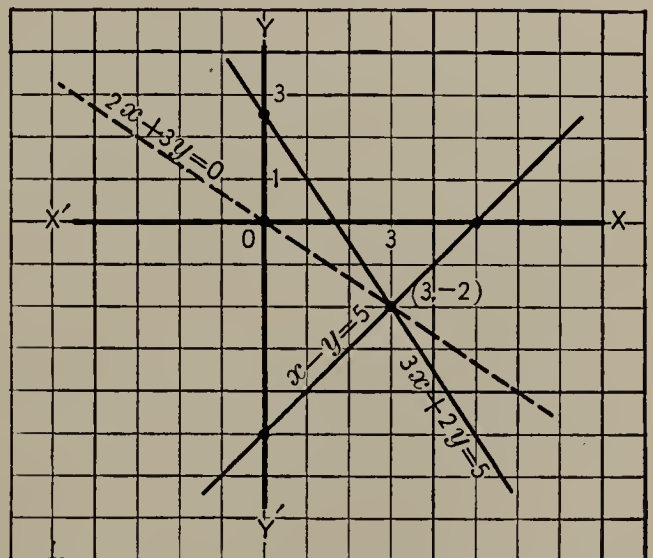
$3x + 2y = 5$

x	0	3	5
y	$\frac{5}{2}$	-2	-5

$2x + 3y = 0$

x	0	6	-3
y	0	-4	2

$x = 3, y = -2.$



The third graph passes through the point of intersection of the other two.

11. $3x - 4y = 12$

x	0	4	2
y	-3	0	$-\frac{3}{2}$

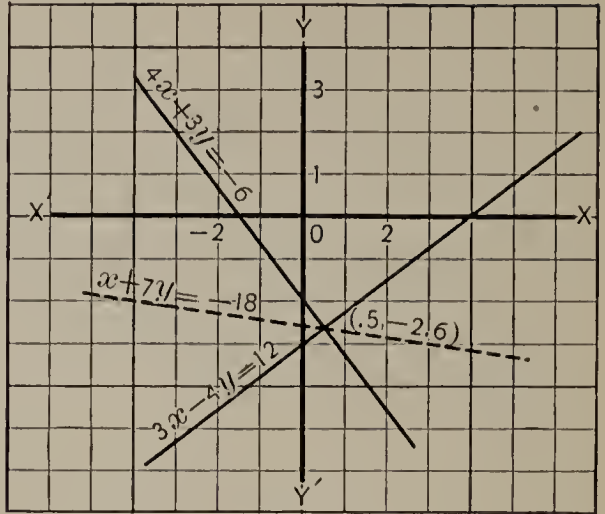
$$4x + 3y = -6$$

x	0	$-\frac{3}{2}$	3
y	-2	0	-6

$$x + 7y = -18$$

x	0	-4	3
y	$-\frac{18}{7}$	-2	-3

$$x = .5, y = -2.6.$$



The third graph passes through the point of intersection of the other two.

12. Yes. A linear equation obtained by adding two given linear equations or by subtracting one from the other is always satisfied by the set of values which satisfies the given equations because its graph passes through the point of intersection of the graphs of the given equations.

13. $x - 2y = 2$

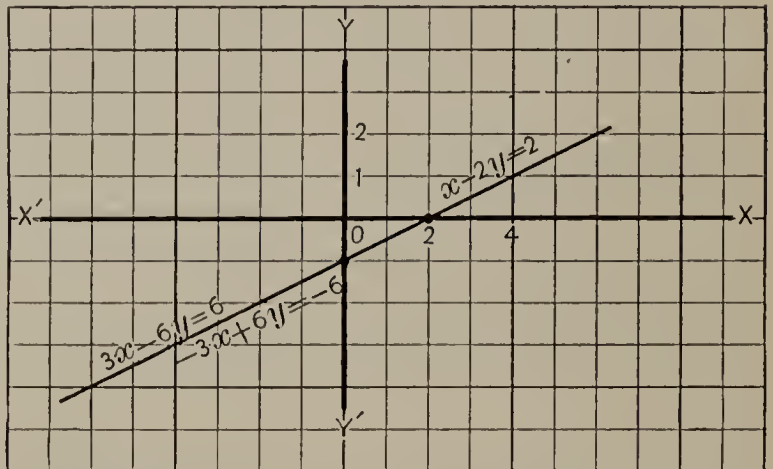
x	0	2	6
y	-1	0	2

$$3x - 6y = 6$$

x	0	2	4
y	-1	0	1

$$-3x + 6y = -6$$

x	0	2	-2
y	-1	0	-2



An infinite number of sets of values will satisfy.

The three graphs are identical.

If one equation is obtained from another by multiplying both members of the given equation by a constant, the given and the resulting equations have identical graphs.

14. $(0, 0)$.

15. Yes. If the graphs of two equations are parallel lines, the system has no graphical solution.

16. Yes, different scales could have been used on the two axes. The lines, however, must be plotted to the same scales on each axis, otherwise the point of intersection would have no meaning related to the two equations.

17. (a) $y = c$. (b) $x = d$.

19. (a) $3x - y = 6$, $y - 3x = 18$.

18. $ax + by = 0$.

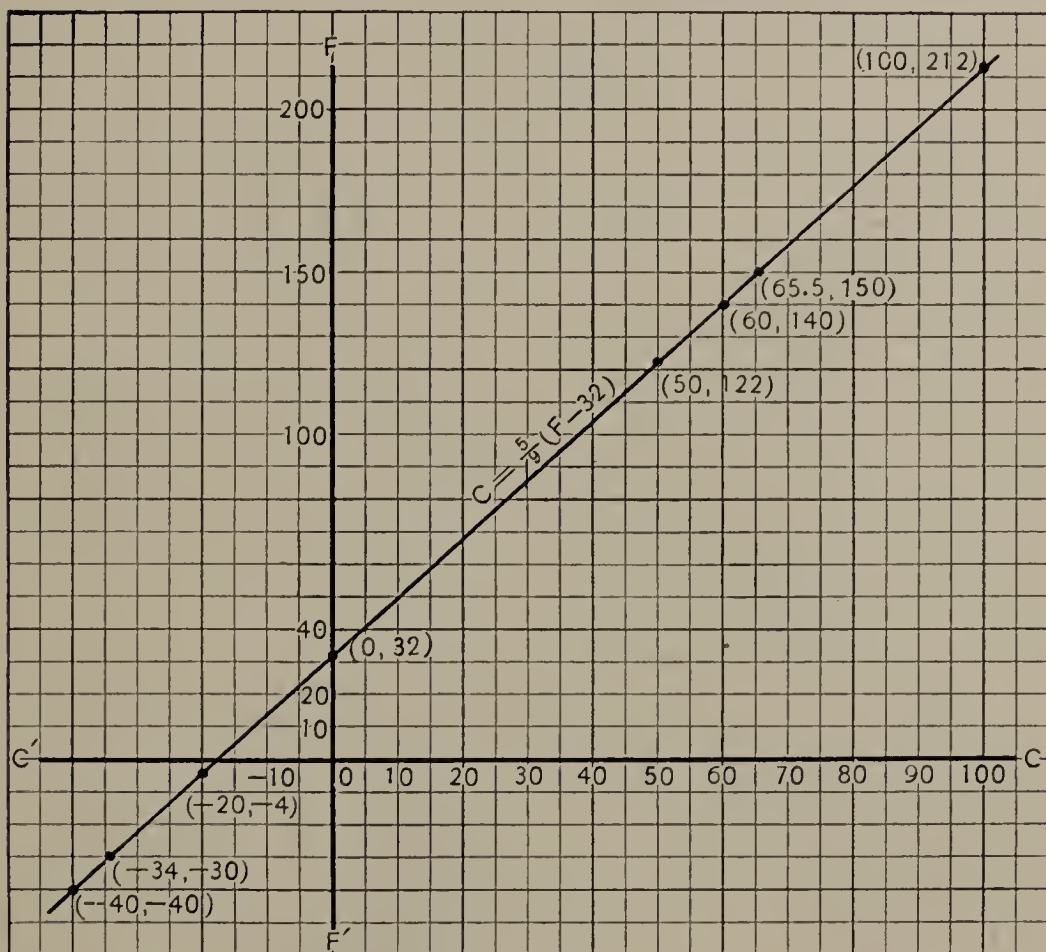
(b) $3x - 4y = 12$, $6x - 8y = 24$.

20. The number of degrees from the freezing point of water to its boiling point is 100 on the Centigrade scale and $212 - 32$, or 180, on the Fahrenheit. Therefore one degree on the Fahrenheit scale equals $\frac{5}{9}$ of a degree on the Centigrade scale. To get a reading above the freezing point on the Fahrenheit scale we must subtract 32 degrees from the given reading, while on the Centigrade scale all readings are made with reference to the freezing point. Hence, if C is degrees Centigrade and F degrees Fahrenheit, $C = \frac{5}{9}(F - 32)$.

$$C = \frac{5}{9}(F - 32).$$

Yes.

C	0	50	100
F	32	122	212



21. (a) $60^\circ \text{ C} = 140^\circ \text{ F}$.

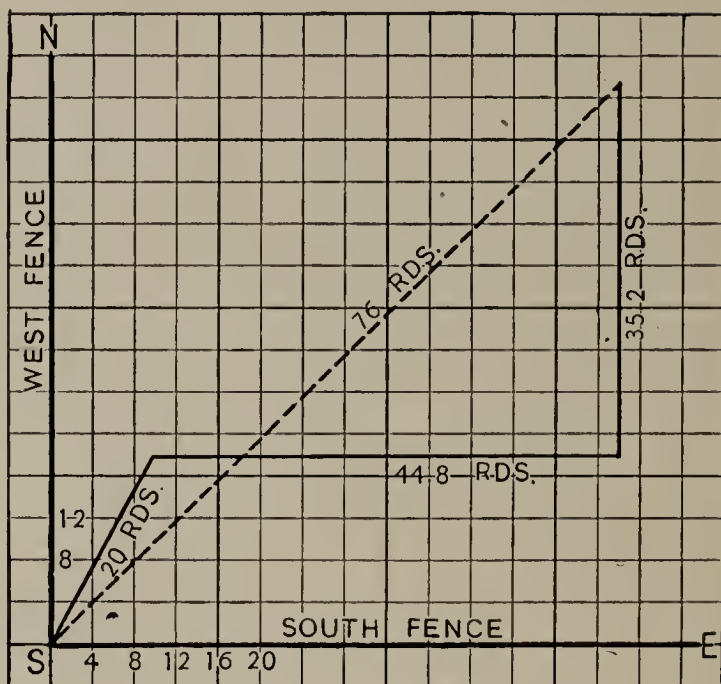
(c) $-20^\circ \text{ C} = -4^\circ \text{ F}$.

(b) $150^\circ \text{ F} = 65.5^\circ \text{ C}$.

(d) $-30^\circ \text{ F} = -34^\circ \text{ C}$.

22. $-40^\circ \text{ C} = -40^\circ \text{ F}$.

23. First distance is 20 rods.
 Second distance is 44.8 rods.
 Third distance is 35.2 rods.
 Distance from starting point is 76 rods.



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1. See pages 203 and 458.
2. For (a), (b), (c), (e), (f), and (g), see pages 203-204.
 For (d) see page 308.
 For (h) see page 206.
3. (a) A straight line.
 (b) Two or more straight lines.
 (c) Two or more straight lines.
 (d) Two or more identical lines.
 (e) Two or more intersecting lines.
 (f) One straight line.
 (g) Two coincident lines.
 (h) Two parallel lines.
4. $ax + by = c$.
5. $ax + by = c$,
 $ax + by = d$.
6. $ax + by = c$,
 $kax + kby = kc$.
7. $2x + 5y = 8$, (1)
 $x - 10y = 9$. (2)
 $(2) \cdot 2$, $2x - 20y = 18$. (3)
 $(1) - (3)$, $25y = -10$.
 Then $y = -\frac{2}{5}$.
 Substituting $-\frac{2}{5}$ for y in (2),
 $x + 4 = 9$.
 Whence $x = 5$.
8. $5x - 12y = -38$, (1)
 $3x + 8y = 0$. (2)
 $(1) \cdot 2$, $10x - 24y = -76$. (3)
 $(2) \cdot 3$, $9x + 24y = 0$. (4)
 $(3) + (4)$, $19x = -76$.
 Whence $x = -4$.
 Substituting -4 for x in (2),
 $-12 + 8y = 0$.
 Whence $y = \frac{3}{2}$.

9. $9t - 2n = 18,$ (1)
 $20t - 7n = 63.$ (2)
 $(1) \cdot 7,$ $63t - 14n = 126.$ (3)
 $(2) \cdot 2,$ $40t - 14n = 126.$ (4)
 $(3) - (4),$ $23t = 0.$
 Whence $t = 0.$
 Substituting 0 for t in (1), $n = -9.$
10. $11m + 18n = 10,$ (1)
 $9m + 12n = -15.$ (2)
 $[(1) \cdot 2 - (2) \cdot 3],$ $-5m = +65.$
 $\therefore m = -13.$
 Substituting -13 for m in (2),
 $-117 + 12n = -15.$
 Whence $n = 8\frac{1}{2}.$
11. $3x - 2y = 18,$ (1)
 $5x - 8y = 30.$ (2)
 $(1) \cdot 4,$ $12x - 8y = 72.$ (3)
 $(2) - (3),$ $-7x = -42.$
 $\therefore x = 6.$
 Substituting 6 for x in (1), $18 - 2y = 18.$
 Whence $y = 0.$
12. $3r - 8s = 13,$ (1)
 $r + 6s = 0.$ (2)
 From (2), $r = -6s.$ (3)
 Substituting $-6s$ for r in (1),
 $-18s - 8s = 13,$ or $s = -\frac{1}{4}.$
 Substituting $-\frac{1}{4}$ for s in (2), $r = 3.$
13. $2(x + y) + 3y = 4,$ $2x + 5y = 4,$ (1)
 $5 = x + y;$ or $x + y = 5.$ (2)
 Substituting $5 - x$ for y from (2) in (1),
 $2x + 25 - 5x = 4.$
 Whence $x = 7.$
 Substituting 7 for x in (2), $y = -2.$
14. $16x + 7 = 15y,$ (1)
 $4x + 5y = 0.$ (2)
 Substituting $-\frac{4x}{5}$ for y from (2) in (1),
 $16x + 7 = -12x.$ (3)
 Whence $x = -\frac{1}{4}.$ (4)
 Then from (2) and (4), $y = \frac{1}{5}.$

$$15. \quad 6x + \frac{14}{3} - 12y = 0, \quad \text{or} \quad 18x - 36y + 14 = 0, \quad (1)$$

$$7y - 3x - 4 = 0; \quad 7y - 3x - 4 = 0. \quad (2)$$

$$\text{From (2),} \quad x = \frac{7y - 4}{3}. \quad (3)$$

$$\text{Substituting } \frac{7y - 4}{3} \text{ for } x \text{ in (1),}$$

$$42y - 24 - 36y + 14 = 0. \quad (4)$$

$$\text{Whence} \quad 6y = 10, \text{ or } y = \frac{5}{3}. \quad (5)$$

$$\text{Then from (3) and (5),} \quad x = \frac{23}{9}.$$

$$16. \quad \frac{8m - 3n}{2} + 6n = -9, \quad 8m + 9n = -18, \quad (1)$$

$$4m - 1 = 3n; \quad \text{or} \quad 4m - 1 = 3n. \quad (2)$$

$$\text{From (2),} \quad n = \frac{4m - 1}{3}. \quad (3)$$

$$\text{Substituting } \frac{4m - 1}{3} \text{ for } n \text{ in (1),}$$

$$8m + 12m - 3 = -18. \quad (4)$$

$$\text{Whence} \quad 20m = -15, \text{ or } m = -\frac{3}{4}. \quad (5)$$

$$\text{From (3) and (5),} \quad n = -\frac{4}{3}.$$

$$17. \quad \frac{2x}{3} + y = -7, \quad 2x + 3y = -21, \quad (1)$$

$$\text{or}$$

$$5x - 3y = 105; \quad 5x - 3y = 105. \quad (2)$$

$$(1) + (2), \quad 7x = 84. \quad (3)$$

$$\text{Whence} \quad x = 12. \quad (4)$$

$$\text{From (1) and (4),} \quad y = -15.$$

$$18. \quad \frac{3r}{4} - \frac{7}{2} = \frac{s}{12}, \quad (1) \quad 19. \quad \frac{3y + 1}{13} - \frac{z + 20}{12} = -3. \quad (1)$$

$$r + 8 = -2s. \quad (2) \quad \frac{z}{8} - \frac{2y}{9} = 1. \quad (2)$$

$$\text{From (1),} \quad 9r - s = 42. \quad (3) \quad \text{From (1), } 36y - 13z = -220. \quad (3)$$

$$\text{From (2),} \quad r + 2s = -8. \quad (4) \quad \text{From (2), } -16y + 9z = 72. \quad (4)$$

$$(4) \cdot 9, \quad 9r + 18s = -72. \quad (5) \quad (3) \cdot 4, \quad 144y - 52z = -880. \quad (5)$$

$$(3) - (5), \quad -19s = 114. \quad (5) \quad (4) \cdot 9, \quad -144y + 81z = 648. \quad (6)$$

$$(3) - (5), \quad -19s = 114. \quad (5) \quad (5) + (6), \quad 29z = -232. \quad (6)$$

$$s = -6. \quad z = -8.$$

$$\text{Substituting } -6 \text{ for } s \text{ in (4),} \quad \text{Substituting } -8 \text{ for } z \text{ in (4),}$$

$$r = 4. \quad y = -9.$$

$$20. \quad 5x + .7y = -1.25, \quad (1)$$

$$.12x - .08y = 3\frac{3}{5}. \quad (2)$$

$$(1) \cdot 100,$$

$$500x + 70y = -125. \quad (3)$$

$$(2) \cdot 100, \quad 12x - 8y = 360. \quad (4)$$

$$(3) \cdot 8, \quad 4000x + 560y = -1000. \quad (5)$$

$$(4) \cdot 70,$$

$$840x - 560y = 25,200. \quad (6)$$

$$(5) + (6), \quad 4840x = 24,200.$$

$$x = 5.$$

Substituting 5 for x in (4),

$$60 - 8y = 360.$$

$$\therefore y = -37\frac{1}{2}.$$

21.

$$\frac{1}{x} + \frac{1}{y} = -\frac{1}{6}, \quad (1)$$

$$\frac{2}{x} - \frac{3}{y} = -\frac{4}{3}. \quad (2)$$

$$\frac{2}{x} + \frac{2}{y} = -\frac{1}{3}. \quad (3)$$

$$(1) \cdot 2,$$

$$(2) - (3),$$

$$-\frac{5}{y} = -1.$$

$$\therefore y = 5.$$

Substituting 5 for y in (1),

$$\frac{1}{x} + \frac{1}{5} = -\frac{1}{6}.$$

$$\therefore x = -\frac{30}{11}.$$

22.

$$\frac{2m + 3n - 2}{m + n + 6} = \frac{4}{3}, \quad (1)$$

$$\frac{1}{m} + \frac{1}{n} = \frac{9}{m}. \quad (2)$$

From (1),

$$2m + 5n = 30. \quad (3)$$

From (2),

$$m - 8n = 0. \quad (4)$$

$$(4) \cdot 2,$$

$$2m - 16n = 0. \quad (5)$$

$$(3) - (5),$$

$$21n = 30.$$

$$n = \frac{10}{7}.$$

Substituting $\frac{10}{7}$ for n in (4),

$$m = \frac{80}{7}.$$

23.

$$\frac{.25h + 8 + .1k}{k - 10 + h} = 2.5, \quad (1)$$

$$\frac{1}{.8h - 2.2} + \frac{20}{35 - 5.5k} = 0. \quad (2)$$

From (1),

$$225h + 240k = 3300. \quad (3)$$

From (2),

$$160h - 55k = 90. \quad (4)$$

$$(3) \cdot 32,$$

$$7200h + 7680k = 105,600. \quad (5)$$

$$(4) \cdot 45,$$

$$7200h - 2475k = 4,050. \quad (6)$$

$$(5) - (6),$$

$$10,155k = 101,550. \quad (7)$$

$$k = 10. \quad (8)$$

From (4) and (8),

$$h = 4.$$

24.

$$\frac{2}{3}(2h + 5k) = 39, \quad (1)$$

$$2h - \frac{3k}{4} = 3(k - h). \quad (2)$$

From (1),

$$4h + 10k = 117. \quad (3)$$

From (2),

$$4h - 3k = 0. \quad (4)$$

$$(3) - (4),$$

$$13k = 117. \quad (5)$$

$$\therefore k = 9. \quad (6)$$

From (6) and (4),

$$h = 6\frac{3}{4}.$$

$$25. \frac{3(x+y)}{\frac{3}{5}} + \frac{x-y}{-\frac{11}{5}} = 0, \quad (1)$$

$$2x + y = 7. \quad (2)$$

$$\text{From (1), } 5x + 6y = 0. \quad (3)$$

$$(2) \cdot 6, \quad 12x + 6y = 42. \quad (4)$$

$$(4) - (3), \quad 7x = 42. \quad (5)$$

$$\therefore x = 6. \quad (6)$$

$$\text{From (2) and (6), } y = -5.$$

$$26. \quad \frac{m+5}{n} = \frac{m-5}{n-10}, \quad (1)$$

$$\frac{m - \frac{2}{5}(3m - 2n) - \frac{1}{3}}{m - 2n - 1} + \frac{1}{3} = 0. \quad (2)$$

$$\text{From (1), } -m + n = 5. \quad (3)$$

$$\text{From (2), } m + n = 5. \quad (4)$$

$$(3) + (4), \quad 2n = 10, \quad n = 5. \quad (5)$$

$$(3) - (4), \quad -2m = 0, \quad m = 0.$$

$$27. \quad 2cx - y = 5ac, \quad (1)$$

$$\frac{2x}{3} - \frac{y}{c} = a. \quad (2)$$

$$(1) \cdot 1, \quad 2cx - y = 5ac. \quad (3)$$

$$\text{From (2), } 2cx - 3y = 3ac. \quad (4)$$

$$30. \quad ay + (a+b)x = xy, \quad (1)$$

$$\frac{a+b}{x} + \frac{b}{y} = \frac{4}{3}. \quad (2)$$

$$(1) \div xy, \quad \frac{a}{x} + \frac{a+b}{y} = 1. \quad (3)$$

$$(3) - (2), \quad -\frac{b}{x} + \frac{a}{y} = \frac{-1}{3}. \quad (4)$$

$$(3) \cdot b, \quad \frac{ab}{x} + \frac{ab+b^2}{y} = b. \quad (5)$$

$$(4) \cdot a, \quad -\frac{ab}{x} + \frac{a^2}{y} = \frac{-a}{3}. \quad (6)$$

$$(5) + (6), \quad \frac{a^2 + ab + b^2}{y} = -\frac{a}{3} + b.$$

$$y = \frac{3a^2 + 3ab + 3b^2}{-a + 3b}.$$

$$\text{From (3), } \frac{a}{x} + (a+b) \cdot \frac{-a+3b}{3a^2 + 3ab - 3b^2} = 1.$$

$$x = \frac{3a^2 + 3ab + 3b^2}{b + 4a}.$$

$$(3) - (4), \quad 2y = 2ac. \quad (5)$$

$$y = ac. \quad (6)$$

$$\text{From (3) and (6), } x = 3a.$$

$$28. \quad \frac{a}{x} + \frac{3a}{y} = 1, \quad (1)$$

$$\frac{3a}{x} + \frac{a}{y} = \frac{1}{2}. \quad (2)$$

$$(1) \cdot 3, \quad \frac{3a}{x} + \frac{9a}{y} = 3. \quad (3)$$

$$(2) - (3), \quad -\frac{8a}{y} = -\frac{5}{2}. \quad (4)$$

$$y = \frac{16a}{5}. \quad (5)$$

$$\text{From (1) and (5), } x = 16a.$$

$$29. \quad \frac{x+y}{10a} = 2 - \frac{x-y}{4a}, \quad (1)$$

$$x = y. \quad (2)$$

$$\text{Substituting } x \text{ for } y \text{ in (1),}$$

$$\frac{2x}{10a} = 2 - 0. \quad (3)$$

$$x = 10a. \quad (4)$$

$$\text{From (2) and (4), } y = 10a.$$

$$\begin{aligned} 31. \quad & (c + b)y + ax = 1, & (1) \\ & ay + (c + b)x = 1. & (2) \end{aligned}$$

$$(1) \cdot a, \quad (ac + ab)y + a^2x = a. \quad (3)$$

$$(2) \cdot (b + c), \quad (ac + ab)y + (c + b)^2x = b + c. \quad (4)$$

$$(4) - (3), \quad [(b + c)^2 - a^2]x = b + c - a. \quad (5)$$

$$x = \frac{b + c - a}{(b + c)^2 - a^2} = \frac{1}{b + c + a}. \quad (6)$$

$$\text{From (1) and (6),} \quad (c + b)y + \frac{a}{b + c + a} = 1.$$

$$\therefore y = \frac{1}{b + c + a}.$$

$$32. \quad a^4 \cdot a^{y-7} = a^{12}, \quad (1)$$

$$c^4 \cdot c^{x-3} = c^y. \quad (2)$$

$$\text{From (1),} \quad 4 + y - 7 = 12. \quad (3)$$

$$\text{From (2),} \quad 4 + x - 3 = y. \quad (4)$$

$$\text{From (3),} \quad y = 15. \quad (5)$$

$$\text{From (4) and (5),} \quad x = 14.$$

$$33. \quad ax + by = c, \quad (1)$$

$$dx + ey = f. \quad (2)$$

$$(1) \cdot d, \quad adx + bdy = cd. \quad (3)$$

$$(2) \cdot a, \quad adx + aey = af. \quad (4)$$

$$(3) - (4), \quad (bd - ae)y = cd - af. \quad (5)$$

$$\text{From (5),} \quad y = \frac{cd - af}{bd - ae}. \quad (6)$$

$$\text{From (2) and (6),} \quad ax + b\left(\frac{cd - af}{bd - ae}\right) = c. \quad (7)$$

$$\text{From (7),} \quad x = \frac{bf - ce}{bd - ae}.$$

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$$1. \begin{vmatrix} 4 & 1 \\ 3 & 5 \end{vmatrix} = 20 - 3 = 17. \quad 3. \begin{vmatrix} 2 & 3 \\ 4 & -3 \end{vmatrix} = 3(-6 - 12) = -54.$$

$$2. \begin{vmatrix} 6 & -2 \\ 8 & 1 \end{vmatrix} = 6 + 16 = 22. \quad 4. \begin{vmatrix} -3 & 2 \\ 7 & 8 \end{vmatrix} = -24 - 14 = -38.$$

$$5. \begin{vmatrix} \frac{1}{2} & 6 \\ \frac{1}{3} & 8 \end{vmatrix} = 3(4 - 2) = 6.$$

$$6. \begin{vmatrix} -3 & 5 \\ 7 & 8 \end{vmatrix} = 4(-24 - 35) = -236.$$

$$7. \begin{vmatrix} 2a & -10b \\ 5a & 3b \end{vmatrix} = 6ab + 50ab = 56ab.$$

$$8. \frac{5}{2a} \begin{vmatrix} a^3 & 0 \\ a^2 & 2a \end{vmatrix} = \frac{5}{2a} (2a^4 - 0) = 5a^3.$$

$$9. \frac{7}{3} \begin{vmatrix} 3e & 0 \\ 9d & -14e^2 \end{vmatrix} = \frac{7}{3} (-42e^3 - 0) = -98e^3.$$

$$10. ax - cr = \begin{vmatrix} a & r \\ c & x \end{vmatrix}.$$

$$11. mz - 3d = \begin{vmatrix} m & d \\ 3 & z \end{vmatrix}.$$

$$12. \frac{8}{3} - ar = \begin{vmatrix} 8 & r \\ a & \frac{1}{3} \end{vmatrix}.$$

$$13. hk - c = \begin{vmatrix} h & c \\ 1 & k \end{vmatrix}.$$

$$14. ab + cd = \begin{vmatrix} a & d \\ -c & b \end{vmatrix}.$$

$$15. a - b = \begin{vmatrix} a & b \\ 1 & 1 \end{vmatrix}.$$

$$19. \frac{\begin{vmatrix} c & (1-c) \\ 7c & (7-12c) \end{vmatrix}}{\begin{vmatrix} c & 3 \\ 7c & 36 \end{vmatrix}} = \frac{7c - 12c^2 - 7c + 7c^2}{36c - 21c} = \frac{-5c^2}{15c} = \frac{-c}{3}.$$

$$20. \frac{st - cd}{3cx - 5r} = \frac{\begin{vmatrix} s & d \\ c & t \end{vmatrix}}{\begin{vmatrix} 3c & r \\ 5 & x \end{vmatrix}}.$$

$$21. \frac{a + \frac{10}{c}}{a^2 - 12} = \frac{\begin{vmatrix} a & -\frac{1}{c} \\ 10 & 1 \end{vmatrix}}{\begin{vmatrix} a & 4 \\ 3 & a \end{vmatrix}}.$$

$$22. \frac{ax - 6}{2r - 5t} = \frac{\begin{vmatrix} a & 3 \\ 2 & x \end{vmatrix}}{\begin{vmatrix} 2 & t \\ 5 & r \end{vmatrix}}.$$

$$16. \frac{\begin{vmatrix} 5 & 1 \\ 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-5 - 3}{-1 - 1} = 4.$$

$$17. \frac{\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix}} = \frac{3 - 8}{9 - 8} = -5.$$

$$18. \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \frac{ce - fb}{ae - db}.$$

$$23. \frac{\frac{m}{3} - 7}{2m - 1} = \frac{\begin{vmatrix} m & 7 \\ 1 & \frac{1}{3} \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & m \end{vmatrix}}.$$

$$24. \frac{6x - hm}{3a - 3r} = \frac{\begin{vmatrix} 6 & m \\ h & x \end{vmatrix}}{\begin{vmatrix} 3 & r \\ 3 & a \end{vmatrix}}.$$

$$25. \frac{\frac{x^2 + 0}{4} + \frac{15}{a}}{\frac{x^2 + 0}{4} + \frac{15}{a}} = \frac{\begin{vmatrix} x & 0 \\ -3 & x \end{vmatrix}}{\begin{vmatrix} a^2 & -\frac{1}{a} \\ 15 & \frac{1}{4} \end{vmatrix}}.$$

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$$1. \begin{array}{l} 2x + 3y = 7, \\ 3x - 2y = 4. \end{array} \quad x = \frac{\begin{vmatrix} 7 & 3 \\ 4 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}} = \frac{-14 - 12}{-4 - 9} = 2.$$

$$\therefore y = \frac{\begin{vmatrix} 2 & 7 \\ 3 & 4 \end{vmatrix}}{-13} = \frac{8 - 21}{-13} = 1.$$

$$2. \quad \begin{array}{l} 4x = 3y + 8, \\ 5y + 6 = 3x; \end{array} \text{ or } \begin{array}{l} 4x - 3y = 8, \\ 3x - 5y = 6. \end{array} \quad x = \frac{\begin{vmatrix} 8 & -3 \\ 6 & -5 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix}} = \frac{-40 + 18}{-20 + 9} = 2.$$

$$\therefore y = \frac{\begin{vmatrix} 4 & 8 \\ 3 & 6 \end{vmatrix}}{-11} = \frac{24 - 24}{-11} = 0.$$

$$3. \quad \begin{array}{l} 5x + 4y = 10a + 4, \\ x - 2ay = 0. \end{array} \quad x = \frac{\begin{vmatrix} (10a + 4) & 4 \\ 0 & -2a \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 1 & -2a \end{vmatrix}} = \frac{-20a^2 - 8a}{-10a - 4} = 2a.$$

$$\therefore y = \frac{\begin{vmatrix} 5 & (10a + 4) \\ 1 & 0 \end{vmatrix}}{-10a - 4} = \frac{-10a - 4}{-10a - 4} = 1.$$

$$4. \quad \begin{array}{l} .3x + .02y = 185, \\ .5x + .04y = 335. \end{array} \quad x = \frac{\begin{vmatrix} 185 & .02 \\ 335 & .04 \end{vmatrix}}{\begin{vmatrix} .3 & .02 \\ .5 & .04 \end{vmatrix}} = \frac{7.4 - 6.7}{.012 - .01} = \frac{.7}{.002} = 350.$$

$$\therefore y = \frac{\begin{vmatrix} .3 & 185 \\ .5 & 335 \end{vmatrix}}{.002} = \frac{100.5 - 92.5}{.002} = 4000.$$

$$5. \quad \begin{array}{l} 4x + 3y = 6, \\ \frac{3x}{4} + \frac{3y}{4} = 3. \end{array} \quad x = \frac{\begin{vmatrix} 6 & 3 \\ 3 & \frac{3}{4} \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ \frac{3}{4} & \frac{3}{4} \end{vmatrix}} = \frac{4\frac{1}{2} - 9}{3 - 2\frac{1}{4}} = \frac{-\frac{9}{2}}{\frac{3}{4}} = -6.$$

$$\therefore y = \frac{\begin{vmatrix} 4 & 6 \\ \frac{3}{4} & 3 \end{vmatrix}}{\frac{3}{4}} = \frac{12 - 4\frac{1}{2}}{\frac{3}{4}} = \frac{1\frac{5}{2}}{\frac{3}{4}} = 10.$$

$$6. \quad \begin{array}{l} \frac{x}{2} - \frac{y}{3} = 6, \\ \frac{3x}{2} + \frac{2y}{3} = 8. \end{array} \quad x = \frac{\begin{vmatrix} 6 & -\frac{1}{3} \\ 8 & \frac{2}{3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{2} & -\frac{1}{3} \\ \frac{3}{2} & \frac{2}{3} \end{vmatrix}} = \frac{4 + 2\frac{2}{3}}{\frac{1}{3} + \frac{1}{2}} = 8.$$

$$\therefore y = \frac{\begin{vmatrix} \frac{1}{2} & 6 \\ \frac{3}{2} & 8 \end{vmatrix}}{\frac{5}{6}} = \frac{4 - 9}{\frac{5}{6}} = -6.$$

$$7. \quad \begin{aligned} 7x + 5y &= 21c, \\ \frac{x}{c} - \frac{y}{2c} &= 3. \end{aligned}$$

$$x = \frac{\begin{vmatrix} 21c & 5 \\ 3 & -\frac{1}{2c} \end{vmatrix}}{\begin{vmatrix} 7 & 5 \\ \frac{1}{c} & -\frac{1}{2c} \end{vmatrix}} = \frac{-\frac{21}{2} - 15}{-\frac{7}{2c} - \frac{5}{c}} = \frac{-\frac{51}{2}}{-\frac{17}{2c}} = 3c.$$

$$y = \frac{\begin{vmatrix} 7 & 21c \\ \frac{1}{c} & 3 \end{vmatrix}}{\frac{-17}{2c}} = \frac{21 - 21}{\frac{-17}{2c}} = 0.$$

$$8. \quad \begin{aligned} \frac{x}{a} + \frac{y}{b} &= \frac{a+b}{ab}, \\ x - y &= \frac{a^2 - b^2}{ab}. \end{aligned}$$

$$x = \frac{\begin{vmatrix} \frac{a+b}{ab} & \frac{1}{b} \\ \frac{a^2 - b^2}{ab} & -1 \end{vmatrix}}{\begin{vmatrix} \frac{1}{a} & \frac{1}{b} \\ 1 & -1 \end{vmatrix}} = \frac{\frac{-a-b}{ab} - \frac{a^2 - b^2}{ab^2}}{-\frac{1}{a} - \frac{1}{b}} = \frac{a}{b}.$$

$$y = \frac{\begin{vmatrix} \frac{1}{a} & \frac{a+b}{ab} \\ 1 & \frac{a^2 - b^2}{ab} \end{vmatrix}}{-\frac{1}{a} - \frac{1}{b}} = \frac{\frac{a^2 - b^2}{a^2b} - \frac{a+b}{ab}}{-\frac{1}{a} - \frac{1}{b}} = \frac{b}{a}.$$

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1. $x - 2y + z = 6.$
 Let $x = 0$ and $y = 0$, then $z = 6.$
 Let $x = 0$ and $z = 0$, then $y = -3.$
 Let $y = 0$ and $z = 0$, then $x = 6.$
 Let $x = 1$ and $y = 5$, then $z = 15.$
 Let $x = -3$ and $z = 7$, then $y = -1.$
2. $m + n - p = 8, \quad (1)$
 $3m - 2n + 4p = 6. \quad (2)$
 $(1) \cdot 2, \quad 2m + 2n - 2p = 16. \quad (3)$
 $(3) + (2), \quad 5m + 2p = 22. \quad (4)$
 From (4), $p = \frac{22 - 5m}{2}. \quad (5)$

In (5), let m be 0, 2, 4 respectively; then $p = 11, 6, 1.$

Substituting these in (1), $n = 19, 12, 5.$

$m = 0, 2, 4; p = 11, 6, 1; n = 19, 12, 5.$

$$\begin{array}{rcl}
 3. & 2x + 3y + 4z = -14, & (1) \\
 & x - y + 3z = 0, & (2) \\
 & 5x + 2y + z = 14. & (3) \\
 (2) \cdot 2, & 2x - 2y + 6z = 0. & (4) \\
 (3) + (4), & 7x + 7z = 14. & (5) \\
 (2) \cdot 3, & 3x - 3y + 9z = 0. & (6) \\
 (1) + (6), & 5x + 13z = -14. & (7) \\
 (5) \cdot \frac{5}{7}, & 5x + 5z = 10. & (8) \\
 (7) - (8), & 8z = -24. & (9) \\
 & \therefore z = -3. & (10) \\
 \text{From (10) and (5),} & x = 5. & (11) \\
 \text{From (10), (11), and (2),} & y = -4. & (12)
 \end{array}$$

$$\begin{array}{rcl}
 4. & x + 2y + 3z = 14, & (1) \\
 & 4x - 5y + 6z = 12, & (2) \\
 & x + 15y + 9z = 58. & (3) \\
 (1) \cdot 4, & 4x + 8y + 12z = 56. & (4) \\
 (4) - (2), & 13y + 6z = 44. & (5) \\
 (3) - (1), & 13y + 6z = 44. & (6) \\
 (5) \text{ and } (6) \text{ show that the system is indeterminate.} \\
 (3) \text{ really} = (1) \cdot 5 - (2). \text{ Hence (1), (2), and (3) are not independent.}
 \end{array}$$

$$\begin{array}{rcl}
 5. & .4r + .3s - 8t = 4, & (1) \\
 & .5r + .8s + t = 1.2, & (2) \\
 & -r - .5s + 2.6t = -.3. & (3) \\
 (3) \cdot .4, & -.4r - .2s + 1.04t = -.12. & (4) \\
 (1) + (4), & .1s - 6.96t = 3.88. & (5) \\
 (3) \cdot .5, & -.5r - .25s + 1.3t = -.15. & (6) \\
 (2) + (6), & .55s + 2.3t = 1.05. & (7) \\
 (5) \cdot 5.5, & .55s - 38.28t = 21.34. & (8) \\
 (7) - (8), & 40.58t = -20.29. & (9) \\
 & \therefore t = -\frac{1}{2}. & (10) \\
 \text{From (5) and (10),} & s = 4. & (11) \\
 \text{From (3), (10), and (11),} & r = -3. \\
 & r = -3, s = 4, t = -\frac{1}{2}.
 \end{array}$$

$$\begin{array}{rcl}
 6. & .25x + .05y - .10z = -1, & (1) \\
 & .50x - .30y = 0, & (2) \\
 & .05y + .04z = 3. & (3) \\
 (2) \cdot \frac{1}{2}, & .25x - .15y = 0. & (4) \\
 (1) - (4), & .2y - .1z = -1. & (5) \\
 (3) \cdot 4, & .2y + .16z = 12. & (6) \\
 (5) - (6), & -.26z = -13. & (7) \\
 & \therefore z = 50. & (8)
 \end{array}$$

$$\text{From (3) and (8),} \quad y = 20. \quad (9)$$

$$\text{From (2) and (9),} \quad x = 12.$$

$$7. \quad \frac{r}{a} + \frac{3s}{2a} - \frac{t}{3a} = 6, \quad (1)$$

$$7r + 4t = 6s, \quad (2)$$

$$\frac{t}{9} - \frac{s}{6} = 0. \quad (3)$$

$$(1) \cdot 6a, \quad 6r + 9s - 2t = 36a. \quad (4)$$

$$(3) \cdot 54, \quad -9s + 6t = 0. \quad (5)$$

$$(4) + (5), \quad 6r + 4t = 36a. \quad (6)$$

$$(2) \cdot 3, \quad 21r - 18s + 12t = 0. \quad (7)$$

$$(5) \cdot 2, \quad -18s + 12t = 0. \quad (8)$$

$$(7) - (8), \quad 21r = 0. \quad (9)$$

$$\therefore r = 0. \quad (10)$$

$$\text{From (6) and (10),} \quad t = 9a. \quad (11)$$

$$\text{From (5) and (11),} \quad s = 6a. \quad (12)$$

$$8. \quad h + 2k - l = 3b + c, \quad (1)$$

$$5h - 4k - 4l = a + b - 8c, \quad (2)$$

$$\frac{h}{2} - \frac{k}{3} = \frac{3a + b}{6} - \frac{2c}{6}. \quad (3)$$

$$(1) \cdot 4, \quad 4h + 8k - 4l = 12b + 4c. \quad (4)$$

$$(2) - (4), \quad h - 12k = a - 11b - 12c. \quad (5)$$

$$(3) \cdot 6, \quad 3h - 2k = 3a + b - 2c. \quad (6)$$

$$(5) \cdot 3, \quad 3h - 36k = 3a - 33b - 36c. \quad (7)$$

$$(6) - (7), \quad 34k = 34b + 34c. \quad (8)$$

$$k = b + c. \quad (9)$$

$$\text{From (9) and (6),} \quad h = a + b. \quad (10)$$

$$\text{From (10), (9), and (1)} \quad l = a + c.$$

$$9. \quad x + 2y + z = -1, \quad (1)$$

$$2x - y + z = -20, \quad (2)$$

$$-x - y - 5z = 13. \quad (3)$$

$$(1) + (3), \quad y - 4z = 12. \quad (4)$$

$$[(2) + (3) \cdot 2] \div 3, \quad -y - 3z = 2. \quad (5)$$

$$(4) + (5), \quad z = -2. \quad (6)$$

$$\text{From (6) and (5),} \quad y = 4. \quad (7)$$

$$\text{From (6), (7), and (1),} \quad x = -7.$$

$$10. \quad x + y = 10, \quad (1)$$

$$x + z = 12, \quad (2)$$

$$y + z = 14. \quad (3)$$

$$\begin{array}{ll}
 (1) - (2), & y - z = -2. \quad (4) \\
 (3) + (4), & 2y = 12. \quad (5) \\
 & \therefore y = 6. \quad (6) \\
 (3) - (4), & 2z = 16. \quad (7) \\
 & z = 8. \quad (8) \\
 \text{From (6) and (1),} & x = 4.
 \end{array}$$

$$11. \quad \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 3, \quad (1)$$

$$\frac{a}{x} - \frac{b}{y} + \frac{2c}{z} = 2, \quad (2)$$

$$\frac{5a}{x} - \frac{3b}{y} - \frac{2c}{z} = 0. \quad (3)$$

$$(2) \cdot 5, \quad \frac{5a}{x} - \frac{5b}{y} + \frac{10c}{z} = 10. \quad (4)$$

$$(1) - (2), \quad \frac{2b}{y} - \frac{c}{z} = 1. \quad (5)$$

$$(3) - (4), \quad \frac{2b}{y} - \frac{12c}{z} = -10. \quad (6)$$

$$(5) - (6), \quad \frac{11c}{z} = 11. \quad (7)$$

$$z = c. \quad (8)$$

$$\text{From (5) and (8),} \quad \frac{2b}{y} - \frac{c}{c} = 1. \quad (9)$$

$$\therefore y = b. \quad (10)$$

$$\text{From (1), (8), and (10),} \quad \frac{x}{a} + 1 + 1 = 3.$$

$$\therefore x = a.$$

$$12. \quad r + s + t + u = 6, \quad (1)$$

$$2r - s + 3t - u = -16, \quad (2)$$

$$5r + 9s - 6t + 4u = 81, \quad (3)$$

$$r + 5s + 9t - 7u = -54. \quad (4)$$

$$(4) \cdot 2, \quad 2r + 10s + 18t - 14u = -108. \quad (5)$$

$$(4) \cdot 5, \quad 5r + 25s + 45t - 35u = -270. \quad (6)$$

$$(1) - (4), \quad -4s - 8t + 8u = 60. \quad (7)$$

$$(2) - (5), \quad -11s - 15t + 13u = 92. \quad (8)$$

$$(3) - (6), \quad -16s - 51t + 39u = 351. \quad (9)$$

$$(7) \div -4, \quad s + 2t - 2u = -15. \quad (10)$$

$$(10) \cdot 11, \quad 11s + 22t - 22u = -165. \quad (11)$$

$$(10) \cdot 16, \quad 16s + 32t - 32u = -240. \quad (12)$$

$$(8) + (11), \quad 7t - 9u = -73. \quad (13)$$

$$(9) + (12), \quad -19t + 7u = 111. \quad (14)$$

$$(13) \cdot 7, \quad 49t - 63u = -511. \quad (15)$$

$$(14) \cdot 9, \quad -171t + 63u = 999. \quad (16)$$

$$(15) + (16), \quad -122t = 488. \quad (17)$$

$$\therefore t = -4. \quad (18)$$

$$\text{From (18) and (13),} \quad -28 - 9u = -73. \quad (19)$$

$$\therefore u = 5. \quad (20)$$

$$\text{From (20), (18), and (10),} \quad s - 8 - 10 = -15. \quad (21)$$

$$\therefore s = 3. \quad (22)$$

$$\text{From (20), (18), (22), and (1), } r + 3 - 4 + 5 = 6.$$

$$\therefore r = 2.$$

$$13. \quad 4x - 3y + 2z = 20, \quad (1)$$

$$5x + 4y - 10z = 3, \quad (2)$$

$$-7x - 18y + 34z = 31. \quad (3)$$

$$(1) \cdot 5, \quad 20x - 15y + 10z = 100. \quad (4)$$

$$(4) + (2), \quad 25x - 11y = 103. \quad (5)$$

$$(1) \cdot 17 - (3), \quad 75x - 33y = 309. \quad (6)$$

But (6) = 3 · (5). Therefore the system is indeterminate.

$$14. \quad 4h - k + m = 0, \quad (1)$$

$$7k + 2m + x = 0, \quad (2)$$

$$8h + 4m + x = 0, \quad (3)$$

$$16h + 5k - x = 4. \quad (4)$$

$$(2) - (3), \quad -8h + 7k - 2m = 0. \quad (5)$$

$$(2) + (4), \quad 16h + 12k + 2m = 4. \quad (6)$$

$$(1) \cdot 2, \quad 8h - 2k + 2m = 0. \quad (7)$$

$$(5) + (6), \quad 8h + 19k = 4. \quad (8)$$

$$(5) + (7), \quad 5k = 0. \quad (9)$$

$$k = 0. \quad (10)$$

$$\text{From (10) and (8),} \quad 8h = 4. \quad (11)$$

$$\therefore h = \frac{1}{2}. \quad (12)$$

$$\text{From (10), (12), and (1),} \quad 2 + m = 0. \quad (13)$$

$$\therefore m = -2. \quad (14)$$

$$\text{From (10), (14), and (2),} \quad -4 + x = 0.$$

$$\therefore x = 4.$$

$$15. \quad ax + by + cz = p, \quad (1)$$

$$dx + ey + fz = q, \quad (2)$$

$$gx + hy + iz = r. \quad (3)$$

$$(1) \cdot f, \quad afx + bfy + cfz = pf. \quad (4)$$

$$(2) \cdot c, \quad cdx + cey + cfz = cq. \quad (5)$$

$$(4) - (5), \quad x(af - cd) + y(bf - ce) = pf - cq. \quad (6)$$

$$(2) \cdot i, \quad dix + eiy + fiz = qi. \quad (7)$$

$$(3) \cdot f, \quad fgx + fhy + fiz = fr. \quad (8)$$

$$(7) - (8), \quad x(di - fg) + y(ei - fh) = qi - fr. \quad (9)$$

$$(6) \cdot (ei - fh), \quad x(af - cd)(ei - fh) + y(bf - ce)(ei - fh) \\ = (pf - cq)(ei - fh). \quad (10)$$

$$(9) \cdot (bf - ce), \quad x(di - fg)(bf - ce) + y(bf - ce)(ei - fh) \\ = (qi - fr)(bf - ce). \quad (11)$$

$$(10) - (11), \quad x[(af - cd)(ei - fh) - (di - fg)(bf - ce)] \\ = (pf - cq)(ei - fh) - (qi - fr)(bf - ce).$$

$$x(afci - af^2h + cd fh - bf di + bf^2g - cefg) \\ = pfei - pf^2h + cqfh - bfqi + bf^2r - cefr.$$

$$x = \frac{pf(ei - fh) + qf(ch - bi) + rf(bf - ce)}{af(ei - fh) + df(ch - bi) + gf(bf - ce)} \\ = \frac{p(ei - fh) + q(ch - bi) + r(bf - ce)}{a(ei - fh) + d(ch - bi) + g(bf - ce)}.$$

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$$1. \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 4 & 1 \end{vmatrix} = 1 \cdot 3 \cdot 1 + 2 \cdot 4 \cdot 1 + 2 \cdot 1 \cdot 1 - 1 \cdot 3 \cdot 2 - 1 \cdot 4 \cdot 1 - 1 \cdot 2 \cdot 1 \\ = 3 + 8 + 2 - 6 - 4 - 2 = 1.$$

$$2. \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1 \cdot 5 \cdot 9 + 4 \cdot 8 \cdot 3 + 7 \cdot 6 \cdot 2 - 3 \cdot 5 \cdot 7 - 6 \cdot 8 \cdot 1 - 9 \cdot 4 \cdot 2 \\ = 45 + 96 + 84 - 105 - 48 - 72 = 0.$$

$$3. \begin{vmatrix} 1 & -2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot 2 \cdot 1 + 3 \cdot 1 \cdot 3 + 1 \cdot 1 \cdot (-2) - 3 \cdot 2 \cdot 1 - 1 \cdot 1 \cdot 1 - 1 \cdot 3 \cdot (-2) \\ = 2 + 9 - 2 - 6 - 1 + 6 = 8.$$

$$4. \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ a & b & c \end{vmatrix} = a \cdot 2 \cdot c + 1 \cdot b \cdot c + a \cdot 3 \cdot b - c \cdot 2 \cdot a - 3 \cdot b \cdot a - c \cdot 1 \cdot b \\ = 2ac + bc + 3ab - 2ac - 3ab - bc = 0.$$

$$5. \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 5 \\ -3 & 1 & 1 \end{vmatrix} = 1 \cdot 4 \cdot 1 + 2 \cdot 1 \cdot 0 + (-3) \cdot 5 \cdot 0 - 0 \cdot 4 \cdot (-3) - 5 \cdot 1 \cdot 1 - 1 \cdot 2 \cdot 0 \\ = 4 + 0 + 0 - 0 - 5 - 0 = -1.$$

$$6. \begin{vmatrix} 1 & 1 & 1 \\ a & 1 & a \\ -a & 5 & 6 \end{vmatrix} = 1 \cdot 1 \cdot 6 + a \cdot 5 \cdot 1 + (-a) \cdot a \cdot 1 - 1 \cdot 1 \cdot (-a) - a \cdot 5 \cdot 1 - 6 \cdot a \cdot 1 \\ = 6 + 5a - a^2 + a - 5a - 6a = -a^2 - 5a + 6.$$

$$7. - \begin{vmatrix} 2 & -1 & 3 \\ -3 & 1 & 2 \\ 4 & 5 & 1 \end{vmatrix} = -[2 \cdot 1 \cdot 1 + (-3) \cdot 5 \cdot 3 + 4 \cdot 2 \cdot (-1) - 3 \cdot 1 \cdot 4 \\ - 2 \cdot 5 \cdot 2 - 1 \cdot (-3) \cdot (-1)] \\ = -(2 - 45 - 8 - 12 - 20 - 3) = 86.$$

$$8. \begin{vmatrix} a & 2 & 7 \\ b & 3 & 8 \\ c & 4 & 9 \end{vmatrix} = a \cdot 3 \cdot 9 + b \cdot 4 \cdot 7 + c \cdot 8 \cdot 2 - 7 \cdot 3 \cdot c - 8 \cdot 4 \cdot a - 9 \cdot b \cdot 2 \\ = 27a + 28b + 16c - 21c - 32a - 18b = 10b - 5a - 5c.$$

$$9. \begin{vmatrix} x & y & 1 \\ 2 & 2 & 1 \\ 3 & 4 & 1 \end{vmatrix} = x \cdot 2 \cdot 1 + 2 \cdot 4 \cdot 1 + 3 \cdot 1 \cdot y - 1 \cdot 2 \cdot 3 - 1 \cdot 4 \cdot x - 1 \cdot 2 \cdot y \\ = 2x + 8 + 3y - 6 - 4x - 2y = y - 2x + 2.$$

$$10. \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + dhc + gbf - ceg - fha - idb.$$

$$11. \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = xy_1 + x_1y_2 + x_2y - y_1x_2 - y_2x - x_1y.$$

$$12. c \begin{vmatrix} 5 & 1 & b \\ c & 0 & 0 \\ 8 & 6 & a \end{vmatrix} = c(5 \cdot 0 \cdot a + c \cdot 6 \cdot b + 8 \cdot 0 \cdot 1 - b \cdot 0 \cdot 8 - 0 \cdot 6 \cdot 5 - a \cdot c \cdot 1) \\ = c(0 + 0 + 6bc - 0 - 0 - ac) = 6bc^2 - ac^2.$$

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$$1. x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{1 - 2 - 3 - 3 - 1 - 2}{1 - 1 - 1 - 1 - 1 - 1} = \frac{-10}{-4} = \frac{5}{2}.$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix}}{-4} = \frac{2 + 3 - 1 - 2 + 3 - 1}{-4} = \frac{4}{-4} = -1.$$

Substituting $\frac{5}{2}$ for x and -1 for y in the first equation, $\frac{5}{2} - 1 + z = 1$.
Whence $z = -\frac{1}{2}$.

$$2. x = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix}} = \frac{-1 + 0 + 0 - 0 + 2 - 0}{-1 + 4 - 2 - 1 + 2 + 4} = \frac{1}{6}.$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 1 & 0 & -1 \end{vmatrix}}{6} = \frac{0 + 0 - 1 + 0 + 0 + 2}{6} = \frac{1}{6}.$$

Substituting $\frac{1}{6}$ for x and $\frac{1}{6}$ for y in the first equation, $\frac{1}{6} + \frac{1}{6} + z = 1$.
Whence $z = \frac{1}{2}$.

$$3. x = \frac{\begin{vmatrix} 5 & 1 & -1 \\ 6 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & 3 & 2 \end{vmatrix}} = \frac{0 - 18 + 0 - 0 + 30 - 12}{0 - 3 + 2 - 0 + 12 - 2} = \frac{0}{9} = 0.$$

Substituting 0 for x in $x - 2z = 6$, $z = -3$.

Substituting 0 for x and -3 for z in $3y + 2z = x$, $y = 2$.

$$4. x = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}} = \frac{0 + 0 + 3 - 0 - 1 - 2}{0 + 0 + 0 - 0 - 1 - 1} = \frac{0}{-2} = 0.$$

Substituting 0 for x in $x + y = 1$, $y = 1$.

Substituting 0 for x in $x + z = 2$, $z = 2$.

$$5. x = \frac{\begin{vmatrix} 3a & 1 & 0 \\ 4a & 0 & 1 \\ 5a & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}} = \frac{0 + 0 + 5a - 0 - 3a - 4a}{0 + 0 + 0 - 0 - 1 - 1} = \frac{-2a}{-2} = a.$$

Substituting a for x in $x + y = 3a$, $y = 2a$.

Substituting a for x in $x + z = 4a$, $z = 3a$.

$$6. x = \frac{\begin{vmatrix} 9 & \frac{1}{2} & 0 \\ 8 & 0 & \frac{1}{3} \\ 13 & \frac{1}{3} & \frac{1}{2} \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} \end{vmatrix}} = \frac{0 + 0 + \frac{1^3}{6} - 0 - 1 - 2}{0 + 0 + 0 - 0 - \frac{1}{2} - \frac{1}{8}} = \frac{\frac{-5}{6}}{\frac{-35}{216}} = \frac{36}{7}.$$

Substituting $\frac{36}{7}$ for x in first equation, $\frac{12}{7} + \frac{y}{2} = 9$, or $y = \frac{102}{7}$.

Substituting $\frac{36}{7}$ for x in second equation, $\frac{18}{7} + \frac{z}{3} = 8$, or $z = \frac{114}{7}$.

$$7. x = \frac{\begin{vmatrix} 0 & b & 0 \\ 2bc & 0 & -b \\ b^2 & -c & a \end{vmatrix}}{\begin{vmatrix} a & b & 0 \\ c & 0 & -b \\ b & -c & a \end{vmatrix}} = \frac{0 + 0 - b^4 - 0 - 0 - 2ab^2c}{0 + 0 - b^3 - 0 - abc - abc} = \frac{-b^4 - 2ab^2c}{-b^3 - 2abc} = b.$$

Substituting b for x in $ax + by = 0$, $y = -a$.

Substituting b for x in $cx - bz = 2bc$, $z = -c$.

$$8. x = \frac{\begin{vmatrix} 2hk & k & -l \\ 2kl & k & l \\ 2hl & -k & l \end{vmatrix}}{\begin{vmatrix} h & k & -l \\ -h & k & l \\ h & -k & l \end{vmatrix}} = \frac{2hk^2l + 2k^2l^2 + 2hkl^2 + 2hkl^2 + 2hk^2l - 2k^2l^2}{hkl - hkl + hkl + hkl + hkl + hkl} = \frac{4hk^2l + 4hkl^2}{4hkl} = k + l.$$

$$y = \frac{\begin{vmatrix} h & 2hk & -l \\ -h & 2kl & l \\ h & 2hl & l \end{vmatrix}}{4hkl} = \frac{2hkl^2 + 2h^2l^2 + 2h^2kl + 2hkl^2 - 2h^2l^2 + 2h^2kl}{4hkl} = \frac{4hkl^2 + 4h^2kl}{4hkl} = l + h.$$

Then from the first equation, $hk + hl + kl + hk - lz = 2kh$. Whence $z = h + k$.

$$9. x = \frac{\begin{vmatrix} 0 & m & 1 \\ (ma-a) & 1 & m \\ 4ma & -3m & 1 \end{vmatrix}}{\begin{vmatrix} m & m & 1 \\ m & 1 & m \\ m & -3m & 1 \end{vmatrix}} = \frac{0 - 3m^2a + 3ma + 4m^3a - 4ma - 0 - m^2a + ma}{m - 3m^2 + m^3 - m + 3m^3 - m^2} = \frac{ma(4m^2 - 4m)}{m(4m^2 - 4m)} = a.$$

$$x_1 = \frac{\begin{vmatrix} m & 0 & 1 \\ m & (ma-a) & m \\ m & 4ma & 1 \end{vmatrix}}{m(4m^2 - 4m)} = \frac{m^2a - ma + 4m^2a + 0 - m^2a + ma - 4m^3a - 0}{m(4m^2 - 4m)} = \frac{ma(-4m^2 + 4m)}{m(4m^2 - 4m)} = -a.$$

Then from the first equation, $ma - ma + x_2 = 0$. Whence $x_2 = 0$.

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1. Let a = the number of days A requires alone,
and b = the number of days B requires alone.

$$\text{Then} \quad \frac{1}{a} + \frac{1}{b} = \frac{7}{24}, \quad (1)$$

$$\text{and} \quad \frac{2}{a} + \frac{2}{b} + \frac{5}{2a} = 1. \quad (2)$$

$$(1) \cdot 2, \quad \frac{2}{a} + \frac{2}{b} = \frac{7}{12}. \quad (3)$$

$$(2) - (3), \quad \frac{5}{2a} = \frac{5}{12}. \quad (4)$$

$$\text{Whence} \quad a = 6.$$

$$\text{From (1) and (4),} \quad \frac{1}{6} + \frac{1}{b} = \frac{7}{24}.$$

$$\text{Whence} \quad b = 8.$$

2. Let m = the number of days the man requires to do the work,
and b = the number of days the boy requires to do the work.

$$\text{Then} \quad \frac{1}{m} + \frac{1}{b} = \frac{1}{18}, \quad (1)$$

$$\frac{5}{m} + \frac{9}{b} = \frac{1}{3}. \quad (2)$$

$$(1) \cdot 5, \quad \frac{5}{m} + \frac{5}{b} = \frac{5}{18}. \quad (3)$$

$$(2) - (3), \quad \frac{4}{b} = \frac{1}{18}. \quad (4)$$

$$\text{Whence} \quad b = 72. \quad (5)$$

$$\text{From (1) and (5),} \quad \frac{1}{m} + \frac{1}{72} = \frac{1}{18}.$$

$$\therefore m = 24.$$

$$3. \quad ax + by = 2.$$

$$2a + 3b = 2. \quad (1)$$

$$6a + 5b = 2. \quad (2)$$

$$(1) \cdot 3, \quad 6a + 9b = 6. \quad (3)$$

$$(3) - (2), \quad 4b = 4. \quad (4)$$

$$\text{Whence} \quad b = 1. \quad (5)$$

$$\text{From (1) and (5),} \quad 2a + 3 = 2.$$

$$\text{Whence} \quad a = -\frac{1}{2}.$$

4. Let x = the number of hours required to go upstream.

Then $24 - x$ = the number of hours required to go downstream.

$$\text{Therefore} \quad x \cdot 10 = 14(24 - x).$$

$$10x = 336 - 14x.$$

Whence

$$x = 14.$$

Then

$$24 - x = 10.$$

5. Let
and

x = in dollars the sum at 5%,

y = in dollars the sum at 6%.

Then

$$.05x + .06y = 100. \quad (1)$$

$$.06x + .05y = 98. \quad (2)$$

$$(1) \cdot 600, \quad 30x + 36y = 60,000. \quad (3)$$

$$(2) \cdot 500, \quad 30x + 25y = 49,000. \quad (4)$$

$$(3) - (4), \quad 11y = 11,000. \quad (5)$$

$$\text{Whence} \quad y = 1000. \quad (6)$$

$$\text{From (3) and (6),} \quad 30x + 36,000 = 60,000.$$

$$\text{Whence} \quad x = 800.$$

6. Let
and

x = in dollars the sum at 5%,

y = in dollars the sum at 6%.

Then

$$x + y = 4000, \quad (1)$$

$$\text{and} \quad .05\left(\frac{x}{.90}\right) + .06\left(\frac{y}{1.10}\right) = 220. \quad (2)$$

$$(2) \cdot 990, \quad 55x + 54y = 217,800.$$

$$(1) \cdot 55, \quad 55x + 55y = 220,000.$$

$$\text{Whence} \quad y = 2200.$$

$$\text{From (1),} \quad x + 2200 = 4000.$$

$$\text{Whence} \quad x = 1800.$$

7. Let
and

d = the distance in miles from M to P,

r = the usual rate in miles per hour.

Then

$$\frac{d}{r} = \text{scheduled time in hours.}$$

Therefore

$$\frac{d}{3r} = \frac{d}{r} - 2, \quad (1)$$

and

$$\frac{d}{25} = \frac{d}{r} - 2 + \frac{4}{5}. \quad (2)$$

From (1),

$$\frac{d}{3r} = 2, \text{ or } d = 6r. \quad (3)$$

From (3) and (2),

$$\frac{6r}{25} = 6 - 2 + \frac{4}{5}.$$

Whence

$$r = 20.$$

Then

$$d = 6r, \text{ or } 120.$$

8. Let d = the distance from M to N in miles,
and r = the usual rate from M to N in miles per hour.

Then $\frac{d}{r}$ = the scheduled time in hours.

Therefore
$$\frac{d}{\frac{6r}{5}} = \frac{d}{r} - \frac{1}{2} + \frac{1}{10}, \quad (1)$$

and
$$\frac{15}{r} + \frac{d-15}{\frac{6r}{5}} = \frac{d}{r} - \frac{1}{2} + \frac{1}{5}. \quad (2)$$

From (1),
$$\frac{d}{6r} = \frac{4}{10}, \text{ or } d = \frac{12r}{5}. \quad (3)$$

From (2) and (3), $d = 60$, and $r = 25$.

9. Let p = the rate of the passenger train in feet per second,
and f = the rate of the freight train in feet per second.

Then
$$\frac{1430 + 550}{p + f} = 18,$$

and
$$\frac{1430 + 550}{p - f} = 90.$$

Whence $p = 66$ feet per second, or 45 miles per hour,
and $f = 44$ feet per second, or 30 miles per hour.

10. Let f = the rate of the first in feet per second,
and s = the rate of the second in feet per second.

Then
$$\frac{1320}{s} = \frac{1120}{f} + 4,$$

and
$$\frac{1320}{s} = \frac{1200}{f}.$$

Whence $f = 20$, and $s = 22$.

11. Let x = number of gallons of 95% alcohol,
and y = number of gallons of 15% alcohol.

Then
$$\frac{.95x + .15y}{10} = \frac{45}{100}, \text{ and } x + y = 10.$$

Whence $x = 3\frac{3}{4}$, and $y = 6\frac{1}{4}$.

12. Let g and s in pounds be the respective weights of the gold and of the silver.

Then
$$g + s = 20,$$

and
$$\frac{18\frac{1}{4}}{19\frac{1}{4}}g + \frac{9\frac{1}{2}}{10\frac{1}{2}}s = 18\frac{3}{4}.$$

Whence $g = 15\frac{1}{8}$, and $s = 4\frac{7}{8}$.

13. Let x , y , and z be respectively the number of degrees in each angle.

Then $x = 2y$,

$$x + y = z,$$

and $x + y + z = 180$.

Whence $z = 90$, $x = 60$, and $y = 30$.

14. Let f , s , and t equal respectively the three numbers.

Then $f + s + t = 217$,

$$\frac{f}{s} = 5,$$

and $\frac{s}{t} = 5$.

Whence $f = 175$, $s = 35$, and $t = 7$.

15. Let h , t , and u be respectively the hundreds', tens', and units' digits of the number.

Then $100h + 10u + t = 100h + 10t + u - 27$, (1)

$$100t + 10h + u = 100h + 10t + u - 180, \quad (2)$$

and $h + t + u = 14$. (3)

From (1), $u - t = -3$. (4)

From (2), $h - t = 2$. (5)

From (3), (4), and (5), $h = 7$, $t = 5$, and $u = 2$.

Hence the number is 752.

16. Let f , s , and t be the number of hours respectively that the first, second, and third pipes require to fill the tank.

Then $\frac{1}{f} + \frac{1}{s} + \frac{1}{t} = \frac{7}{8}$,

$$\frac{1}{f} + \frac{1}{s} = \frac{3}{4},$$

and $\frac{1}{s} + \frac{1}{t} = \frac{3}{8}$.

Whence $f = 2$, $s = 4$, and $t = 8$.

17. Let f , s , t , and x in feet be the length of the sides of the quadrilateral.

Then $f + s = 80$,

$$s + t = 108,$$

$$t + x = 116,$$

$$\text{and } x - f = 24;$$

Whence $f = 32$,

$$s = 48,$$

$$t = 60,$$

$$\text{and } x = 56;$$

$$\begin{cases} f + s = 80, \\ s + t = 108, \\ t + x = 116, \\ f - x = 24. \end{cases} \text{ or } \begin{cases} f = 56, \\ s = 24, \\ t = 84, \\ x = 32. \end{cases}$$

18. Let f = the cost of the first chair in cents,

and s = the cost of the second chair in cents.

Then $f + s = 100h$,

and $f - s = m$.

Whence $f = 50h + \frac{m}{2}$,

and $s = 50h - \frac{m}{2}$.

19. Let a = the number of dollars A has,
 and b = the number of dollars B has.
 Then $a + b = d$,
 and $a - c + m = \frac{1}{3}(b + c - m)$.
 Whence $a = \frac{4c - 4m + d}{4}$, and $b = \frac{3d - 4c + 4m}{4}$.

20. Let a = the number of days A requires to do the work,
 and b = the number of days B requires to do the work.
 Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{m}$,
 and $b = \frac{a}{c}$.
 Whence $a = m + cm$, and $b = \frac{m}{c} + m$.

21. Let x = his rate in miles per hour in still water,
 and r = the rate in miles per hour of the river.
 Then $\frac{m}{x + r} = t$,
 and $\frac{m}{x - r} = a$.
 Whence $x = \frac{ma + mt}{2at}$, and $r = \frac{ma - mt}{2at}$.

22. Since each of two children has borrowed more than his share of the estate, he receives nothing. The note of each, however, is collectible up to the amount of his share in the estate, as will be clear from the following solution :

Let x = the number of dollars in each child's share.
 Then $5x$ = the number of dollars in the widow's share,
 and $10x$ = the number of dollars in the entire estate.
 But two children have received their share (and more).
 Hence the estate equals $(1200 + 2x)$ dollars.
 Therefore $10x = 1200 + 2x$.
 Whence $x = 150$.
 Then the widow receives \$750.
 Each of the three children receives \$150.
 Each of the two children receives nothing.

23. When $5x + 2y = 42$,

x	8	6	4	2
y	1	6	11	16

24. $7x + 2y = 36.$
 $\therefore y = \frac{36 - 7x}{2} = 18 - \frac{7x}{2}.$

Hence if x is any even number, y is an integer. Therefore the positive values which satisfy the equation are

x	4	2
y	4	11

For $7x + 2y = 36.$

x	0	$5\frac{1}{2}$	2	4
y	18	0	11	4

25. $5x + 2y = 73.$
 $y = \frac{73 - 5x}{2} = 36 + \frac{1 - 5x}{2}.$

Hence if x is any odd number, y is an integer. Therefore the positive values which satisfy the equation are

x	13	11	9	7	5	3	1
y	4	9	14	19	24	29	34

For $5x + 2y = 73.$

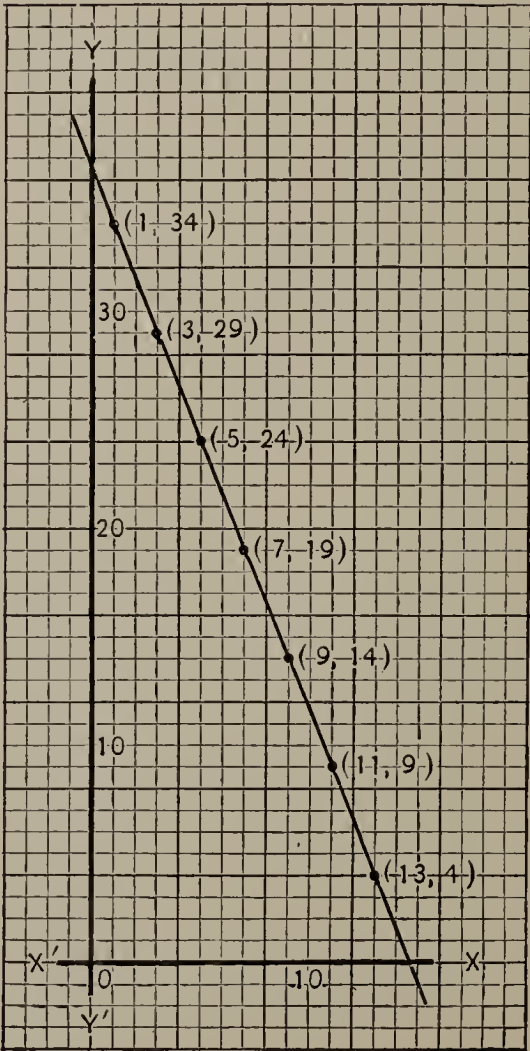
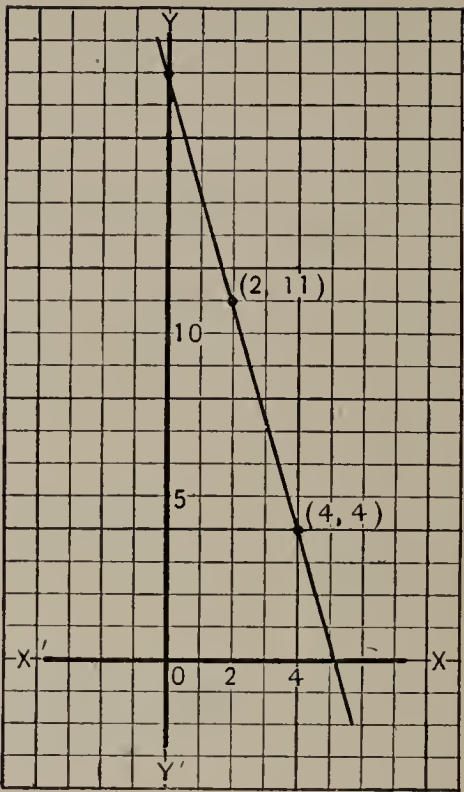
x	14.6	13	9
y	0	4	14

 etc.

26. Let c = the cost of one calf in dollars, and p = the cost of one pig in dollars.
Then $6c + 4p = 72.$
 $p = \frac{72 - 6c}{4} = 18 - \frac{3c}{2}.$

Hence if c is any even number, p is an integer. Therefore the values which satisfy the equation are

c	2	4	6	8	10
p	15	12	9	6	3



27. Let q be a possible number of quarters and n be a corresponding number of nickels.

Then $25q + 5n = 175,$
and $n = 35 - 5q.$

Hence any integral value of q gives an integral value of n . Therefore the values which satisfy the equation are

q	1	2	3	4	5	6
n	30	25	20	15	10	5

Six ways.

28. Let c , p , and l represent the number of the calves, the pigs, and the lambs respectively.

Then $6c + 3p + 4l = 126,$ (1)

and $c + p + l = 32.$ (2)

(2) $\cdot 6,$ $6c + 6p + 6l = 192.$ (3)

(3) $-$ (1), $3p + 2l = 66.$ (4)

Whence $l = \frac{66 - 3p}{2} = 33 - \frac{3p}{2}.$

Hence if p is any *even* integer, l is an integer. Therefore the values which satisfy (4) are

p	2	4	6	8	10	12	14	16	18	20
l	30	27	24	21	18	15	12	9	6	3

Substitution of these values in (2) gives respectively the following for c :

c	0	1	2	3	4	5	6	7	8	9
-----	---	---	---	---	---	---	---	---	---	---

Rejecting $p = 2$, $l = 30$, and $c = 0$, there are nine possible sales.

29. Let n , d , and q represent the number of the nickels, the dimes, and the quarters respectively.

Then $5n + 10d + 25q = 240,$ (1)

and $n = d + q.$ (2)

(2) $\cdot 5,$ $5n - 5d - 5q = 0.$ (3)

(1) $-$ (3), $15d + 30q = 240.$ (4)

(4) $\div 15,$ $d + 2q = 16.$

Whence $d = 16 - 2q.$

If q is any integer whatever, d has an integral value. Therefore the values which satisfy (4) are

q	1	2	3	4	5	6	7
d	14	12	10	8	6	4	2

Substitution of these values in (2) gives respectively the following for n :

n	15	14	13	12	11	10	9
-----	----	----	----	----	----	----	---

Hence there are seven possible groups, all different.

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1-3. See pages 101-103.

4. By extracting the square root of the square root.

5. See page 229.

6. 2, 3.

7. (a) No. (b) No. The result in either case would be the sixth root.

$$8. \quad \begin{array}{r} a^6 - 10a^4 - 4a^3 + 25a^2 + 20a + 4 \sqrt{a^3 - 5a - 2} \\ a^6 \end{array}$$

$$\begin{array}{r} 2a^3 - 5a \sqrt{-10a^4 - 4a^3} \\ -10a^4 \qquad + 25a^2 \\ \hline 2a^3 - 10a - 2 \sqrt{-4a^3 \qquad + 20a + 4} \\ -4a^3 \qquad + 20a + 4 \end{array}$$

Therefore the square roots are $\pm (a^3 - 5a - 2)$.

$$9. \quad \begin{array}{r} 4a^8 + 12a^4 - 7 - 24a^{-4} + 16a^{-8} \sqrt{2a^4 + 3 - 4a^{-4}} \\ 4a^8 \end{array}$$

$$\begin{array}{r} 4a^4 + 3 \sqrt{12a^4 - 7} \\ 12a^4 + 9 \\ \hline 4a^4 + 6 - 4a^{-4} \sqrt{-16 - 24a^{-4} + 16a^{-8}} \\ -16 - 24a^{-4} + 16a^{-8} \end{array}$$

Therefore the square roots are $\pm (2a^4 + 3 - 4a^{-4})$.

$$10. \quad \begin{array}{r} 49c^{-6} - 28c^{-4} + 74c^{-2} - 20 + 25c^2 \sqrt{7c^{-3} - 2c^{-1} + 5c} \\ 49c^{-6} \end{array}$$

$$\begin{array}{r} 14c^{-3} - 2c^{-1} \sqrt{-28c^{-4} + 74c^{-2}} \\ -28c^{-4} + 4c^{-2} \\ \hline 14c^{-3} - 4c^{-1} + 5c \sqrt{70c^{-2} - 20 + 25c^2} \\ 70c^{-2} - 20 + 25c^2 \end{array}$$

Therefore the square roots are $\pm (7c^{-3} - 2c^{-1} + 5c)$.

$$11. \quad \begin{array}{r} 9x^4 - 6x^{\frac{7}{2}} + x^3 - 66x^{\frac{5}{2}} + 22x^2 + 121x \sqrt{3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}}} \\ 9x^4 \end{array}$$

$$\begin{array}{r} 6x^2 - x^{\frac{3}{2}} \sqrt{-6x^{\frac{7}{2}} + x^3} \\ -6x^{\frac{7}{2}} + x^3 \\ \hline 6x^2 - 2x^{\frac{3}{2}} - 11x^{\frac{1}{2}} \sqrt{-66x^{\frac{5}{2}} + 22x^2 + 121x} \\ -66x^{\frac{5}{2}} + 22x^2 + 121x \end{array}$$

Therefore the square roots are $\pm (3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}})$.

$$\begin{array}{r|l}
 12. \quad 16m^{-7} - 8m^{-4} + m^{-1} + 104m - 26m^4 + 169m^9 & \underline{4m^{-\frac{7}{2}} - m^{-\frac{1}{2}} + 13m^{\frac{9}{2}}} \\
 16m^{-7} & \\
 \hline
 8m^{-\frac{7}{2}} - m^{-\frac{1}{2}} & \begin{array}{l} -8m^{-4} + m^{-1} \\ -8m^{-4} + m^{-1} \end{array} \\
 \hline
 8m^{-\frac{7}{2}} - 2m^{-\frac{1}{2}} + 13m^{\frac{9}{2}} & \begin{array}{l} 104m - 26m^4 + 169m^9 \\ 104m - 26m^4 + 169m^9 \end{array} \\
 \hline
 \end{array}$$

Therefore the square roots are $\pm (4m^{-\frac{7}{2}} - m^{-\frac{1}{2}} + 13m^{\frac{9}{2}})$.

$$\begin{array}{r|l}
 13. \quad \frac{a^2}{9c^2} + \frac{4a}{3c} + \frac{20}{3} + \frac{16c}{a} + \frac{16c^2}{a^2} & \underline{\frac{a}{3c} + 2 + \frac{4c}{a}} \\
 \frac{a^2}{9c^2} & \\
 \hline
 \frac{2a}{3c} + 2 & \begin{array}{l} \frac{4a}{3c} + \frac{20}{3} \\ \frac{4a}{3c} + 4 \end{array} \\
 \hline
 \frac{2a}{3c} + 4 + \frac{4c}{a} & \begin{array}{l} \frac{8}{3} + \frac{16c}{a} + \frac{16c^2}{a^2} \\ \frac{8}{3} + \frac{16c}{a} + \frac{16c^2}{a^2} \end{array} \\
 \hline
 \end{array}$$

Therefore the square roots are $\pm \left(\frac{a}{3c} + 2 + \frac{4c}{a} \right)$.

$$\begin{array}{r|l}
 14. \quad \frac{1}{4a^4} - \frac{3x}{a^3} + \frac{9x^2}{a^2} + \frac{1}{5x} - \frac{6a}{5} + \frac{a^4}{25x^2} & \underline{\frac{1}{2a^2} - \frac{3x}{a} + \frac{a^2}{5x}} \\
 \frac{1}{4a^4} & \\
 \hline
 \frac{1}{a^2} - \frac{3x}{a} & \begin{array}{l} -\frac{3x}{a^3} + \frac{9x^2}{a^2} \\ -\frac{3x}{a^3} + \frac{9x^2}{a^2} \end{array} \\
 \hline
 \frac{1}{a^2} - \frac{6x}{a} + \frac{a^2}{5x} & \begin{array}{l} \frac{1}{5x} - \frac{6a}{5} + \frac{a^4}{25x^2} \\ \frac{1}{5x} - \frac{6a}{5} + \frac{a^4}{25x^2} \end{array} \\
 \hline
 \end{array}$$

Therefore the square roots are $\pm \left(\frac{1}{2a^2} - \frac{3x}{a} + \frac{a^2}{5x} \right)$.

$$\begin{array}{r}
 15. \qquad 1 + 2x \overline{) 1 + x - \frac{x^2}{2} + \frac{x^3}{2}} \\
 \qquad \qquad \qquad \frac{1}{2 + x} \overline{) 2x} \\
 \qquad \qquad \qquad \qquad \qquad \frac{2x + x^2}{2 + 2x - \frac{x^2}{2}} \overline{) -x^2} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \frac{-x^2 - x^3 + \frac{x^4}{4}}{2 + 2x - x^2} \overline{) x^3 - \frac{x^4}{4}}
 \end{array}$$

The required terms of the square roots are $\pm \left(1 + x - \frac{x^2}{2} + \frac{x^3}{2}\right)$.

$$\begin{array}{r}
 16. \qquad \frac{25}{9} + a^3 \overline{) \frac{5}{3} + \frac{3a^3}{10} - \frac{27a^6}{1000} + \frac{50,000}{243a^9}} \\
 \qquad \qquad \qquad \frac{25}{9} \overline{) \frac{10}{3} + \frac{3a^3}{10}} \overline{) a^3} \\
 \qquad \qquad \qquad \qquad \qquad \frac{a^3 + \frac{9a^6}{100}}{\frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{1000}} \overline{) -\frac{9a^6}{100}} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \frac{-\frac{9a^6}{100} - \frac{81a^9}{5000} + \frac{1,000,000}{729a^{12}}}{\frac{10}{3} + \frac{8a^3}{5} - \frac{27a^6}{500}} \overline{) \frac{81a^9}{5000}}
 \end{array}$$

The required terms of the square roots are

$$\pm \left(\frac{5}{3} + \frac{3a^3}{10} - \frac{27a^6}{1000} + \frac{243a^9}{50,000} \right).$$

$$\begin{array}{r}
 17. \qquad 7c^{-3} - 2c^{-1} + 5c \overline{) 7^{\frac{1}{2}}c^{-\frac{3}{2}} - 7^{-\frac{1}{2}}c^{\frac{1}{2}} + 17 \cdot 7^{-\frac{3}{2}}c^{\frac{5}{2}}} \\
 \qquad \qquad \qquad \frac{7c^{-3}}{2 \cdot 7^{\frac{1}{2}}c^{-\frac{3}{2}} - 7^{-\frac{1}{2}}c^{\frac{1}{2}}} \overline{) -2c^{-1} + 5c} \\
 \qquad \qquad \qquad \qquad \qquad \frac{-2c^{-1} + \frac{c}{7}}{2 \cdot 7^{\frac{1}{2}} \cdot c^{-\frac{3}{2}} - 2 \cdot 7^{-\frac{1}{2}}c^{\frac{1}{2}} + 17c^{\frac{5}{2}}} \overline{) \frac{34c}{7}}
 \end{array}$$

The required terms of the fourth roots are $\pm (7^{\frac{1}{2}}c^{-\frac{3}{2}} - 7^{-\frac{1}{2}}c^{\frac{1}{2}} + 17 \cdot 7^{-\frac{3}{2}}c^{\frac{5}{2}})$.

Page 340 (First set)

1. (a) 4.4; (b) 6.7; (c) 7.7; (d) 8.3.
2. (a) 2.9; (b) 2.6; (c) 3.7; (d) 5.4; (e) 5.3.
3. (a) 5.7; (b) 37; (c) 63; (d) 69.
4. (a) 32.8; (b) 59; (c) 22; (d) 175; (e) 111.

Page 340 (Second set)

$$\begin{array}{r}
 1. \quad 68'89' \overline{)83} \\
 \quad \quad 64 \\
 \hline
 163 \overline{)489} \\
 \quad \quad \underline{489} \quad \text{Ans. } \pm 83.
 \end{array}$$

$$\begin{array}{r}
 2. \quad 5'61'69' \overline{)237} \\
 \quad \quad 4 \\
 \hline
 43 \overline{)161} \\
 \quad \quad \underline{129} \\
 467 \overline{)3269} \\
 \quad \quad \underline{3269} \quad \text{Ans. } \pm 237.
 \end{array}$$

$$\begin{array}{r}
 3. \quad .67'24' \overline{).82} \\
 \quad \quad 64 \\
 \hline
 162 \overline{)324} \\
 \quad \quad \underline{324} \quad \text{Ans. } \pm .82.
 \end{array}$$

$$\begin{array}{r}
 4. \quad 1'.46'41' \overline{)1.21} \\
 \quad \quad 1 \\
 \hline
 22 \overline{)46} \\
 \quad \quad \underline{44} \\
 241 \overline{)241} \\
 \quad \quad \underline{241} \quad \text{Ans. } \pm 1.21.
 \end{array}$$

$$\begin{array}{r}
 5. \quad 4'.20'25' \overline{)2.05} \\
 \quad \quad 4 \\
 \hline
 405 \overline{)2025} \\
 \quad \quad \underline{2025} \quad \text{Ans. } \pm 2.05.
 \end{array}$$

$$\begin{array}{r}
 6. \quad .04'02'80'49' \overline{).2007} \\
 \quad \quad 04 \\
 \hline
 4007 \overline{)028049} \\
 \quad \quad \underline{28049} \quad \text{Ans. } \pm .2007.
 \end{array}$$

$$\begin{array}{r}
 7. \quad 5'00'00'00'00' \overline{)2.2360} \\
 \quad \quad 4 \\
 \hline
 42 \overline{)100} \\
 \quad \quad \underline{84} \\
 443 \overline{)1600} \\
 \quad \quad \underline{1329} \\
 4466 \overline{)27100} \\
 \quad \quad \underline{26796} \\
 44720 \overline{)30400} \quad \text{Ans. } \pm 2.2360.
 \end{array}$$

$$\begin{array}{r}
 8. \quad .07'00'00'00' \overline{).2645} \\
 \quad \quad 04 \\
 \hline
 46 \overline{)300} \\
 \quad \quad \underline{276} \\
 524 \overline{)2400} \\
 \quad \quad \underline{2096} \\
 5285 \overline{)30400} \quad \text{Ans. } \pm .2645.
 \end{array}$$

$$\begin{array}{r}
 9. \quad 7' \overline{)13} \\
 \quad \quad 1.85'71'42'85' \overline{)1.3627} \\
 \quad \quad \quad 1 \\
 \hline
 23 \overline{)85} \\
 \quad \quad \underline{69} \\
 266 \overline{)1671} \\
 \quad \quad \underline{1596} \\
 2722 \overline{)7542} \\
 \quad \quad \underline{5444} \\
 27247 \overline{)209885} \quad \text{Ans. } \pm 1.3627.
 \end{array}$$

$$\begin{array}{r}
 10. \quad 93' \overline{)237} \overline{)2.54838709} \\
 \quad \quad 186 \\
 \hline
 510 \\
 465 \\
 \hline
 450 \\
 372 \\
 \hline
 780 \\
 744 \\
 \hline
 360 \\
 279 \\
 \hline
 810 \\
 744 \\
 \hline
 660 \\
 651 \\
 \hline
 900
 \end{array}$$

$$2.54'83'87'09' \overline{)1.5963}$$

$$\begin{array}{r}
 1 \\
 \hline
 25 \overline{)154} \\
 \quad \quad \underline{125} \\
 309 \overline{)2983} \\
 \quad \quad \underline{2781} \\
 3186 \overline{)20287} \\
 \quad \quad \underline{19116} \\
 31923 \overline{)117109} \quad \text{Ans. } \pm 1.5963.
 \end{array}$$

11. $\sqrt{183^2 + 264^2} = 321.22$, hypotenuse.

12. $\sqrt{90^2 + 90^2} = 127.27$.

13. $\sqrt{207^2 - 83^2} = 189.63$ feet.

14. $\sqrt{292849^2 - 207000^2} = 207,151$ feet.

15. Let x represent the altitude in inches.

Then $x = \sqrt{(11)^2 - (\frac{11}{2})^2} = \frac{11}{2} \sqrt{3}$.

Whence $x = 9.5$ inches.

16. $x^2 - \frac{x^2}{4} = 100$.

$$x = \frac{20}{3} \sqrt{3} = 11.547 \text{ inches.}$$

17.
$$\begin{aligned} \text{Area} &= \sqrt{37 \cdot 25 \cdot 10 \cdot 2} \\ &= \sqrt{37 \cdot 5^2 \cdot 5 \cdot 2^2} \\ &= 10 \sqrt{185} \\ &= 136.014 \text{ square feet.} \end{aligned}$$

18.
$$\begin{aligned} \text{Area} &= \sqrt{33 \cdot 11 \cdot 11 \cdot 11} \\ &= \sqrt{(11)^4 3} \\ &= 121 \sqrt{3} = 209.57 \text{ square inches.} \end{aligned}$$

19. $\pi r^2 = 40$.

Then $r = \sqrt{\frac{40}{\pi}} = 3.56$ feet.

20. Diagonal $= \sqrt{14^2 + 20^2 + 30^2} = \sqrt{1496} = 38.67$.

21. Diagonal $= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} = 1.732$ feet.

22. Let $ABCD$ be the floor (AB being 24), DKC one end wall, and AKC an adjoining wall. Let DKC be folded back about DC , to the plane of the floor. Fold ADK back about AD . Then the required line is the shorter of the two lines BK .

BK (across end wall) $= \sqrt{24^2 + (40 + 14)^2} = 59.09$ feet.

BK (across side wall) $= \sqrt{(24 + 14)^2 + 40^2} = 55.17$ feet, required length.

23. Let the integers be 2 and 3.

Then
$$\begin{aligned} 2^2 + 3^2 &= 13, & 5^2 + 12^2 &= 13^2 = 169. \\ 3^2 - 2^2 &= 5, \end{aligned}$$

and $2 \cdot 3 \cdot 2 = 12$.

One can thus take any two integers and find three integers which will be the three sides of a right triangle.

24. $28 = 2 \cdot 7 \cdot 2 = 2 \cdot 1 \cdot 14$.

Now using 7 and 2, and 1 and 14, as in Exercise 23, we obtain:

$$\begin{aligned} 7^2 + 2^2 &= 53, & 14^2 + 1^2 &= 197, \\ 7^2 - 2^2 &= 45, & 14^2 - 1^2 &= 195, \\ \overline{28}^2 + \overline{45}^2 &= \overline{53}^2, & \overline{28}^2 + \overline{195}^2 &= \overline{197}^2. \end{aligned}$$

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1. $x^{\frac{3}{5}} = \sqrt[5]{x^3}$.
2. $(ac)^{\frac{2}{3}} = \sqrt[3]{a^2c^2}$.
3. $(4x)^{\frac{1}{3}} = \sqrt[3]{4x}$.
4. $4x^{\frac{1}{3}} = 4\sqrt[3]{x}$.
5. $6cx^{\frac{3}{4}} = 6c\sqrt[4]{x^3}$.
6. $a^{\frac{2}{3}}c^{\frac{3}{5}} = \sqrt[3]{a^2}\sqrt[5]{c^3}$.
7. $3a^{\frac{1}{2}}(c^2x)^{\frac{2}{3}} = 3\sqrt{a}\sqrt[3]{c^4x^2}$.
8. $\frac{4a^{\frac{2}{3}}(c-x)^{\frac{4}{3}}}{a^{\frac{1}{2}}(c-x)} = 4a^{\frac{1}{6}}(c-x)^{\frac{1}{3}}$
 $= 4\sqrt[6]{a(c-x)^2}$.
9. $4^{\frac{1}{2}} = 2$.
10. $36^{\frac{1}{2}} = 6$.
11. $64^{\frac{1}{3}} = 4$.
12. $81^{\frac{1}{4}} = 3$.
13. $4^{\frac{3}{2}} = 2^3 = 8$.
14. $27^{\frac{2}{3}} = 3^2 = 9$.
15. $(-8)^{\frac{2}{3}} = (-2)^2 = 4$.
16. $(\frac{1}{9})^{\frac{1}{2}} = \frac{1}{3}$.
17. $(-32)^{\frac{3}{5}} = (-2)^3 = -8$.
18. $(\frac{4}{25})^{\frac{1}{2}} \cdot (\frac{125}{8})^{\frac{1}{3}} = \frac{2}{5} \cdot \frac{5}{2} = 1$.
19. $(-125)^{\frac{2}{3}} \cdot (\frac{1}{25})^{\frac{3}{2}} = 25 \cdot \frac{1}{25} = \frac{1}{5}$.
20. $(-243)^{\frac{2}{5}} \cdot (81)^{\frac{3}{4}} = 9 \cdot 27 = 243$.
21. $\sqrt[3]{x^4} = x^{\frac{4}{3}}$.
22. $\sqrt{ac^3} = a^{\frac{1}{2}}c^{\frac{3}{2}}$.
23. $3\sqrt{x^5} = 3x^{\frac{5}{2}}$.
24. $4\sqrt[3]{4x^2} = 2^{\frac{8}{3}}x^{\frac{2}{3}}$.
25. $5\sqrt[3]{8x^4} = 10x^{\frac{4}{3}}$.
26. $6\sqrt[3]{64x^5} = 24x^{\frac{5}{3}}$.
27. $5\sqrt[3]{-125c^2} = -25c^{\frac{2}{3}}$.
28. $c\sqrt{(x+a)^3} = (cx+ca)(x+a)^{\frac{1}{2}}$.
29. $\frac{5c\sqrt{ax^3} \cdot \sqrt[3]{5x^4}}{\sqrt[3]{5x^2}\sqrt{ax}} = 5c\sqrt{x^2} \cdot \sqrt[3]{x^2}$
 $= 5cx^{\frac{5}{3}}$.
30. $\sqrt[n]{x^a} \cdot \sqrt[n]{c} = x^{\frac{a}{n}}c^{\frac{1}{n}}$.
31. $\sqrt[n]{x^{4a}} \cdot \sqrt[n]{x^{2a}} = x^{\frac{4a}{n}}x^{\frac{2a}{n}} = x^{\frac{6a}{n}}$.
32. (a) 5. (g) 3 in $\sqrt[3]{5}$.
 (b) $3\sqrt{-2}$. (h) ax in $3\sqrt{ax}$.
 (c) $\frac{3}{4}$. (i) $+2$ of $\sqrt[3]{8}$.
 (d) $\sqrt{3}$. (j) 8 of $\sqrt{64}$.
 (e) $\sqrt{7}$. (k) -3 of $\sqrt[3]{-27}$.
 (f) $\sqrt{8}$.
33. See pp. 237-238.
34. The first four are rational; the last two are irrational.
35. See p. 238.
36. No. See pp. 238-239.
37. Yes. See pp. 238-239.
38. See p. 238.
39. $\sqrt{3}$, $\sqrt[3]{6}$ only are surds; all are radicals.
40. 2, 2, -2.
41. Second, third, third, third, and fourth.
42. One; one.
43. Two; none.

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1. $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$.
2. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$.
3. $2\sqrt{75} = 10\sqrt{3}$.
4. $4\sqrt[3]{-54} = -12\sqrt[3]{2}$.
5. $\sqrt{\sqrt{4}} = \sqrt{2}$.
6. $\sqrt[3]{3\sqrt{9}} = \sqrt[3]{9}$.
7. $\sqrt{\frac{3}{5}} = \frac{1}{5}\sqrt{15}$.
8. $\sqrt{\frac{5}{8}} = \frac{1}{4}\sqrt{10}$.

$$9. \sqrt[3]{-\frac{3}{4}} = \frac{1}{2} \sqrt[3]{-6}.$$

$$12. \sqrt{3^2 - \left(\frac{3}{2}\right)^2} = \sqrt{9 - \frac{9}{4}} = \frac{3}{2} \sqrt{3}.$$

$$10. \sqrt[3]{\frac{1}{7}} = \frac{1}{7} \sqrt[3]{49}.$$

$$13. \sqrt{7^2 - \left(\frac{7}{2}\right)^2} = \sqrt{49 - \frac{49}{4}} = \frac{7}{2} \sqrt{3}.$$

$$11. 6 \sqrt[3]{-\frac{1}{9}} = 2 \sqrt[3]{-3}.$$

$$14. \sqrt{4 - 8\sqrt{3}} = \sqrt{4(1 - 2\sqrt{3})} = 2 \sqrt{1 - 2\sqrt{3}}.$$

$$15. \sqrt{36 + 18\sqrt{5}} = \sqrt{9(4 + 2\sqrt{5})} = 3 \sqrt{4 + 2\sqrt{5}}.$$

$$16. \sqrt[3]{81 - 3\sqrt{243}} = \sqrt[3]{81 - 27\sqrt{3}} = 3 \sqrt[3]{3 - \sqrt{3}}.$$

$$17. \sqrt{R^2 - 3R^2\sqrt{5}} = R \sqrt{1 - 3\sqrt{5}}.$$

$$18. \sqrt{\frac{R^2 - R^2\sqrt{6}}{3}} = \frac{R}{3} \sqrt{3 - 3\sqrt{6}}.$$

$$19. \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \frac{R}{2} \sqrt{3}.$$

$$20. \sqrt{R^2 + \left(\frac{R}{3}\right)^2\sqrt{3}} = \sqrt{R^2 + \frac{R^2}{9}\sqrt{3}} = \frac{R}{3} \sqrt{9 + \sqrt{3}}.$$

$$21. 3\sqrt{5} = \sqrt{45}.$$

$$23. 2c \sqrt[3]{c^2} = \sqrt[3]{8c^5}.$$

$$25. \frac{a}{3} \sqrt[3]{\frac{9}{a^2}} = \sqrt[3]{\frac{a}{3}}.$$

$$22. 2\sqrt[3]{8} = \sqrt[3]{64}.$$

$$24. 4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{16}.$$

$$26. (2a+1) \sqrt{\frac{2}{4a^2-1}} = \sqrt{\frac{4a+2}{2a-1}}. \quad 27. \frac{x-3a}{5} \sqrt[3]{\frac{125}{(x-3a)^2}} = \sqrt[3]{x-3a}.$$

$$\begin{aligned} 28. \frac{c}{2} \sqrt{a^2 - \left(\frac{a^2 + c^2 - b^2}{2c}\right)^2} &= \frac{c}{2} \sqrt{\left(a + \frac{a^2 + c^2 - b^2}{2c}\right) \left(a - \frac{a^2 + c^2 - b^2}{2c}\right)} \\ &= \frac{c}{2} \sqrt{\left(\frac{a^2 + 2ac + c^2 - b^2}{2c}\right) \left(\frac{-a^2 + 2ac - c^2 + b^2}{2c}\right)} \\ &= \frac{c}{2} \sqrt{\frac{(a+b+c)(a-b+c)(a+b-c)(-a+b+c)}{4c^2}} \\ &= \sqrt{\frac{a+b+c}{2} \cdot \frac{a+b-c}{2} \cdot \frac{a-b+c}{2} \cdot \frac{-a+b+c}{2}} \\ &= \sqrt{s(s-a)(s-b)(s-c)}. \end{aligned}$$

$$29. \sqrt{\sqrt{x}} = \sqrt{x^{\frac{1}{2}}} = x^{\frac{1}{4}} = \sqrt[4]{x}.$$

$$31. \sqrt[3]{\sqrt{x}} = \sqrt[3]{x^{\frac{1}{2}}} = x^{\frac{1}{6}} = \sqrt[6]{x}.$$

$$30. \sqrt{\sqrt[3]{x}} = \sqrt{x^{\frac{1}{3}}} = x^{\frac{1}{6}} = \sqrt[6]{x}.$$

$$32. \sqrt[3]{\sqrt{8a^2x}} = \sqrt[3]{(8a^2x)^{\frac{1}{2}}} = \sqrt[6]{8a^2x}.$$

$$33. \sqrt{3\sqrt{3}} = \sqrt{\sqrt{27}} = \sqrt[4]{27}.$$

$$34. \sqrt{3\sqrt{3\sqrt{3}}} = \sqrt{\sqrt{27\sqrt{3}}} = \sqrt{\sqrt{\sqrt{3^7}}} = \sqrt[8]{2187}.$$

$$35. 2\sqrt[3]{2\sqrt[3]{2}} = 2\sqrt[3]{\sqrt[3]{16}} = 2\sqrt[9]{16}.$$

$$36. \sqrt[n]{\sqrt[n]{x^c}} = \sqrt[n]{x^{\frac{c}{n}}} = x^{\frac{c}{n^2}} = \sqrt[n^2]{x^c}.$$

$$37. \sqrt[n]{x^{\frac{a}{n}}} = x^{\left(\frac{a}{n} \div n\right)} = x^{\frac{a}{n^2}} = \sqrt[n^2]{x^a}.$$

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1. $\sqrt{50} + \sqrt{98} - \sqrt{32} = 5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2} = 8\sqrt{2}.$
2. $\sqrt[3]{192} - 4\sqrt[3]{24} + \sqrt[3]{375} = 4\sqrt[3]{3} - 8\sqrt[3]{3} + 5\sqrt[3]{3} = \sqrt[3]{3}.$
3. $10\sqrt{\frac{6}{5}} - \sqrt{\frac{3}{10}} + 4\sqrt{\frac{15}{2}} = 2\sqrt{30} - \frac{1}{10}\sqrt{30} + 2\sqrt{30} = \frac{39}{10}\sqrt{30}.$
4. $3\sqrt{\frac{2}{7}} + 3\sqrt{\frac{7}{2}} - 2\sqrt{\frac{1}{14}} = \frac{3}{7}\sqrt{14} + \frac{3}{2}\sqrt{14} - \frac{1}{7}\sqrt{14} = \frac{25}{14}\sqrt{14}.$
5. $\sqrt{x^3} + \sqrt[4]{x^2} - 12\sqrt[6]{x^3} = x\sqrt{x} + \sqrt{x} - 12\sqrt{x} = (x - 11)\sqrt{x}.$
6. $\sqrt{\frac{3a}{x}} + \sqrt{\frac{3x}{a}} - \sqrt{\frac{ax}{3}} = \frac{1}{x}\sqrt{3ax} + \frac{1}{a}\sqrt{3ax} - \frac{1}{3}\sqrt{3ax}$
 $= \frac{3a + 3x - ax}{3ax}\sqrt{3ax}.$
7. $\sqrt{\frac{a}{x^3}} - \sqrt{\frac{a}{x^5}} + \sqrt{\frac{5x^3}{a}} = \frac{1}{x^2}\sqrt{ax} - \frac{1}{x^3}\sqrt{ax} + \frac{x}{a}\sqrt{5ax}$
 $= \frac{x-1}{x^3}\sqrt{ax} + \frac{x}{a}\sqrt{5ax}.$
8. $\sqrt[4]{32x^5} + \sqrt[4]{1250x} - \sqrt[4]{512x} - \sqrt[4]{2x}$
 $= 2x\sqrt[4]{2x} + 5\sqrt[4]{2x} - 4\sqrt[4]{2x} - \sqrt[4]{2x}$
 $= 2x\sqrt[4]{2x}.$
9. $\sqrt[3]{(a-c)^4} + c\sqrt[6]{a^2 - 2ac + c^2} + (a+c)\sqrt[3]{a-c}$
 $= (a-c)\sqrt[3]{a-c} + c\sqrt[3]{a-c} + (a+c)\sqrt[3]{a-c}$
 $= (2a+c)\sqrt[3]{a-c}.$
10. $\sqrt{\frac{a}{c}} - \sqrt{\frac{c}{a}} + \sqrt{\frac{a^2 + c^2}{ac}} + 2 - \sqrt{\frac{a^2 + c^2}{ac}} - 2$
 $= \frac{1}{c}\sqrt{ac} - \frac{1}{a}\sqrt{ac} + \frac{a+c}{ac}\sqrt{ac} - \frac{a-c}{ac}\sqrt{ac}$
 $= \frac{a-c + a+c - a+c}{ac}\sqrt{ac} = \frac{a+c}{ac}\sqrt{ac}.$
11. $\sqrt[3]{24} + \sqrt[3]{(3a+9)(a+3)^2} - \sqrt[3]{81} + a\sqrt[6]{9} - 4\sqrt[3]{3}$
 $= 2\sqrt[3]{3} + (a+3)\sqrt[3]{3} - 3\sqrt[3]{3} + a\sqrt[3]{3} - 4\sqrt[3]{3}$
 $= (2a-2)\sqrt[3]{3}.$
12. $2\sqrt{9a^3 - 9a^2b} - 3\sqrt{9ab^2 - 9b^3} + \sqrt{(a^2 - b^2)(a+b)}$
 $= 6a\sqrt{a-b} - 9b\sqrt{a-b} + (a+b)\sqrt{a-b}$
 $= (7a-8b)\sqrt{a-b}.$

$$\begin{aligned}
 13. (a-b) \sqrt{\frac{a+b}{a-b}} + \sqrt{25a^2 - 25b^2} + \frac{a+b}{a-b} \sqrt{\frac{36ab^2 - 36b^3}{a+b}} \\
 = \sqrt{a^2 - b^2} + 5\sqrt{a^2 - b^2} + \frac{6b}{a-b} \sqrt{a^2 - b^2} \\
 = 6 + \frac{6b}{a-b} \sqrt{a^2 - b^2} = \frac{6a}{a-b} \sqrt{a^2 - b^2}.
 \end{aligned}$$

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1. $\sqrt{3} \cdot \sqrt{27} = \sqrt{81} = 9.$
2. $\sqrt{\frac{4}{5}} \cdot \sqrt{\frac{5}{6}} \cdot \sqrt{\frac{3}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{2} \sqrt{2}.$
3. $\sqrt{3} \cdot \sqrt[3]{2} = 3^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} = 3^{\frac{3}{6}} \cdot 2^{\frac{2}{6}} = \sqrt[6]{3^3 \cdot 2^2} = \sqrt[6]{108}.$
4. $\sqrt[4]{6} \cdot \sqrt{2} = \sqrt[4]{6} \cdot \sqrt[4]{4} = \sqrt[4]{24}.$
5. $\sqrt[3]{12} \cdot \sqrt{\frac{1}{8}} = \sqrt[6]{(12)^2 (\frac{1}{8})^3} = \frac{1}{2} \sqrt[6]{18}.$
6. $\sqrt[3]{a^2} \cdot \sqrt{a^3} = \sqrt[6]{a^4 \cdot a^9} = a^2 \sqrt[6]{a}.$
7. $\sqrt[3]{\frac{x}{a}} \cdot \sqrt{\frac{a}{x}} = \sqrt[6]{\frac{x^2}{a^2} \cdot \frac{a^3}{x^3}} = \sqrt[6]{\frac{a}{x}} = \frac{1}{x} \sqrt[6]{ax^5}.$
8. $\sqrt[4]{2x^3} \cdot \sqrt{3x} = \sqrt[4]{18x^5} = x \sqrt[4]{18x}.$
9. $(\sqrt{x-3})^2 = x-3.$
10. $(2\sqrt{3x-1})^2 = 12x-4.$
11. $(3\sqrt[4]{x-2})^2 = 9\sqrt{x-2}.$
12. $3\sqrt{x-3} \cdot \sqrt{4x-8} = 6\sqrt{x^2-5x+6}.$
13. $(\sqrt{x} - \sqrt{x-3})^2 = x + x - 3 - 2\sqrt{x^2-3x} = 2x-3-2\sqrt{x^2-3x}.$
14. $(\sqrt{x-3} - \sqrt{4x-7})^2 = 5x-10-2\sqrt{4x^2-19x+21}.$
15. $(\sqrt{x} - \sqrt{3x})4\sqrt{x} = 4x-4x\sqrt{3}.$
16. $\left(\frac{6-2\sqrt{5}}{3}\right)^2 \frac{[(\sqrt{5}+1)(\sqrt{5}+1)]^2}{2} \cdot \frac{(\sqrt{2}+1)(9\sqrt{2}-9)}{16}$
 $= \frac{56-24\sqrt{5}}{9} (28+12\sqrt{5}) \frac{9}{16} = (14-6\sqrt{5})(7+3\sqrt{5}) = 8.$
18. $\sqrt[3]{6} = \sqrt[6]{6^2} = \sqrt[6]{36}.$
 $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}. \quad \therefore \sqrt[3]{6} > \sqrt{3}.$
19. $\sqrt[3]{19} = \sqrt[6]{(19)^2} = \sqrt[6]{361}.$
 $\sqrt{7} = \sqrt[6]{7^3} = \sqrt[6]{343}. \quad \therefore \sqrt[3]{19} > \sqrt{7}.$
20. $\sqrt{3} = \sqrt[4]{9}.$
 $\sqrt[4]{6} = \sqrt[4]{6}. \quad \therefore \sqrt{3} > \sqrt[4]{6}.$
21. $2\sqrt{5} = \sqrt{20} = \sqrt[6]{8000}.$
 $\sqrt[3]{89} = \sqrt[6]{(89)^2} = \sqrt[6]{7921}. \quad \therefore 2\sqrt{5} > \sqrt[3]{89}.$

$$22. \quad 3\sqrt{3} = \sqrt{27} = \sqrt[6]{(27)^3} = \sqrt[6]{19683}. \quad \therefore 3\sqrt{3} > 2\sqrt[3]{10}.$$

$$2\sqrt[3]{10} = \sqrt[3]{80} = \sqrt[6]{(80)^2} = \sqrt[6]{6400}.$$

$$23. \quad \sqrt[4]{48} = 2\sqrt[4]{3} = 2\sqrt[20]{243}. \quad \therefore \sqrt[4]{48} > \sqrt[5]{64}.$$

$$\sqrt[5]{64} = 2\sqrt[5]{2} = 2\sqrt[20]{16}.$$

$$24. \quad \sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}.$$

$$\sqrt[3]{6} = \sqrt[6]{6^2} = \sqrt[6]{36}. \quad \therefore \sqrt[9]{125} < \sqrt{3} < \sqrt[3]{6}.$$

$$\sqrt[9]{125} = \sqrt[3]{5} = \sqrt[6]{25}.$$

$$25. \quad 4\sqrt[4]{6} = 4\sqrt[12]{216} = \sqrt[12]{(4^{12})216}.$$

$$3\sqrt[6]{25} = 3\sqrt[12]{625} = \sqrt[12]{(3^{12})625}.$$

$$4\sqrt[9]{64} = 4\sqrt[3]{4} = 4\sqrt[12]{4^4} = \sqrt[12]{(4^{12})256}.$$

$$(3^{12})625 < (4^{12})216 < (4^{12})256.$$

$$\therefore 3\sqrt[6]{25} < 4\sqrt[4]{6} < 4\sqrt[9]{64}.$$

$$26. \quad \sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}.$$

$$\sqrt[3]{3x^2} = \sqrt[6]{(3x^2)^2} = \sqrt[6]{9x^4}.$$

$$27. \quad \sqrt[3]{a+b} = \sqrt[6]{(a+b)^2}.$$

$$\sqrt{a-b} = \sqrt[6]{(a-b)^3}.$$

$$28. \quad 2x\sqrt[3]{5xy} = 2x\sqrt[12]{625x^4y^4}.$$

$$5x\sqrt[4]{3xy} = 5x\sqrt[12]{27x^3y^3}.$$

$$29. \quad \sqrt[3]{xy} = \sqrt[60]{x^{20}y^{20}}.$$

$$\sqrt[4]{xy^2} = \sqrt[60]{x^{15}y^{30}}.$$

$$\sqrt[5]{x^2y} = \sqrt[60]{x^{24}y^{12}}.$$

$$30. \quad (\sqrt[3]{3})^2 = \sqrt[3]{9}.$$

$$31. \quad (2\sqrt[3]{4})^2 = 4\sqrt[3]{16} = 8\sqrt[3]{2}.$$

$$32. \quad (\sqrt{5} - \sqrt{5})^2 = 5 - \sqrt{5}.$$

$$33. \quad (4\sqrt{3} - \sqrt{5})^2 = 48 - 16\sqrt{5}.$$

$$34. \quad (\sqrt[3]{4} - 4\sqrt{3})^3 = \sqrt[3]{64} - 32\sqrt{3} = 2\sqrt[3]{8} - 4\sqrt{3}.$$

$$35. \quad (\sqrt[4]{6} - 3\sqrt{2})^2 = \sqrt{6} - 3\sqrt{2}.$$

$$36. \quad (3\sqrt{5})^3 = 27\sqrt{125} = 135\sqrt{5}.$$

$$37. \quad (3\sqrt[3]{2} - 2\sqrt{3})^3 = 54 - 54\sqrt[6]{432} + 108\sqrt[3]{2} - 24\sqrt{3}.$$

$$38. \quad ((\sqrt[4]{1} - \sqrt{2})^3)^3 = (\sqrt[4]{7} - 5\sqrt{2})^3 = \sqrt[4]{1393} - 985\sqrt{2}.$$

$$39. \quad (5\sqrt{5} + 9\sqrt{3} - \sqrt{7} + 2\sqrt{105})(\sqrt{3} + \sqrt{5} - \sqrt{7})$$

$$= 5\sqrt{15} + 27 - \sqrt{21} + 6\sqrt{35} + 25 + 9\sqrt{15} - \sqrt{35}$$

$$+ 10\sqrt{21} - 5\sqrt{35} - 9\sqrt{21} + 7 - 14\sqrt{15} = 59.$$

$$40. \quad (\sqrt{a} - \sqrt[4]{ac} + \sqrt{c})(\sqrt{a} + \sqrt[4]{ac} + \sqrt{c})(a + c + \sqrt{ac})$$

$$= (a + \sqrt{ac} + c)(a + \sqrt{ac} + c)$$

$$= a^2 + (a + c)2\sqrt{ac} + 3ac + c^2.$$

$$41. \quad (\sqrt{2x-1} - \sqrt{5})(2\sqrt{2x-1} + \sqrt{45})(4x - \sqrt{10x-5} - 17)$$

$$= [2(2x-1) - 2\sqrt{10x-5} + 3\sqrt{10x-5} - 15](4x - \sqrt{10x-5} - 17)$$

$$= (4x + \sqrt{10x-5} - 17)(4x - \sqrt{10x-5} - 17).$$

$$= 16x^2 - 146x + 294.$$

42. $\sqrt{R^2 - \left(\frac{R}{3}\sqrt{5} - \frac{R}{3}\right)^2} = \sqrt{R^2 - \frac{5R^2 - 2R^2\sqrt{5} + R^2}{9}}$
 $= \sqrt{\frac{2R^2\sqrt{5} + 3R^2}{9}} = \frac{R}{3}\sqrt{2\sqrt{5} + 3}.$
43. $\left[\left(R - \frac{R}{6}\sqrt{3}\right)^2 + \left(\frac{R}{6}\right)^2\right]^{\frac{1}{2}} = \sqrt{R^2 - \frac{2R^2\sqrt{3}}{6} + \frac{3R^2}{36} + \frac{R^2}{36}}$
 $= \sqrt{\frac{40R^2 - 12R^2\sqrt{3}}{36}} = \frac{R}{3}\sqrt{10 - 3\sqrt{3}}.$
44. $\sqrt{R^2 - \left(\frac{R\sqrt{4 - \sqrt{2}}}{2}\right)^2} = \sqrt{R^2 - \frac{4R^2 - R^2\sqrt{2}}{4}}$
 $= \sqrt{\frac{4R^2 - 4R^2 + R^2\sqrt{2}}{4}}$
 $= \frac{R}{2}\sqrt{\sqrt{2}} = \frac{R\sqrt[4]{2}}{2}.$
45. $\left(\frac{R}{2}\sqrt{3 + \sqrt{3}}\right)\left(\frac{R}{2}\sqrt{3 - \sqrt{3}}\right) = \frac{R^2}{4}\sqrt{9 - 3} = \frac{R^2}{4}\sqrt{6}.$
46. $\sqrt{\left(e^x + \frac{2}{e^x}\right)^2 - (e^x - 2e^{-x})(e^x - 2e^{-x}) + e^{2x} + e^{-2x} - 6}$
 $= \sqrt{e^{2x} + 4 + 4e^{-2x} - e^{2x} + 4 - 4e^{-2x} + e^{2x} + e^{-2x} - 6}$
 $= \sqrt{e^{2x} + 2 + e^{-2x}} = e^x + e^{-x}.$

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1. $\sqrt{7}.$ 3. $\sqrt[4]{2}.$ 5. $\sqrt{5} + 7.$ 7. $\sqrt{3a} + \sqrt{2x}.$
 2. $\sqrt[3]{2}.$ 4. $\sqrt[3]{4}.$ 6. $3\sqrt{7} + 2\sqrt{13}.$ 8. $\sqrt{x - c} + \sqrt{a}.$
9. $(\sqrt{2} + \sqrt{3} + \sqrt{5})\sqrt{6}, \text{ or } 2\sqrt{3} + 3\sqrt{2} + \sqrt{30}.$
10. $\sqrt{8} \div \sqrt{24} = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3}.$ 12. $24 \div 3\sqrt{3} = \frac{24\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{8}{3}\sqrt{3}.$
11. $8 \div 4\sqrt{3} = \frac{8\sqrt{3}}{4\sqrt{3}\sqrt{3}} = \frac{2}{3}\sqrt{3}.$ 13. $a \div c\sqrt{x} = \frac{a\sqrt{x}}{c\sqrt{x}\sqrt{x}} = \frac{a\sqrt{x}}{cx}.$
14. $(\sqrt{12} - \sqrt{18}) \div 2\sqrt{3} = \frac{(\sqrt{12} - \sqrt{18})\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{\sqrt{36} - \sqrt{54}}{6} = 1 - \frac{\sqrt{6}}{2}.$
15. $(12 - 3\sqrt{6} - 4\sqrt{24}) \div 3\sqrt{2} = \frac{(12 - 3\sqrt{6} - 4\sqrt{24})\sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}}$
 $= 2\sqrt{2} - \sqrt{3} - \frac{8}{3}\sqrt{3} = 2\sqrt{2} - \frac{11}{3}\sqrt{3}.$

$$\begin{aligned}
 16. \quad \frac{\sqrt{6}}{\sqrt[3]{2}} &= \frac{\sqrt{6} \cdot \sqrt[3]{4}}{\sqrt[3]{2} \cdot \sqrt[3]{4}} \\
 &= \frac{\sqrt[6]{3^3 \cdot 2^3 \cdot 2^4}}{2} \\
 &= \sqrt[6]{3^3 \cdot 2} = \sqrt[6]{54}.
 \end{aligned}$$

$$17. \quad \frac{\sqrt[4]{8}}{\sqrt{2}} = \frac{\sqrt[4]{8}}{\sqrt[4]{4}} = \sqrt[4]{2}.$$

$$\begin{aligned}
 18. \quad \frac{\sqrt{32}}{\sqrt[4]{2}} &= \frac{\sqrt[4]{32^2}}{\sqrt[4]{2}} \\
 &= \sqrt[4]{512} = 4\sqrt[4]{2}.
 \end{aligned}$$

$$19. \quad \frac{\sqrt[3]{\frac{1}{4}}}{\sqrt{\frac{1}{2}}} = \frac{\sqrt[6]{(\frac{1}{4})^2}}{\sqrt[6]{(\frac{1}{2})^3}} = \sqrt[6]{\frac{1}{16} \cdot 8} = \sqrt[6]{\frac{1}{2}} = \frac{1}{2} \sqrt[6]{32}.$$

$$20. \quad \frac{3}{2 - \sqrt{3}} = \frac{3(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{6 + 3\sqrt{3}}{4 - 3} = 6 + 3\sqrt{3}.$$

$$21. \quad \frac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{\sqrt{3}(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} = \frac{\sqrt{6} - 3}{2 - 3} = 3 - \sqrt{6}.$$

$$22. \quad \frac{\sqrt{7}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{7}(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})} = \frac{\sqrt{14 + 7\sqrt{3}}}{\sqrt{4 - 3}} = \sqrt{14 + 7\sqrt{3}}.$$

$$\begin{aligned}
 23. \quad \frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}} &= \frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6} - 3\sqrt{3} + 2\sqrt{2} - \sqrt{6}}{\sqrt{9 - 2}} \\
 &= \frac{\sqrt{6} - 3\sqrt{3} + 2\sqrt{2} - \sqrt{6}}{\sqrt{7}} = \frac{\sqrt{42 - 21\sqrt{3} + 14\sqrt{2} - 7\sqrt{6}}}{7}.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \frac{\sqrt{7} + \sqrt{5}}{2\sqrt{7} - \sqrt{5}} &= \frac{(\sqrt{7} + \sqrt{5})(2\sqrt{7} + \sqrt{5})}{(2\sqrt{7} - \sqrt{5})(2\sqrt{7} + \sqrt{5})} = \frac{14 + 3\sqrt{35} + 5}{28 - 5} = \frac{19 + 3\sqrt{35}}{23} \\
 &= \frac{19 + 3\sqrt{35}}{23(19 - 3\sqrt{35})} = \frac{(19 + 3\sqrt{35})^2}{23(19 - 3\sqrt{35})(19 + 3\sqrt{35})} \\
 &= \frac{361 + 114\sqrt{35} + 315}{23(361 - 315)} = \frac{676 + 114\sqrt{35}}{23 \cdot 46} = \frac{338 + 57\sqrt{35}}{529}.
 \end{aligned}$$

$$25. (a) \quad 6 + 3\sqrt{3} = 6 + 3(1.73205) = 11.1961.$$

$$(b) \quad 3 - \sqrt{6} = 3 - 2.44948 = .55051.$$

$$(c) \quad \sqrt{14 + 7\sqrt{3}} = \sqrt{14 + 7(1.73205)} = \sqrt{26.12435} = 5.1111.$$

$$26. \quad \frac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{1.73205}{1.41421 + 1.73205} = \frac{1.73205}{3.14626} = .55050.$$

There is a difference of one one-hundred-thousandth.

27. In general the result will be far more easily obtained and will be more nearly correct if rationalization is used.

$$28. \quad \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{7 + 2\sqrt{10}}{3}.$$

$$29. \frac{2\sqrt{5} + 3\sqrt{7}}{3\sqrt{5} - 2\sqrt{7}} = \frac{(2\sqrt{5} + 3\sqrt{7})(3\sqrt{5} + 2\sqrt{7})}{(3\sqrt{5} - 2\sqrt{7})(3\sqrt{5} + 2\sqrt{7})} = \frac{30 + 42 + 13\sqrt{35}}{17} \\ = \frac{72 + 13\sqrt{35}}{17}.$$

$$30. \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b})}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{a + b + 2\sqrt{ab}}{a - b}.$$

$$31. \frac{\sqrt{x} - 2\sqrt{c}}{\sqrt{x} + \sqrt{c}} = \frac{(\sqrt{x} - 2\sqrt{c})(\sqrt{x} - \sqrt{c})}{(\sqrt{x} + \sqrt{c})(\sqrt{x} - \sqrt{c})} = \frac{x + 2c - 3\sqrt{cx}}{x - c}.$$

$$32. \frac{\sqrt{x-3} + \sqrt{3}}{\sqrt{x-3} - \sqrt{3}} = \frac{(\sqrt{x-3} + \sqrt{3})(\sqrt{x-3} + \sqrt{3})}{(\sqrt{x-3} - \sqrt{3})(\sqrt{x-3} + \sqrt{3})} = \frac{x + 2\sqrt{3x-9}}{x-6}.$$

$$33. \frac{4}{\sqrt[4]{2} - \sqrt{2}} = \frac{4\sqrt[4]{2} + \sqrt{2}}{\sqrt[4]{2} - \sqrt{2} \sqrt[4]{2} + \sqrt{2}} = \frac{4\sqrt[4]{2} + \sqrt{2}}{\sqrt{2}} = \frac{4\sqrt[4]{2} + \sqrt{2}\sqrt[4]{8}}{\sqrt{2} \cdot \sqrt[4]{8}} \\ = 2\sqrt[4]{16} + 8\sqrt{2}.$$

$$34. \frac{\sqrt{10} - \sqrt{5}}{\sqrt{10} + \sqrt{5}} = \frac{(\sqrt{10} - \sqrt{5})(\sqrt{10} - \sqrt{5})}{(\sqrt{10} + \sqrt{5})(\sqrt{10} - \sqrt{5})} = \frac{15 - 10\sqrt{2}}{5} = 3 - 2\sqrt{2}.$$

$$35. \frac{x - \sqrt{c}}{x - 3\sqrt{c}} = \frac{(x - \sqrt{c})(x + 3\sqrt{c})}{(x - 3\sqrt{c})(x + 3\sqrt{c})} = \frac{x^2 - 3c + 2x\sqrt{c}}{x^2 - 9c}.$$

$$36. \frac{\sqrt{a+c} - \sqrt{x}}{\sqrt{a+c} + \sqrt{x}} = \frac{(\sqrt{a+c} - \sqrt{x})(\sqrt{a+c} - \sqrt{x})}{(\sqrt{a+c} + \sqrt{x})(\sqrt{a+c} - \sqrt{x})} \\ = \frac{a+c+x-2\sqrt{ax+cx}}{a+c-x}.$$

$$37. \frac{\sqrt{3} + \sqrt{2}}{2 - \sqrt{3} + \sqrt{2}} = \frac{(\sqrt{3} + \sqrt{2})(2 + \sqrt{3} + \sqrt{2})}{(2 - \sqrt{3} + \sqrt{2})(2 + \sqrt{3} + \sqrt{2})} \\ = \frac{2\sqrt{3} + 2\sqrt{2} + 5 + 2\sqrt{6}}{3 + 4\sqrt{2}} = \frac{(2\sqrt{3} + 2\sqrt{2} + 5 + 2\sqrt{6})(3 - 4\sqrt{2})}{(3 + 4\sqrt{2})(3 - 4\sqrt{2})} \\ = \frac{10\sqrt{3} + 14\sqrt{2} + 1 + 2\sqrt{6}}{23}.$$

$$38. \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} + \sqrt{7} - \sqrt{2}} = \frac{(\sqrt{5} - \sqrt{7})(\sqrt{5} + \sqrt{7} + \sqrt{2})}{(\sqrt{5} + \sqrt{7} - \sqrt{2})(\sqrt{5} + \sqrt{7} + \sqrt{2})} \\ = \frac{\sqrt{10} - \sqrt{14} - 2}{10 + 2\sqrt{35}} = \frac{(\sqrt{10} - \sqrt{14} - 2)(10 - 2\sqrt{35})}{(10 + 2\sqrt{35})(10 - 2\sqrt{35})} \\ = \frac{24\sqrt{10} - 20\sqrt{14} - 20 + 4\sqrt{35}}{-40} = \frac{5\sqrt{14} + 5 - \sqrt{35} - 6\sqrt{10}}{10}.$$

Page 349 (First set)

1. $6 - 2\sqrt{8} = 4 - 2\sqrt{8} + 2 = (2 - \sqrt{2})^2.$
2. $7 + 2\sqrt{10} = 5 + 2\sqrt{10} + 2 = (\sqrt{5} + \sqrt{2})^2.$
3. $13 + \sqrt{48} = 12 + 2\sqrt{12} + 1 = (2\sqrt{3} + 1)^2.$
4. $8 - \sqrt{60} = 8 - 2\sqrt{15} = 5 - 2\sqrt{15} + 3 = (\sqrt{5} - \sqrt{3})^2.$
5. $11 - 4\sqrt{7} = 7 - 2\sqrt{28} + 4 = (\sqrt{7} - 2)^2.$
6. $17 + 12\sqrt{2} = 8 + 2\sqrt{72} + 9 = (2\sqrt{2} + 3)^2.$
7. $11 - 3\sqrt{8} = 9 - 2\sqrt{18} + 2 = (3 - \sqrt{2})^2.$
8. $65x - 20\sqrt{3}x^2 = 60x - 2\sqrt{300}x^2 + 5x = (2\sqrt{15}x - \sqrt{5}x)^2.$
9. $126a - 10a\sqrt{5} = a - 2\sqrt{125a^2} + 125a = (5\sqrt{5}a - \sqrt{a})^2.$
10. $\frac{13a}{4} - \sqrt{3a^2} = 3a - 2\sqrt{\frac{3a^2}{4}} + \frac{a}{4} = (\sqrt{3}a - \frac{1}{2}\sqrt{a})^2.$
11. $2x + 2\sqrt{x^2 - 49} = x - 7 + 2\sqrt{x^2 - 49} + x + 7 = (\sqrt{x-7} + \sqrt{x+7})^2.$
12. $a + \sqrt{a^2 - 1} = \frac{a+1}{2} + 2\sqrt{\frac{a^2-1}{4}} + \frac{a-1}{2}$
 $= (\frac{1}{2}\sqrt{2a+2} + \frac{1}{2}\sqrt{2a-2})^2.$
13. $\sqrt{9 + 3\sqrt{8}} = \sqrt{6 + 3\sqrt{8} + 3} = \sqrt{6} + \sqrt{3}.$
14. $\sqrt{15 - 5\sqrt{8}} = \sqrt{10 - 2\sqrt{50} + 5} = (\sqrt{10} - \sqrt{5}).$
15. $\sqrt{m^2 + m + 2n + 2m\sqrt{m+2n}} = \sqrt{m^2 + 2m\sqrt{m+2n} + m + 2n}$
 $= (m + \sqrt{m+2n}).$

Page 349 (Second set)

1. $x^2 - 11 = (x + \sqrt{11})(x - \sqrt{11}).$
2. $3x^2 - 16 = (x\sqrt{3} + 4)(x\sqrt{3} - 4).$
3. $x^3 + 2 = (x + \sqrt[3]{2})(x^2 - x\sqrt[3]{2} + \sqrt[3]{4}).$

$$4. x^3 - 12 = (x - \sqrt[3]{12})(x^2 + x\sqrt[3]{12} + 2\sqrt[3]{18}).$$

$$5. 3x^3 - 27 = 3(x^3 - 9) = 3(x - \sqrt[3]{9})(x^2 + x\sqrt[3]{9} + 3\sqrt[3]{3}).$$

$$6. 5x^3 + 125 = 5(x^3 + 25) = 5(x + \sqrt[3]{25})(x^2 - x\sqrt[3]{25} + 5\sqrt[3]{5}).$$

$$7. \frac{2\sqrt{b}}{a-b} + \frac{2}{\sqrt{a} + \sqrt{b}} = \frac{2\sqrt{b}}{a-b} + \frac{2\sqrt{a} - 2\sqrt{b}}{a-b} = \frac{2\sqrt{a}}{a-b}.$$

$$8. \frac{x+c}{\sqrt{x}-\sqrt{c}} - \frac{x^{\frac{3}{2}}+c^{\frac{3}{2}}}{x-c} = \frac{x^{\frac{3}{2}}+cx^{\frac{1}{2}}+xc^{\frac{1}{2}}+c^{\frac{3}{2}}-x^{\frac{3}{2}}-c^{\frac{3}{2}}}{x-c} \\ = \frac{cx^{\frac{1}{2}}+c^{\frac{1}{2}}x}{x-c}.$$

$$9. x^2 - 5 = 0.$$

$$(x - \sqrt{5})(x + \sqrt{5}) = 0. \therefore x = \sqrt{5} \text{ and } -\sqrt{5}.$$

$$10. 2x^2 - 3 = 0.$$

$$(x\sqrt{2} + \sqrt{3})(x\sqrt{2} - \sqrt{3}) = 0.$$

$$\therefore x = -\frac{\sqrt{3}}{\sqrt{2}} \text{ and } \frac{\sqrt{3}}{\sqrt{2}} \text{ or } -\frac{1}{2}\sqrt{6} \text{ and } \frac{1}{2}\sqrt{6}.$$

$$11. x^4 + 144 = 26x^2.$$

$$x^4 - 26x^2 + 144 = 0.$$

$$(x^2 - 8)(x^2 - 18) = 0. \therefore x = 2\sqrt{2}, -2\sqrt{2}, 3\sqrt{2}, -3\sqrt{2}.$$

$$12. 4x^4 + c = x^2 + 4cx^2 \text{ or } 4x^4 - 4cx^2 - x^2 + c = 0.$$

$$(4x^2 - 1)(x^2 - c) = 0. (2x - 1)(2x + 1)(x + \sqrt{c})(x - \sqrt{c}) = 0.$$

$$x = \frac{1}{2}, -\frac{1}{2}, -\sqrt{c}, \sqrt{c}.$$

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$$1. \overline{12}^2 - 6^2 = a^2. \therefore a = 6\sqrt{3}.$$

$$2. s^2 - \left(\frac{s}{2}\right)^2 = a^2. \therefore a = \frac{s}{2}\sqrt{3}. \text{ Area} = \frac{s \cdot \frac{s}{2}\sqrt{3}}{2} = \frac{s^2}{4}\sqrt{3}.$$

$$3. \overline{20}^2 = s^2 - \left(\frac{s}{2}\right)^2. \therefore s = \frac{40}{3}\sqrt{3}. \text{ Area} = \frac{20 \cdot \frac{40}{3}\sqrt{3}}{2} = \frac{400}{3}\sqrt{3}.$$

$$4. s^2 - \left(\frac{s}{2}\right)^2 = a^2. \text{ Whence } s = \frac{2a}{3}\sqrt{3}.$$

$$5. \quad x^2 + y^2 = 100. \quad (1)$$

$$(x + 9)^2 + y^2 = 289. \quad (2)$$

$$(2) - (1),$$

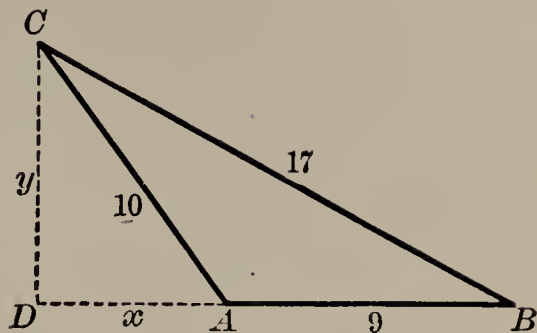
$$18x + 81 = 189.$$

$$18x = 108.$$

$$x = 6.$$

$$\therefore y = \sqrt{100 - 36} = 8.$$

$$\text{Area} = \frac{8 \cdot 9}{2} = 36.$$



$$6. \quad y^2 + x^2 = 100. \quad (1)$$

$$y^2 + (16 - x)^2 = 144. \quad (2)$$

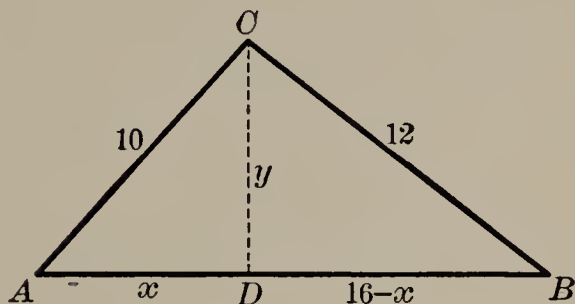
$$(2) - (1),$$

$$-32x + 256 = 44.$$

$$32x = 212.$$

$$x = 6\frac{5}{8}.$$

$$\therefore y = \sqrt{100 - (6\frac{5}{8})^2} \\ = \sqrt{\frac{3591}{64}} = \frac{3}{8} \sqrt{399}.$$



7. (a) In $\triangle AOB$, $AO = 15$ and $AK = \frac{15}{2}$.

Therefore $OK = \frac{15}{2} \sqrt{3}$ and the area of triangle AOB is $\frac{225}{4} \sqrt{3}$.

Since the hexagon contains six such triangles, its area $= \frac{675}{2} \sqrt{3}$.

(b) As in (a), $OK = \frac{s}{2} \sqrt{3}$, the area of $\triangle AOB = \frac{s^2}{4} \sqrt{3}$, and the area of the hexagon $= \frac{3s^2}{2} \sqrt{3}$.

8. (a) Let $AO = x$. Then $AK = \frac{x}{2}$.

$$x^2 - \left(\frac{x}{2}\right)^2 = 625. \quad \text{Whence } x = \frac{50}{3} \sqrt{3}.$$

$$\therefore \text{the area of } \triangle AOB = \frac{25 \cdot \frac{50}{3} \sqrt{3}}{2} = \frac{625}{3} \sqrt{3}.$$

$$\therefore \text{the area of hexagon} = \frac{6 \cdot 625}{3} \sqrt{3} = 1250 \sqrt{3}.$$

(b) As in (a), $x^2 - \frac{x^2}{4} = h^2$. $\therefore x = \frac{2h}{3} \sqrt{3}$.

$$\text{Then the area of } \triangle AOB = \frac{h \cdot \frac{2h}{3}}{2} \sqrt{3} = \frac{h^2}{3} \sqrt{3}.$$

$$\therefore \text{the area of the hexagon} = 6 \cdot \frac{h^2}{3} \sqrt{3} = 2h^2 \sqrt{3}.$$

9. Let $ABCD$ be the base of the pyramid, K the vertex, and R the point of intersection of the diagonals of the base. Then $AR = 5\sqrt{2}$. Therefore

$$KR = \sqrt{20^2 - (5\sqrt{2})^2} = 5\sqrt{14}.$$

Hence the volume is
$$\frac{100 \cdot 5\sqrt{14}}{3} = \frac{500}{3}\sqrt{14}.$$

10. As in Exercise 9, $AR = \sqrt{97}$, and therefore

$$KR = \sqrt{16^2 - (\sqrt{97})^2} = \sqrt{159}.$$

11. Let a = the altitude. Then $a^2 = 18^2 - (\frac{18}{2})^2$. Whence $a = 9\sqrt{3}$. Then $\frac{2}{3} \cdot 9\sqrt{3} = 6\sqrt{3}$, and $\frac{1}{3} \cdot 9\sqrt{3} = 3\sqrt{3}$.

12. As in Exercise 11, $CR = 6\sqrt{3}$, $CK = 4\sqrt{3}$, and therefore

$$DK = \sqrt{12^2 - (4\sqrt{3})^2} = \sqrt{144 - 48} = \sqrt{96} = 4\sqrt{6}.$$

13. As in Exercise 12, $CR = \frac{15}{2}\sqrt{3}$, $CK = 5\sqrt{3}$, $DK = 5\sqrt{6}$, and area of $ABC = \frac{225}{4}\sqrt{3}$. Therefore the volume of the pyramid

$$= \frac{5\sqrt{6} \cdot \frac{225}{4}\sqrt{3}}{3} = \frac{1125}{4}\sqrt{2}.$$

14. As in Exercise 12, $CR = \frac{e}{2}\sqrt{3}$, $CK = \frac{e}{3}\sqrt{3}$.

$$\therefore DK = \sqrt{e^2 - \left(\frac{e}{3}\sqrt{3}\right)^2} = \frac{e}{3}\sqrt{6}.$$

$$\text{Area of } ABC = \frac{e^2}{4}\sqrt{3}. \quad \text{Volume} = \frac{\frac{e}{3}\sqrt{6} \cdot \frac{e^2}{4}\sqrt{3}}{3} = \frac{e^3}{12}\sqrt{2}.$$

15. Let $ABCDEF$ be the base, V the vertex, and OK the apothem. Then $VO = \sqrt{16^2 - 10^2} = 2\sqrt{39}$. As in Exercise 7, area $ABCDEF = 150\sqrt{3}$. Therefore the volume of the pyramid

$$= \frac{2\sqrt{39} \cdot 150\sqrt{3}}{3} = 300\sqrt{13}.$$

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1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

2. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}.$

3. $2^{-4} \cdot 3^0 = \frac{1}{2^4} \cdot 1 = \frac{1}{16}.$

4. $2^{-2} \cdot 3^{-4} = \frac{1}{2^2 \cdot 3^4} = \frac{1}{324}.$

5. $7 \cdot 7^0 \cdot 0 = 0.$

6. $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 8.$

$$7. \left(\frac{2}{3}\right)^{-2} \cdot 4^0 = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\frac{4}{9}} = \frac{9}{4}.$$

$$8. \left(\frac{4}{5}\right)^{-3} \left(\frac{10}{3}\right)^{-2} = \frac{1}{\left(\frac{4}{5}\right)^3 \left(\frac{10}{3}\right)^2} \\ = \frac{1}{\frac{64 \cdot 100}{125 \cdot 9}} = \frac{1}{\frac{256}{45}} = \frac{45}{256}.$$

$$9. \frac{2}{3^{-2}} = 2 \cdot 3^2 = 18.$$

$$10. \frac{3}{3^0} = \frac{3}{1} = 3.$$

$$11. \frac{12}{4^{-1}} = 12 \cdot 4^1 = 48.$$

$$12. 5 \cdot 2^0 - (5 \cdot 2)^0 = 5 - 1 = 4.$$

$$13. \frac{4^{-2} \cdot 3^{-2}}{6^{-2}} = \frac{6^2}{4^2 \cdot 3^2} = \frac{2^2}{4^2} = \frac{1}{4}.$$

$$14. (m - n)^0 = 1 \text{ if } m \neq n.$$

$$24. (-8)^{-\frac{2}{3}} = \frac{1}{(-8)^{\frac{2}{3}}} = \frac{1}{(-2)^2} = \frac{1}{4}.$$

$$25. (-64)^{-\frac{2}{3}} = \frac{1}{(-64)^{\frac{2}{3}}} = \frac{1}{(-4)^2} = \frac{1}{16}.$$

$$26. (-32)^{\frac{1}{5}} = -2.$$

$$27. (32)^{-\frac{1}{5}} = \frac{1}{(32)^{\frac{1}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$$

$$28. (-125)^{-\frac{2}{3}} = \frac{1}{(-125)^{\frac{2}{3}}} = \frac{1}{(-5)^2} = \frac{1}{25}.$$

$$29. \sqrt[3]{27^{-2}} = \sqrt[3]{3^{-6}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$$

$$30. \sqrt[3]{8^{-2}} = \sqrt[3]{2^{-6}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}.$$

$$31. (\sqrt[3]{-8})^2 = (-2)^2 = 4.$$

$$32. \left(\frac{1}{2}\right)^{-4} \cdot \left(\frac{1}{3}\right)^{-3} \cdot \left(\frac{1}{2}\right)^0 = \frac{1}{\left(\frac{1}{2}\right)^4 \left(\frac{1}{3}\right)^3} = \frac{1}{\frac{1}{16 \cdot 27}} = 432.$$

$$33. \left(\frac{1}{6}\right)^{-2} = \frac{1}{\left(\frac{1}{6}\right)^2} = \frac{1}{\frac{1}{36}} = 36.$$

$$34. (.04)^{\frac{3}{2}} = (.2)^3 = .008.$$

$$15. 32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$$

$$16. 0^5 \cdot 5^0 = 0.$$

$$17. 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}.$$

$$18. 8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}.$$

$$19. 16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}.$$

$$20. 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{2^2} = \frac{1}{4}.$$

$$21. 16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{4^3} = \frac{1}{64}.$$

$$22. 25^{1.5} = 25^{\frac{3}{2}} = 5^3 = 125.$$

$$23. 0^3 \cdot 0^{\frac{3}{4}} = 0.$$

$$35. (.027)^{-\frac{2}{3}} = \frac{1}{(.027)^{\frac{2}{3}}} = \frac{1}{(.3)^2} = \frac{1}{.09} = \frac{100}{9}.$$

$$36. (.064)^{-\frac{1}{3}} = \frac{1}{(.064)^{\frac{1}{3}}} = \frac{1}{.4} = \frac{5}{2}.$$

$$37. (.00032)^{\frac{2}{5}} = (.2)^2 = .04.$$

$$38. \frac{\sqrt[3]{9^{-3}} \cdot \sqrt[2]{9^{-2}}}{3^{-4}} = \frac{9^{-1} \cdot 9^{-1}}{3^{-4}} = \frac{3^4}{9 \cdot 9} = 1.$$

$$39. \frac{2^{-1}}{2^{-2} - 2^{-3}} = \frac{\frac{1}{2}}{\frac{1}{2^2} - \frac{1}{2^3}} = \frac{\frac{1}{2}}{\frac{1}{8}} = 4.$$

$$40. \frac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{3^2} - \frac{1}{2^2}}{\frac{1}{3} - \frac{1}{2}} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}.$$

$$41. \frac{2^{-1} + 3^{-1}}{2^{-3} + 3^{-3}} = \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2^3} + \frac{1}{3^3}} = \frac{1}{\frac{1}{2^2} - \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3^2}} = \frac{1}{\frac{7}{36}} = \frac{36}{7}.$$

$$42. \frac{3^{-3} - 2^{-3}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{3^3} - \frac{1}{2^3}}{\frac{1}{3} - \frac{1}{2}} = \frac{1}{3^2} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{2^2} = \frac{19}{36}.$$

$$43. m^{-3} = \frac{1}{m^3}.$$

$$44. 2a^{-3} = \frac{2}{a^3}.$$

$$45. 3ab^{-2} = \frac{3a}{b^2}.$$

$$46. 7x^2y^{-2} = \frac{7x^2}{y^2}.$$

$$47. x^{-1}y^{-2}z = \frac{z}{xy^2}.$$

$$48. 4a^3b^{-2}c^2 = \frac{4a^3c^2}{b^2}.$$

$$49. \frac{3}{a^{-2}} = 3a^2.$$

$$50. \frac{4x}{y^{-3}} = 4xy^3.$$

$$51. \frac{4c^0}{xy^{-2}} = \frac{4y^2}{x}.$$

$$52. \frac{4a^{-2}b^0}{y^{-5}} = \frac{4y^5}{a^2}.$$

$$53. \frac{5^{-1}(ab)^0}{10^{-2}b^2} = \frac{10^2}{5b^2} = \frac{20}{b^2}.$$

$$54. \frac{3a^3b^{-2}c}{4a^{-2}b^0} = \frac{3a^5c}{4b^2}.$$

$$55. \frac{12x^2y^{-1}}{2yx^{-1}} = \frac{6x^3}{y^2}.$$

$$56. \frac{10^{-1}a}{bc^3} = \frac{a}{10bc^3}.$$

$$57. \frac{4a^{-2}bc^{-3}}{6a^{-2}b^{-3}c^0} = \frac{2b^4}{3c^3}.$$

$$58. \frac{4^{-3} \cdot r^{-6} \cdot s^6}{s^{-2}r^{-2}t^3} = \frac{s^8}{4^3 \cdot r^4 \cdot t^3} = \frac{s^8}{64r^4t^3}.$$

$$59. \frac{2s^{-1}}{m^na^{-b}} = \frac{2a^b}{m^ns}.$$

$$60. \frac{5x^2y^a}{x^{-m}y^{-4}} = 5x^{(m+2)}y^{(a+4)}.$$

$$61. \frac{2}{a^{-2} - b^{-2}} = \frac{2}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{2}{\frac{b^2 - a^2}{a^2 b^2}} = \frac{2 a^2 b^2}{b^2 - a^2}.$$

$$62. \frac{3}{a^{-1} + b^{-1}} = \frac{3}{\frac{1}{a} + \frac{1}{b}} = \frac{3}{\frac{b + a}{ab}} = \frac{3 ab}{a + b}.$$

$$63. \frac{a}{a^{-2} - b^{-2}} = \frac{a}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{a}{\frac{b^2 - a^2}{a^2 b^2}} = \frac{a^3 b^2}{b^2 - a^2}.$$

$$64. \frac{5 se^2}{s^{-2} + e^{-2}} = \frac{5 se^2}{\frac{1}{s^2} + \frac{1}{e^2}} = \frac{5 se^2}{\frac{e^2 + s^2}{s^2 e^2}} = \frac{5 s^3 e^4}{e^2 + s^2}.$$

$$65. \frac{a^{-2}}{a^{-2} + b^{-2}} = \frac{1}{a^2 \left(\frac{1}{a^2} + \frac{1}{b^2} \right)} = \frac{1}{\frac{a^2 + b^2}{b^2}} = \frac{b^2}{a^2 + b^2}.$$

$$66. \frac{a^{-3} b^{-3}}{a^{-3} + b^{-3}} = \frac{1}{a^3 b^3 \left(\frac{1}{a^3} + \frac{1}{b^3} \right)} = \frac{1}{b^3 + a^3}.$$

$$67. \frac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}} = \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\frac{1}{a} + \frac{1}{b}} = \frac{1}{a} - \frac{1}{b} = \frac{b - a}{ab}.$$

$$68. \frac{a^{-4} - b^{-4}}{a^{-2} - b^{-2}} = \frac{\frac{1}{a^4} - \frac{1}{b^4}}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2 + a^2}{a^2 b^2}.$$

$$69. \frac{a^{-1} + b^{-1}}{a^{-3} + b^{-3}} = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^3} + \frac{1}{b^3}} = \frac{1}{\frac{1}{a^2} - \frac{1}{ab} + \frac{1}{b^2}} = \frac{1}{\frac{b^2 - ab + a^2}{a^2 b^2}} = \frac{a^2 b^2}{b^2 - ab + a^2}.$$

$$70. \frac{a^{-3} - 27^{-1}}{a^{-1} - 3^{-1}} = a^{-2} + a^{-1} 3^{-1} + 3^{-2} = \frac{1}{a^2} + \frac{1}{3a} + \frac{1}{9} = \frac{a^2 + 3a + 9}{9a^2}.$$

$$71. \frac{2xy}{z^2} = 2xyz^{-2}. \quad 72. \frac{4a^3}{b^3} = 4a^3b^{-3}. \quad 73. \frac{3x}{a^{-2}b^4} = 3a^2b^{-4}x.$$

$$74. \frac{4sc^{-3}}{2^{-1}s^{-2}} = 2^3 \cdot s^3c^{-3} = 8s^3c^{-3}. \quad 76. \frac{7x^{-1}y^2}{2^{-1}y^3} = 14x^{-1}y^{-1}.$$

$$75. \frac{12a^2b^3}{4xy^2} = 3a^2b^3x^{-1}y^{-2}. \quad 77. \frac{5ac^{-2}}{c(x-y)^2} = 5ac^{-3}(x-y)^{-2}.$$

$$78. \frac{7 m^{-3} n^{\frac{1}{2}}}{m^{\frac{2}{3}} (m-n)^0} = 7 m^{-\frac{11}{3}} n^{\frac{1}{2}}.$$

$$79. \frac{1}{5 a^2 (c+d)^{-3}} = 5^{-1} a^{-2} (c+d)^3.$$

$$80. \frac{a (x-y)^{-2}}{bcx (x-y)} = ab^{-1} c^{-1} x^{-1} (x-y)^{-3}.$$

$$81. \frac{42 m^{-n} n^{2m}}{56 m^{-2n} n^{-3n}} = 3 \cdot 4^{-1} m^n n^{2m+3n}.$$

$$82. \frac{r^{-1} s^2}{r^{-2} s^3 (s-r)^3} = rs^{-1} (s-r)^{-3}.$$

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$$1. x^4 \cdot x^{-2} = x^2.$$

$$2. x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{5}{6}}.$$

$$3. x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} \cdot x^0 = x^{\frac{1}{2} + \frac{1}{3}} = x^{\frac{5}{6}}.$$

$$4. 7 x^0 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{2}} = 7 x^{\frac{13}{6}}.$$

$$5. \sqrt[3]{x} \cdot x^{\frac{1}{4}} = x^{\frac{1}{3} + \frac{1}{4}} = x^{\frac{7}{12}}.$$

$$6. \sqrt[3]{x} \cdot \sqrt[4]{x^3} = x^{\frac{1}{3} + \frac{3}{4}} = x^{\frac{13}{12}}.$$

$$7. a \sqrt{ax} \cdot a^{\frac{1}{2}} \sqrt{ax^{-1}} \\ = aa^{\frac{1}{2}} x^{\frac{1}{2}} a^{\frac{1}{2}} a^{\frac{1}{2}} x^{-\frac{1}{2}} = a^{\frac{5}{2}}.$$

$$8. e^x \cdot e^{-x} = e^0 = 1.$$

$$9. e^{a-3} \cdot e^3 = e^a.$$

$$10. e^{3-2a} \cdot e^{2+3a} = e^{5+a}.$$

$$11. x \cdot x^a \cdot x^b \cdot x^0 \cdot x^{2b-3a} \\ = x^{3b-2a+1}.$$

$$12. (2^3)^3 = 2^9 = 512.$$

$$13. (2^{-3})^{-2} = 2^6 = 64.$$

$$14. (x^2)^3 = x^6.$$

$$15. (x^{\frac{1}{3}})^{\frac{1}{2}} = x^{\frac{1}{6}}.$$

$$16. (3x^{-2})^3 = 27 x^{-6} = \frac{27}{x^6}.$$

$$17. (25 a^4 b^6)^{-\frac{1}{2}} = 5^{-1} a^{-2} b^{-3} = \frac{1}{5 a^2 b^3}.$$

$$18. (a^2 b)^3 (a+b) = a^6 b^3 (a+b) = a^7 b^3 + a^6 b^4.$$

$$19. (a^3)^{2x} (a^2)^{3x} = a^{6x} \cdot a^{6x} = a^{12x}.$$

$$20. (a^3)^{x+y} (a^2)^{y-x} = a^{3x+3y} \cdot a^{2y-2x} = a^{x+5y}.$$

$$21. (x^2 - x^{-2}) x^3 = x^5 - x.$$

$$22. (x^4 - a^4) x^{-2} a^{-2} = x^2 a^{-2} - x^{-2} a^2 = \frac{x^2}{a^2} - \frac{a^2}{x^2}.$$

$$23. (x^{\frac{1}{3}} + y^{\frac{1}{3}}) x^{\frac{2}{3}} y^{\frac{1}{3}} = xy^{\frac{1}{3}} + x^{\frac{2}{3}} y^{\frac{2}{3}}.$$

$$24. (x^{\frac{1}{3}} + y^{\frac{1}{3}}) (x^{\frac{1}{3}} - y^{\frac{1}{3}}) = x^{\frac{2}{3}} - y^{\frac{2}{3}}.$$

$$25. (a^{-2} + 3) (a^{-2} - 5) = a^{-4} - 2 a^{-2} - 15 = \frac{1}{a^4} - \frac{2}{a^2} - 15.$$

$$26. (a^{-1} - a)^2 = a^{-2} - 2 a^{-1} a + a^2 = \frac{1}{a^2} - 2 + a^2.$$

$$27. (a^3 - 2 a^{-2})^3 = a^9 - 3 (a^3)^2 (2 a^{-2}) + 3 (a^3) (2 a^{-2})^2 - (2 a^{-2})^3 \\ = a^9 - 6 a^4 + 12 a^{-1} - 8 a^{-6} \\ = a^9 - 6 a^4 + \frac{12}{a} - \frac{8}{a^6}.$$

$$28. (a^{-1} - 2a + 3a^{-2})^2 = a^{-2} + 4a^2 + 9a^{-4} - 4 + 6a^{-3} - 12a^{-1} \\ = \frac{1}{a^2} + 4a^2 + \frac{9}{a^4} - 4 + \frac{6}{a^3} - \frac{12}{a}.$$

$$29. (e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x} = e^{2x} + 2 + \frac{1}{e^{2x}}.$$

$$30. (a^{-1} + b^{-2})\left(\frac{1}{a} - \frac{1}{b^2}\right) = \left(\frac{1}{a} + \frac{1}{b^2}\right)\left(\frac{1}{a} - \frac{1}{b^2}\right) = \frac{1}{a^2} - \frac{1}{b^4}.$$

$$31. (e^{2x} - 2 + e^{-2x})^2 = e^{4x} + 4 + e^{-4x} - 4e^{2x} + 2 - 4e^{-2x} \\ = e^{4x} + 6 + \frac{1}{e^{4x}} - 4e^{2x} - \frac{4}{e^{2x}}.$$

$$32. (3a^{-\frac{1}{2}} + 2a^{\frac{2}{3}})^2 = 9a^{-1} + 12a^{\frac{1}{6}} + 4a^{\frac{4}{3}} \\ = \frac{9}{a} + 12a^{\frac{1}{6}} + 4a^{\frac{4}{3}}.$$

$$33. (a^{\frac{4}{5}} - 2a^{\frac{2}{5}}x + 4x^2)(a^{\frac{2}{5}} + 2x) = a^{\frac{6}{5}} + 8x^3.$$

$$34. (x^{\frac{1}{3}} + 2y^{\frac{1}{3}})(x^{\frac{2}{3}} - 2x^{\frac{1}{3}}y^{\frac{1}{3}} + 4y^{\frac{2}{3}}) = x + 8y.$$

$$35. (a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}})(a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}) = a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b.$$

$$36. (x - x^{\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1})(x + x^{\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1}) = x^2 + xy^{-1} + y^{-2} \\ = x^2 + \frac{x}{y} + \frac{1}{y^2}.$$

$$37. (\sqrt[3]{a^5} - 3\sqrt{a})(\sqrt[3]{a^5} - 3\sqrt{a}) = \sqrt[3]{a^{10}} - 6\sqrt[3]{a^5} \cdot \sqrt{a} + 9a \\ = a^{\frac{10}{3}} - 6a^{\frac{13}{6}} + 9a.$$

$$38. (5\sqrt{c^{-5}}\sqrt[3]{d^{-3}} - a\sqrt[5]{c^{-2}})^3 = (5c^{-\frac{5}{2}}d^{-1} - ac^{-\frac{2}{5}})^3 \\ = \frac{125}{c^{\frac{15}{2}}d^3} - \frac{75a}{c^{\frac{27}{5}}d^2} + \frac{15a^2}{c^{\frac{33}{10}}d} - \frac{a^3}{c^6}.$$

$$39. \left(\sqrt{a} + \frac{\sqrt{b}}{c}\right)\left(\sqrt{a} + \frac{\sqrt{b}}{c}\right)^2 = \left(a^{\frac{1}{2}} + \frac{b^{\frac{1}{2}}}{c}\right)^3 = a^{\frac{3}{2}} + \frac{3ab^{\frac{1}{2}}}{c} + \frac{3a^{\frac{1}{2}}b}{c^2} + \frac{b^{\frac{3}{2}}}{c^3}.$$

$$40. \begin{array}{r} m^n - 1 - 5m^{n-2}a^n + 25a^{2n} \\ \hline m + 5a^n \\ \hline m^n - 5m^{n-1}a^n + 25ma^{2n} \\ \hline 5m^{n-1}a^n \qquad \qquad \qquad - 25m^{n-2}a^{2n} + 125a^{3n} \\ \hline m^n \qquad \qquad \qquad + 25ma^{2n} - 25m^{n-2}a^{2n} + 125a^{3n} \end{array}$$

$$41. \begin{array}{r} 25x^{-4} + 15x^{-2}y^{-8} + 9y^{-16} \\ \hline 5x^{-2} - 3y^{-8} \\ \hline 125x^{-6} + 75x^{-4}y^{-8} + 45x^{-2}y^{-16} \\ \hline - 75x^{-4}y^{-8} - 45x^{-2}y^{-16} - 27y^{-24} \\ \hline 125x^{-6} \qquad \qquad \qquad - 27y^{-24} = \frac{125}{x^6} - \frac{27}{y^{24}}. \end{array}$$

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$$1. x^4 \div x^6 = x^{-2} = \frac{1}{x^2}.$$

$$2. x^3 \div x^{\frac{1}{3}} = x^{\frac{8}{3}}.$$

$$3. x^{\frac{1}{2}} \div x^2 = x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}}.$$

$$4. \frac{ax^{\frac{2}{3}}}{a^{\frac{1}{2}}x^{\frac{1}{2}}} = a^{\frac{1}{2}}x^{\frac{1}{6}}.$$

$$5. \frac{ax - a^2x^2}{a^2x^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}}}{a} - x^{\frac{3}{2}}.$$

$$6. (x^a - 2x^{2a-1} + 3x^{3a-2}) \div x^{2a-1} = x^{1-a} - 2 + 3x^{a-1}.$$

$$7. (6a^3 + 4n - 9a^{n-2} + 5a^{2-n}) \div 3a^{n-2} = 2a^{5+3n} - 3 + \frac{5}{3}a^{4-2n}.$$

$$8. (x - y) \div (x^{\frac{1}{2}} - y^{\frac{1}{2}}) = x^{\frac{1}{2}} + y^{\frac{1}{2}}.$$

$$9. \begin{array}{r|l} x - 8y & x^{\frac{1}{3}} - 2y^{\frac{1}{3}} \\ x - 2x^{\frac{2}{3}}y^{\frac{1}{3}} & x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{\frac{1}{3}} + 4y^{\frac{2}{3}} \\ \hline & 2x^{\frac{2}{3}}y^{\frac{1}{3}} \\ & 2x^{\frac{2}{3}}y^{\frac{1}{3}} - 4x^{\frac{1}{3}}y^{\frac{2}{3}} \\ \hline & 4x^{\frac{1}{3}}y^{\frac{2}{3}} - 8y \\ & 4x^{\frac{1}{3}}y^{\frac{2}{3}} - 8y \end{array}$$

$$10. \begin{array}{r|l} 16x^2 - 81y^2 & 2x^{\frac{1}{2}} + 3y^{\frac{1}{2}} \\ 16x^2 + 24x^{\frac{3}{2}}y^{\frac{1}{2}} & 8x^{\frac{3}{2}} - 12xy^{\frac{1}{2}} + 18x^{\frac{1}{2}}y - 27y^{\frac{3}{2}} \\ \hline & -24x^{\frac{3}{2}}y^{\frac{1}{2}} \\ & -24x^{\frac{3}{2}}y^{\frac{1}{2}} - 36xy \\ \hline & 36xy \\ & 36xy + 54x^{\frac{1}{2}}y^{\frac{3}{2}} \\ \hline & -54x^{\frac{1}{2}}y^{\frac{3}{2}} - 81y^2 \\ & -54x^{\frac{1}{2}}y^{\frac{3}{2}} - 81y^2 \end{array}$$

$$11. \begin{array}{r|l} a^3 - b^2 & a^{\frac{1}{2}} + b^{\frac{1}{3}} \\ a^3 + a^{\frac{5}{2}}b^{\frac{1}{3}} & a^{\frac{5}{2}} - a^2b^{\frac{1}{3}} + a^{\frac{3}{2}}b^{\frac{2}{3}} - ab + a^{\frac{1}{2}}b^{\frac{4}{3}} - b^{\frac{5}{3}} \\ \hline & -a^{\frac{5}{2}}b^{\frac{1}{3}} \\ & -a^{\frac{5}{2}}b^{\frac{1}{3}} - a^2b^{\frac{2}{3}} \\ \hline & a^2b^{\frac{2}{3}} \\ & a^2b^{\frac{2}{3}} + a^{\frac{3}{2}}b \\ \hline & -a^{\frac{3}{2}}b \\ & -a^{\frac{3}{2}}b - ab^{\frac{4}{3}} \\ \hline & \end{array}$$

$$\begin{array}{r} ab^{\frac{4}{3}} - b^2 \\ ab^{\frac{4}{3}} + a^{\frac{1}{2}}b^{\frac{5}{3}} \\ \hline -a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2 \\ -a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2 \end{array}$$

$$\begin{array}{r}
 12. \quad a^2 + ab^{-1} + b^{-2} \quad \left| \begin{array}{l} a - a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ a^2 - a^{\frac{3}{2}}b^{-\frac{1}{2}} + ab^{-1} \end{array} \right. \\
 \hline
 a^{\frac{3}{2}}b^{-\frac{1}{2}} \\
 a^{\frac{3}{2}}b^{-\frac{1}{2}} - ab^{-1} + a^{\frac{1}{2}}b^{-\frac{3}{2}} \\
 \hline
 ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} \\
 ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 13. \quad e^{2x} - 2 + e^{-2x} \quad \left| \begin{array}{l} e^x - e^{-x} \\ e^{2x} - 1 \end{array} \right. \\
 \hline
 -1 + e^{-2x} \\
 -1 + e^{-2x} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 14. \quad e^{3x} + 3e^x + \frac{3}{e^x} + \frac{1}{e^{3x}} \quad \left| \begin{array}{l} e^x + e^{-x} \\ e^{2x} + 2 + \frac{1}{e^{2x}} \end{array} \right. \\
 \hline
 e^{3x} + e^x \\
 2e^x + \frac{3}{e^x} \\
 2e^x + \frac{2}{e^x} \\
 \hline
 \frac{1}{e^x} + \frac{1}{e^{3x}} \\
 \frac{1}{e^x} + \frac{1}{e^{3x}} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 15. \quad a + a^{\frac{3}{5}}b^{\frac{2}{3}} + 2a^{\frac{2}{5}}b^{\frac{1}{3}} + 2b \quad \left| \begin{array}{l} a^{\frac{3}{5}} + 2b^{\frac{1}{3}} \\ a^{\frac{2}{5}} + b^{\frac{2}{3}} \end{array} \right. \\
 \hline
 a \quad \quad \quad + 2a^{\frac{2}{5}}b^{\frac{1}{3}} \\
 a^{\frac{3}{5}}b^{\frac{2}{3}} \quad \quad \quad + 2b \\
 a^{\frac{3}{5}}b^{\frac{2}{3}} \quad \quad \quad + 2b \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 16. \quad m^4 + 7m^2 + 8 - 7m^{-2} + m^{-4} \quad \left| \begin{array}{l} m^2 + 5 - m^{-2} \\ m^4 + 5m^2 - 1 \end{array} \right. \\
 \hline
 2m^2 + 9 - 7m^{-2} \quad . \\
 2m^2 + 10 - 2m^{-2} \\
 \hline
 -1 - 5m^{-2} + m^{-4} \\
 -1 - 5m^{-2} + m^{-4} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 17. \quad 2x^{2n-1} - x^{3n-2} + 9x^{5n-4} \quad \left| \begin{array}{l} 2x^{n-1} + 3x^{2n-2} \\ 2x^{2n-1} + 3x^{3n-2} \end{array} \right. \\
 \hline
 -4x^{3n-2} + 9x^{5n-4} \\
 -4x^{3n-2} - 6x^{4n-3} \\
 \hline
 6x^{4n-3} + 9x^{5n-4} \\
 6x^{4n-3} + 9x^{5n-4} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 18. \quad -16a^{\frac{19}{5}}b^{\frac{3}{4}} + 40ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} \quad \Big| \quad -4a^{\frac{12}{5}}b^{-\frac{5}{8}} + 5a^{-\frac{2}{5}}b^{-\frac{3}{8}} \\
 \hline
 -16a^{\frac{19}{5}}b^{\frac{3}{4}} + 20ab \quad \Big| \quad 4a^{\frac{7}{5}}b^{\frac{11}{8}} - 5a^{-\frac{7}{5}}b^{\frac{13}{8}} \\
 \hline
 20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} \\
 \hline
 20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}}
 \end{array}$$

$$\begin{array}{r}
 19. \quad x^{3a} + 11x^{2a} + 38x^a + 33 - 28x^{-a} + 2x^{-2a} \quad \Big| \quad x^a + 4 - 2x^{-a} \\
 \hline
 x^{3a} + 4x^{2a} - 2x^a \quad \Big| \quad x^{2a} + 7x^a + 12 - x^{-a} \\
 \hline
 7x^{2a} + 40x^a + 33 \\
 7x^{2a} + 28x^a - 14 \\
 \hline
 12x^a + 47 - 28x^{-a} \\
 12x^a + 48 - 24x^{-a} \\
 \hline
 -1 - 4x^{-a} + 2x^{-2a} \\
 -1 - 4x^{-a} + 2x^{-2a}
 \end{array}$$

$$\begin{array}{r}
 20. \quad 25m^3 - 4mn^{\frac{4}{3}} + 4mn^2 - mn^{\frac{8}{3}} \quad \Big| \quad 5m^{\frac{3}{2}} - 2n^{\frac{2}{3}}m^{\frac{1}{2}} + m^{\frac{1}{2}}n^{\frac{4}{3}} \\
 \hline
 25m^3 - 10m^2n^{\frac{2}{3}} + 5m^2n^{\frac{4}{3}} \quad \Big| \quad 5m^{\frac{3}{2}} + 2m^{\frac{1}{2}}n^{\frac{2}{3}} - m^{\frac{1}{2}}n^{\frac{4}{3}} \\
 \hline
 +10m^2n^{\frac{2}{3}} - 5m^2n^{\frac{4}{3}} - 4mn^{\frac{4}{3}} + 4mn^2 \\
 +10m^2n^{\frac{2}{3}} \quad -4mn^{\frac{4}{3}} + 2mn^2 \\
 \hline
 -5m^2n^{\frac{4}{3}} \quad +2mn^2 - mn^{\frac{8}{3}} \\
 -5m^2n^{\frac{4}{3}} \quad +2mn^2 - mn^{\frac{8}{3}}
 \end{array}$$

$$\begin{array}{r}
 21. \quad e^{3x} - 3e^x + 3e^{-x} - e^{-3x} \quad \Big| \quad e^x - e^{-x} \\
 \hline
 e^{3x} - e^x \quad \Big| \quad e^{2x} - 2 + e^{-2x} \\
 \hline
 -2e^x \\
 -2e^x + 2e^{-x} \\
 \hline
 e^{-x} - e^{-3x} \\
 e^{-x} - e^{-3x}
 \end{array}$$

$$\begin{array}{r}
 22. \quad e^{5x} - 5e^{3x} + 10e^x - 10e^{-x} + 5e^{-3x} - e^{-5x} \quad \Big| \quad e^{2x} - 2 + e^{-2x} \\
 \hline
 e^{5x} - 2e^{3x} + e^x \quad \Big| \quad e^{3x} - 3e^x + 3e^{-x} - e^{-3x} \\
 \hline
 -3e^{3x} + 9e^x - 10e^{-x} \\
 -3e^{3x} + 6e^x - 3e^{-x} \\
 \hline
 3e^x - 7e^{-x} + 5e^{-3x} \\
 3e^x - 6e^{-x} + 3e^{-3x} \\
 \hline
 -e^{-x} + 2e^{-3x} - e^{-5x} \\
 -e^{-x} + 2e^{-3x} - e^{-5x}
 \end{array}$$

23. The equation $e^{2x} = 1$ is equivalent to the two equations $e^x = -1$ and $e^x = +1$. While $x = 0$ is one root of $e^{2x} = 1$, it is not a root of $e^x = -1$. Therefore the error consists in setting the positive square root of a number equal to the negative square root of the same number.

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1. $a^3 \cdot a^2 = a^n$. $\therefore n = 5$.
2. $2^3 \cdot 2^2 = 2^n$. $\therefore n = 5$.
3. $2^4 \cdot 2^3 = 2^n$. $\therefore n = 7$.
4. $2^5 \cdot 2^n = 2^{11}$. $\therefore n = 6$.
5. $3^2 \cdot 2^4 = 9 \cdot 2^n$.
 $2^4 = 2^n$. $\therefore n = 4$.
6. $4^2 \cdot 2^2 = 2^n$.
 $2^4 \cdot 2^2 = 2^n$. $\therefore n = 6$.
7. $4^3 \cdot 2^3 = 2^n$.
 $2^6 \cdot 2^3 = 2^n$. $\therefore n = 9$.
8. $4^2 \cdot 2^4 = 2^{2n}$.
 $2^4 \cdot 2^4 = 2^{2n}$. $\therefore n = 4$.
9. $3^3 \cdot 9^3 = 3^n$.
 $3^3 \cdot 3^6 = 3^n$. $\therefore n = 9$.
10. $9^2 \cdot 3^n = 3^5$.
 $3^4 \cdot 3^n = 3^5$. $\therefore n = 1$.
11. $2^2 \cdot 2^n = 32 = 2^5$. $\therefore n = 3$.
12. $8^2 \cdot 4^3 = 2^n$.
 $2^6 \cdot 2^6 = 2^n$. $\therefore n = 12$.
13. $8^3 \cdot 4^2 = 2^n$.
 $2^9 \cdot 2^4 = 2^n$. $\therefore n = 13$.
14. $9^3 \cdot 27^2 = 3^n$.
 $3^6 \cdot 3^6 = 3^n$. $\therefore n = 12$.
15. $27^n \cdot 9^2 = 3^{10}$.
 $3^{3n} \cdot 3^4 = 3^{10}$. $\therefore n = 2$.
16. $8^n \cdot 4^{2n} = 2^{14}$.
 $2^{3n} \cdot 2^{4n} = 2^{14}$. $\therefore n = 2$.
17. $3^6 \cdot 9^n = 81^2$.
 $9^3 \cdot 9^n = 9^4$. $\therefore n = 1$.
18. $9^n \cdot 3^3 = 27^n$.
 $3^{2n} \cdot 3^3 = 3^{3n}$. $\therefore n = 3$.
19. $2^{2n+2} \cdot 4^{n+2} = 8^{2n}$.
 $2^{2n+2} \cdot 2^{2n+4} = 2^{6n}$.
 $4n + 6 = 6n$. $\therefore n = 3$.
20. $81 \cdot 27^n = (9^n)^2$.
 $3^4 \cdot 3^{3n} = 3^{n^2}$.
 $n^2 = 3n + 4$. $\therefore n = 4$ and -1 .
21. $(25^n)^n = \frac{5^{7n}}{(125)^2}$, or $5^{2n^2} = \frac{5^{7n}}{5^6}$. $2n^2 = 7n - 6$. $\therefore n = 2$ and $\frac{3}{2}$.
22. $2^{6n+3} \cdot 4^{3n+6} = (8^n)^n$, or $2^{6n+3} \cdot 2^{6n+12} = 2^{3n^2}$.
 $\therefore 3n^2 = 12n + 15$. Whence $n = 5$ and -1 .
23. $x^{\frac{2}{3}} = 8$.
 $x^{\frac{1}{3}} = \pm \sqrt[3]{8}$.
 $x = (\pm \sqrt[3]{8})^3 = \pm 16\sqrt{2}$.
24. $x^{-\frac{1}{3}} = 5$.
 $\sqrt[3]{x} = \frac{1}{5}$.
 $x = \frac{1}{125}$.
25. $x^{-\frac{4}{3}} = 256$.
 $x^{\frac{4}{3}} = \frac{1}{256}$.
 $x^{\frac{1}{3}} = \pm \frac{1}{4}$.
 $x = \pm \frac{1}{64}$.
26. $x^{\frac{3}{2}} = -343$.
 $x^{\frac{1}{2}} = -7$.
 $x = 49$.
27. $\frac{1}{2} x^{-\frac{2}{3}} = 2$.
 $x^{-\frac{2}{3}} = 4$.
 $x^{\frac{2}{3}} = \frac{1}{4}$.
 $x^{\frac{1}{3}} = \pm \frac{1}{2}$.
 $x = \frac{1}{8}, -\frac{1}{8}$.

$$28. (x^{-\frac{1}{2}})^{-4} = 49.$$

$$x^2 = 49.$$

$$x = \pm 7.$$

$$29. (ax^{\frac{1}{2}})^{-6} = 27.$$

$$x^{-3} = 27 a^6.$$

$$x = \frac{1}{3 a^2}.$$

$$30. \frac{\sqrt[3]{x^7}}{\sqrt[3]{x^{\frac{1}{5}}}} = \frac{\sqrt[5]{25}}{\sqrt[5]{16}}. \quad x^{\frac{2}{5}} = \frac{25^{\frac{1}{5}}}{16^{\frac{1}{5}}} \text{ or } x^2 = \frac{25}{16}. \text{ Whence } x = \pm \frac{5}{4}.$$

$$31. (\sqrt[3]{-27x^9})^{-2} = \frac{1}{(-3x^3)^2} = \frac{1}{9x^6}.$$

$$32. (\sqrt[4]{16a^4x^8})^{-2} = \frac{1}{(2ax^2)^2} = \frac{1}{4a^2x^4}.$$

$$33. \sqrt{16^{\frac{3}{4}}x^9} = (16)^{\frac{3}{8}}x^{\frac{9}{2}} = 2^{\frac{3}{2}}x^{\frac{9}{2}}.$$

$$35. (x^{\frac{3}{2}}\sqrt{x^{-4}})^{-\frac{2}{3}} = x^{-1}x^{\frac{4}{3}} = x^{\frac{1}{3}}.$$

$$34. (x\sqrt[3]{x^{-2}})^3 = x^3 \cdot x^{-2} = x.$$

$$36. (x^{-2}\sqrt{x^3}\sqrt[4]{4})^4 = \frac{4x^6}{x^8} = \frac{4}{x^2}.$$

$$37. (x^{-2}\sqrt{x^3\sqrt[3]{x^2}})^{-\frac{1}{2}} = x \cdot x^{-\frac{3}{4}} \cdot x^{-\frac{1}{6}} = x^{\frac{1}{12}}.$$

$$38. [(\sqrt[3]{8x^4})^{-2}]^{\frac{4}{3}} = (8^{\frac{1}{3}}x^{\frac{4}{3}})^{-\frac{8}{3}} = \frac{1}{2^{\frac{8}{3}}x^{\frac{32}{9}}}.$$

$$39. (\sqrt{\sqrt[4]{16a^6}})^4 = (16a^6)^{\frac{4}{8}} = 4a^3.$$

$$40. \left[(27x^3)^{-\frac{1}{3}} \cdot \frac{1}{5x^{-4}}\right]^{-3} = 27x^3 \cdot \frac{125}{x^{12}} = \frac{3375}{x^9}.$$

$$41. (a^{\frac{1}{2}}x^{-\frac{1}{3}}\sqrt{ax^{-\frac{1}{4}}\sqrt[3]{x^{\frac{5}{3}}}})^{\frac{1}{2}} = a^{\frac{1}{4}}x^{-\frac{1}{6}}a^{\frac{1}{4}}x^{-\frac{1}{16}}x^{\frac{5}{36}} = \frac{a^{\frac{1}{2}}}{x^{\frac{13}{144}}}.$$

$$42. \sqrt{\sqrt{a}} = a^{\frac{1}{4}}.$$

$$45. \sqrt{2a^4\sqrt[5]{x^3}} = 2^{\frac{1}{2}}a^2x^{\frac{3}{10}}.$$

$$43. \sqrt[3]{\sqrt{x^4}} = x^{\frac{2}{3}}.$$

$$46. \frac{[\sqrt{(25x^2)^2}]^{-\frac{1}{3}}}{\sqrt[3]{5x}} = \frac{5^{-\frac{2}{3}}x^{-\frac{2}{3}}}{5^{\frac{1}{3}}x^{\frac{1}{3}}} = \frac{1}{5x}.$$

$$44. \sqrt[3]{\sqrt[4]{x^5}} = x^{\frac{5}{12}}.$$

$$47. (32)^4 = 32^{\frac{2}{5}} = 4.$$

$$48. 16^{-1.5} = 16^{-\frac{3}{2}} = \frac{1}{64}.$$

$$49. 4 \cdot 2^5 \cdot 2^{n-5} = 2^{n+2}.$$

$$50. \left(\frac{2}{3}\right)^{-2} \cdot \left(\frac{3}{2}\right)^0 \cdot 2^2 - \frac{2(\frac{1}{8})^{\frac{1}{3}}}{8^{-\frac{2}{3}}} = 9 - 4 = 5.$$

$$51. 5 \cdot 2^0 - 1^5 + (5 \cdot 2)^0 = 5.$$

$$52. \frac{4^{-3} - 2^{-3}}{4^{-1} + 2^{-1}} = \frac{-\frac{7}{64}}{\frac{3}{4}} = -\frac{7}{48}.$$

$$53. \frac{3^e \cdot 3 - 3^{4+e}}{9 \cdot 3^{2+e}} = 3^{-3} - 1 = -\frac{26}{27}.$$

$$54. \frac{a^{-1} + 2x^{-1}}{a^{-3} + 8x^{-3}} = \frac{\frac{1}{a} + \frac{2}{x}}{\frac{1}{a^3} + \frac{8}{x^3}} = \frac{x + 2a}{ax} \cdot \frac{a^3x^3}{x^3 + 8a^3} = \frac{a^2x^2}{x^2 - 2xa + 4a^2}.$$

$$55. \frac{x^{-4} + x^{-2} + 1}{x^{-2} - x^{-1} + 1} = \frac{\frac{1}{x^4} + \frac{1}{x^2} + 1}{\frac{1}{x^2} - \frac{1}{x} + 1} = \frac{1 + x^2 + x^4}{x^4} \cdot \frac{x^2}{1 - x + x^2} = \frac{1 + x + x^2}{x^2}.$$

$$56. (a^n - 1)(a^3 - n)^3(a^{2n+5})^{-1} = a^{n-1}a^{9-3n}a^{-2n-5} = a^{3-4n}.$$

$$57. \frac{n+2}{a^{n+1}} \div \frac{1}{a^{n+1}} = \frac{n+1}{a^{n+1}} = a.$$

$$58. \frac{n}{a^{n-1}} \div \frac{n}{a^{n+1}} = a \frac{n^2 + n - n^2 + n}{n^2 - 1} = \frac{2n}{a^{n^2-1}}.$$

$$59. a^{n^2-m^2} \div a^{n-m} = a^{n^2-m^2+m-n}.$$

$$60. \left((a^n + 1)^{\frac{1}{n^2-1}} \right)^{n^2-n} = \left(a^{\frac{1}{n-1}} \right)^{n^2-n} = a^n.$$

$$61. \left(\sqrt[n]{x^{n^2}} \div \sqrt[3]{x} \right)^{\frac{3}{3n-1}} = \left(x^n \div x^{\frac{1}{3}} \right)^{\frac{3}{3n-1}} = \left(x^{\frac{3n-1}{3}} \right)^{\frac{3}{3n-1}} = x.$$

$$62. \frac{a^2 \cdot 16^{\frac{1}{2}} a^{-\frac{3}{4}}}{a^{-\frac{7}{4}} + \frac{4}{a^{\frac{3}{4}}}} = \frac{4a^{\frac{5}{4}}}{\frac{a^{-1} + 4}{a^{\frac{3}{4}}}} = \frac{4a^2}{\frac{1}{a} + 4} = \frac{4a^3}{4a + 1}.$$

$$63. [(x - y)^2]^0 = 1.$$

$$64. \left(x^{\frac{1}{n^2-1}} \right)^{\frac{1}{n+1}} = \left(x^{\frac{1-n^2}{n^2}} \right)^{\frac{1}{n+1}} = x^{\frac{1-n}{n^2}}.$$

$$65. [32^{-\frac{3}{5}} \cdot 9^{\frac{3}{2}}]^{-\frac{1}{3}} = 32^{\frac{1}{5}} \cdot 9^{-\frac{1}{2}} = \frac{2}{3}.$$

$$66. \left(\frac{8x^3}{125y^{-3}} \right)^{-\frac{1}{3}} \left(\frac{m^{-3}n^2}{x^3y^{-2}} \right)^{-2} \left(\frac{2^4x^{-6}}{x^2y^8} \right)^{\frac{1}{2}} \left(\frac{2y^3}{m^2} \right)^{-3} \left(\frac{n^6}{x^3} \right)^{\frac{1}{3}} \\ = \frac{2^{-1}x^{-1}}{5^{-1}y} \cdot \frac{m^{6n-4}}{x^{-6}y^4} \cdot \frac{2^2x^{-3}}{xy^4} \cdot \frac{2^{-3}y^{-9}}{m^{-6}} \cdot \frac{n^2}{x} = \frac{5}{4}y^{-18}m^{12}n^{-2}.$$

$$67. \frac{4^{n+1}}{2^n(4^n-1)^n} \div \frac{8^{n+1}}{(4^n+1)^{n-1}} + 3 + 9x^0 - (9x)^0 - 1^9x + 9^0 \\ = \frac{2^{2n+2}}{2^{2n^2-n}} \cdot \frac{2^{2n^2-2}}{2^{3n+3}} + 3 + 9 - 1 - 1 + 1 \\ = \frac{2^{2n^2+2n}}{2^{2n^2+2n+3}} + 11 = \frac{1}{2^3} + 11 = 11\frac{1}{8}.$$

$$68. [((x-m)^n)^{-p}] \div [((x-n)^m)^p] = x^{mnp} \div x^{-mnp} = x^{2mnp}.$$

$$69. [(2x^2)^0 \cdot (2x)^{\frac{1}{2}} \cdot (2x)^{-\frac{1}{2}}]^{-\frac{1}{5}} = 1.$$

$$70. [(5a)^{2n} \cdot (5a)^{p-3n}]^{\frac{2}{3}} = ((5a)^{p-n})^{\frac{2}{3}} = (5a)^{\frac{2p-2n}{3}}.$$

$$71. (a^{\frac{2}{3}} \cdot a^{\frac{5}{11}})^{\frac{33}{5}} = a^{\frac{37}{5}}.$$

$$72. \left(r^{-\frac{1}{2}} s^{\frac{2}{3}} \sqrt{r^3 s^{-\frac{4}{3}}}\right)^{\frac{2}{5}} = (rs^0)^{\frac{2}{5}} = r^{\frac{2}{5}}.$$

$$73. \sqrt[5]{x^{10}} + 3(x^{-6})^{-\frac{1}{2}} = x^2 + 3x^3.$$

$$74. \frac{3a^{-b}c^{-3}}{7x^{-4}y^{-d}} \cdot \frac{21x^{-2}y^{5-d}}{6a^b c^{d-3}} = \frac{3x^2 y^5}{2a^2 b c^d}.$$

$$75. \frac{2^{2n} \cdot 3^{n+1} \cdot 6^{n+\frac{1}{2}}}{(2 \cdot 3)^n \cdot 3^{n-\frac{1}{2}}} \div \frac{2^{\frac{3}{2}-n} \cdot 12^{2n+2}}{18^{n+2}}$$

$$= \frac{2^{2n} \cdot 3^{n+1} \cdot 2^{n+\frac{1}{2}} \cdot 3^{n+\frac{1}{2}} \cdot 3^{2n+4} \cdot 2^{n+2}}{2^n \cdot 3^n \cdot 3^{n-\frac{1}{2}} \cdot 2^{\frac{3}{2}-n} 2^{4n+4} \cdot 3^{2n+2}}$$

$$= \frac{2^{4n+\frac{5}{2}} \cdot 3^{4n+\frac{11}{2}}}{2^{4n+\frac{11}{2}} \cdot 3^{4n+\frac{3}{2}}} = \frac{3^4}{2^3} = \frac{81}{8}.$$

$$76. \left(\frac{x^{-2}y}{x^{\frac{3}{2}}y^{-\frac{5}{2}}}\right)^{\frac{2}{7}} \div \left(\frac{xy^{-\frac{1}{2}}}{\sqrt{xy^{-1}}}\right)^{-2} = \frac{x^{-\frac{4}{7}}y^{\frac{2}{7}}}{x^{\frac{3}{7}}y^{-\frac{5}{7}}} \cdot \frac{x^{-1}y^2}{x^{-2}y} = y^2.$$

$$77. \frac{m^7 n}{r^{-r} s^{-1}} \cdot \frac{m^s r^{r+2}}{n^{-5} s^{-3}} = m^{7+s} r^{2r+2} s^4 n^6.$$

$$78. \frac{ab^{\frac{1}{2}}c \sqrt{ab} \sqrt[3]{c}}{\sqrt[3]{a^2 b^{-\frac{1}{2}}} a^{\frac{1}{3}} b^{\frac{2}{3}} c^{\frac{4}{3}}} = \frac{a \cdot b^{\frac{1}{2}} \cdot c \cdot a^{\frac{1}{2}} \cdot b \cdot c^{\frac{1}{3}}}{a^{\frac{2}{3}} b^{-\frac{1}{2}} a^{\frac{1}{3}} b^{\frac{2}{3}} c^{\frac{4}{3}}} = \frac{a^{\frac{3}{2}} b^{\frac{3}{2}} c^{\frac{4}{3}}}{ab^{\frac{3}{2}} c^{\frac{4}{3}}} = a^{\frac{1}{2}}.$$

$$79. \frac{xy^{\frac{1}{3}}z \sqrt{xy} \sqrt[5]{z}}{\sqrt{x^3} \cdot y^{-1} z y^3 \sqrt[5]{z^6} x^{-2}} = \frac{xy^{\frac{1}{3}} z x^{\frac{1}{2}} y^{\frac{1}{2}} z^{\frac{1}{5}}}{x^{\frac{3}{2}} y^{-1} z y^3 z^{\frac{6}{5}} x^{-2}} = \frac{x^{\frac{3}{2}} y^{\frac{5}{6}} z^{\frac{6}{5}}}{x^{-\frac{1}{2}} y^2 z^{\frac{11}{5}}} = x^2 y^{-\frac{7}{6}} z^{-1}.$$

$$80. \begin{array}{l} x^6 - 4x^{\frac{9}{2}}y^{\frac{1}{3}} + 4x^3y^{\frac{2}{3}} + 6x^3 - 12x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 \left[x^3 - 2x^{\frac{3}{2}}y^{\frac{1}{3}} + 3 \right. \\ \left. \pm (x^3 - 2x^{\frac{3}{2}}y^{\frac{1}{3}} + 3) \right] \end{array}$$

$$\begin{array}{l} 2x^3 - 2x^{\frac{3}{2}}y^{\frac{1}{3}} \quad \left| \begin{array}{l} -4x^{\frac{9}{2}}y^{\frac{1}{3}} \\ -4x^{\frac{9}{2}}y^{\frac{1}{3}} + 4x^3y^{\frac{2}{3}} \end{array} \right. \\ \hline 2x^3 - 4x^{\frac{3}{2}}y^{\frac{1}{3}} + 3 \quad \left| \begin{array}{l} 6x^3 - 12x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 \\ 6x^3 - 12x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 \end{array} \right. \end{array}$$

$$81. \frac{x^2}{y^2} + 4 + \frac{4y^2}{x^2} + \frac{3x^2}{y^2} - 20 + \frac{12y^2}{x^2} = \frac{4x^2}{y^2} - 16 + \frac{16y^2}{x^2}.$$

$$\frac{4x^2}{y^2} - 16 + \frac{16y^2}{x^2} \left| \begin{array}{l} \frac{2x}{y} - \frac{4y}{x} \\ \hline \frac{4x^2}{y^2} \pm \left(\frac{2x}{y} - \frac{4y}{x} \right) \end{array} \right.$$

$$\frac{4x}{y} - \frac{4y}{x} \left| \begin{array}{l} -16 + \frac{16y^2}{x^2} \\ \hline -16 + \frac{16y^2}{x^2} \end{array} \right.$$

$$82. [(e^x - e^{-x})^2 + 4]^{\frac{1}{2}} = [e^{2x} + 2 + e^{-2x}]^{\frac{1}{2}} = e^x + e^{-x}.$$

$$83. [(e^x + 2^{-1}e^{-x})^2 - 2]^{\frac{1}{2}} = [e^{2x} - 1 + 2^{-2}e^{-2x}]^{\frac{1}{2}} = e^x - 2^{-1}e^{-x}.$$

$$84. \begin{array}{r} e^{2x} + 4e^x + 2 - 4e^{-x} + e^{-2x} \overline{) e^x + 2 - e^{-x}} \\ c^{2x} \\ 2e^x + 2 \overline{) 4e^x} \\ 4e^x + 4 \overline{) 2e^x + 4 - e^{-x}} \\ -2 - 4e^{-x} + e^{-2x} \\ -2 - 4e^{-x} + e^{-2x} \end{array}$$

$$85. \begin{array}{r} 16x^2 - 24 + 17x^{-2} - 6x^{-4} + x^{-6} \overline{) 4x - 3x^{-1} + x^{-3}} \\ 16x^2 \\ 8x - 3x^{-1} \overline{) -24} \\ -24 + 9x^{-2} \\ 8x - 6x^{-1} + x^{-3} \overline{) 8x^{-2}} \\ 8x^{-2} - 6x^{-4} + x^{-6} \end{array}$$

$$86. \begin{array}{r} x^{-1} - 4x^{-\frac{3}{4}} + 6x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 \overline{) x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} + 1} \\ x^{-1} \\ 2x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} \overline{) -4x^{-\frac{3}{4}}} \\ -4x^{-\frac{3}{4}} + 4x^{-\frac{1}{2}} \\ 2x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 \overline{) 2x^{-\frac{1}{2}}} \\ 2x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 \quad \sqrt{x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} + 1} = x^{-\frac{1}{4}} - 1 \end{array}$$

$$87. \begin{array}{r} \frac{4a^{-8}b^8}{9} + a^{-6}b^6 + \frac{9a^{-4}b^4}{16} - \frac{16a^{-4}b^4}{15} - \frac{6a^{-2}b^2}{5} + \frac{16}{25} \overline{) \frac{2a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{4} - \frac{4}{5}} \\ \frac{4a^{-8}b^8}{9} \\ 9 \\ \frac{4a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{4} \overline{) a^{-6}b^6} \\ a^{-6}b^6 + \frac{9a^{-4}b^4}{16} \\ \frac{4a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{2} - \frac{4}{5} \overline{) -\frac{16a^{-4}b^4}{15}} \\ -\frac{16a^{-4}b^4}{15} - \frac{6a^{-2}b^2}{5} + \frac{16}{25} \end{array}$$

$$88. \frac{\frac{x^3 \cdot a \cdot x^0 - a \cdot 3 \cdot x^2}{(x^3)^2}}{\frac{a}{x^3}} = \frac{ax^3 - 3ax^2}{x^6} \cdot \frac{x^3}{a} = \frac{x-3}{x}.$$

$$89. \frac{\frac{x^4(3x^2) - (x^3 + 5)4x^3}{(x^4)^2}}{\frac{x^3 + 20}{x^4}} = \frac{3x^6 - 4x^6 - 20x^3}{x^8} \cdot \frac{x^4}{x^3 + 20} \\ = \frac{-x^6 - 20x^3}{x^4(x^3 + 20)} = -\frac{1}{x}.$$

$$90. \frac{\frac{(x^2 - 1)ax^{a-1} - x^a \cdot 2x}{(x^2 - 1)^2}}{\frac{x^a}{x^2 - 1}} = \frac{ax^{a+1} - ax^{a-1} - 2x^{a+1}}{(x^2 - 1)^2} \cdot \frac{x^2 - 1}{x^a} \\ = \frac{ax - ax^{-1} - 2x}{x^2 - 1}.$$

$$91. \frac{\frac{x^2(ax^{a-1}) - (x^a + 1)2x}{(x^2)^2}}{\frac{x^a + 1}{x^2}} = \frac{ax^{a+1} - 2x^{a+1} - 2x}{x^4} \cdot \frac{x^2}{x^a + 1} \\ = \frac{ax^a - 2x^a - 2}{x(x^a + 1)}.$$

$$92. \frac{\frac{x^{2a}nx^{n-1} - x^n \cdot 2ax^{2a-1}}{(x^{2a})^2}}{\frac{x^n}{x^{2a}}} = \frac{nx^{2a+n-1} - 2ax^{2a+n-1}}{x^{4a}} \cdot \frac{x^{2a}}{x^n} = \frac{n - 2a}{x}.$$

$$93. \frac{\frac{x^{-5}(-2x^{-1}) - (x^2 + 3)(-5x^{-4})}{(x^{-5})^2}}{\frac{x^{-2} + 3}{x^{-5}}} = \frac{-2x^{-6} + 5x^{-2} + 15x^{-4}}{x^{-10}} \cdot \frac{x^{-5}}{x^{-2} + 3} \\ = \frac{-2x^{-1} + 5x^3 + 15x}{x^{-2} + 3} \\ = \frac{5x^5 + 15x^3 - 2x}{1 + 3x^2}.$$

$$94. \frac{\frac{e^{-nx}(ne^{nx}) - (e^{nx} + 1)(-ne^{-nx})}{(e^{-nx})^2}}{\frac{2e^{nx} + 1}{e^{-nx}}} = \frac{n + n + ne^{-nx}}{e^{-2nx}} \cdot \frac{e^{-nx}}{2e^{nx} + 1} \\ = \frac{2n + ne^{-nx}}{e^{-nx}(2e^{nx} + 1)} = \frac{2ne^{nx} + n}{2e^{nx} + 1} = n.$$

$$95. \frac{\frac{(\sqrt[3]{x} + 1)^{\frac{1}{3}}x^{-\frac{2}{3}} - \sqrt[3]{x}(\frac{1}{3}x^{-\frac{2}{3}})}{(\sqrt[3]{x} + 1)^2}}{\frac{\sqrt[3]{x}}{\sqrt[3]{x} + 1}} = \frac{\frac{x^{-\frac{1}{3}}}{3} + \frac{x^{-\frac{2}{3}}}{3} - \frac{x^{-\frac{1}{3}}}{3}}{(x^{\frac{1}{3}} + 1)^2} \cdot \frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}} = \frac{x^{-1}}{3} \\ = \frac{1}{3(x^{\frac{1}{3}} + 1)x} = \frac{1}{3x^{\frac{4}{3}} + 3x}.$$

$$96. \frac{\frac{\sqrt{ax-x^2} \cdot b - bx(ax-x^2)^{-\frac{1}{2}} \cdot (a-2x)}{(\sqrt{ax-x^2})^2}}{\frac{bx}{\sqrt{ax-x^2}}} = \frac{abx - bx^2 - abx + 2bx^2}{(ax-x^2)^{\frac{3}{2}}} \cdot \frac{(ax-x^2)^{\frac{1}{2}}}{bx} = \frac{x}{ax-x^2} = \frac{1}{a-x}.$$

$$97. \frac{\frac{(\sqrt{x^{2n}-1}) 2nx^{2n-1} - x^{2n}(x^{2n}-1)^{-\frac{1}{2}}(nx^{n-1})}{(\sqrt{x^{2n}-1})^2}}{\frac{x^{2n}}{\sqrt{x^{2n}-1}}} = \frac{2nx^{4n-1} - 2nx^{2n-1} - nx^{3n-1}}{(x^{2n}-1)^{\frac{3}{2}}} \cdot \frac{(x^{2n}-1)^{\frac{1}{2}}}{x^{2n}} = \frac{2nx^{2n-1} - 2nx^{-1} - nx^{n-1}}{x^{2n}-1} = \frac{2nx^n - 2n - nx^n}{x^{2n+1} - x}.$$

$$98. \frac{\frac{\sqrt{x^2-10x} \cdot 5 - 5x \cdot \frac{1}{2}(x^2-10x)^{-\frac{1}{2}}(2x-10)}{(\sqrt{x^2-10x})^2}}{\frac{5x}{(x^2-10x)^{\frac{1}{2}}}} = \frac{5x^2 - 50x - 5x^2 + 25x}{(x^2-10x)^{\frac{3}{2}}} \cdot \frac{(x^2-10x)^{\frac{1}{2}}}{5x} = \frac{-5}{x^2-10x} = \frac{5}{10x-x^2}.$$

$$99. \frac{\frac{\sqrt{m-n}}{\sqrt{m}+\sqrt{n}}}{\frac{\sqrt{m}}{\sqrt{m-n}} - \frac{\sqrt{n}}{\sqrt{m-n}}} = \left(\frac{\sqrt{m-n}}{\sqrt{m}+\sqrt{n}} = \frac{\sqrt{m}-\sqrt{n}}{\sqrt{m-n}} \right) \sqrt{m-n}(\sqrt{m}+\sqrt{n}).$$

$$m-n = m-n.$$

$$100. \frac{\frac{a^{a+1}\sqrt{a+2}\sqrt{x}}{\sqrt{a^{a+1}}\sqrt{x}}}{\frac{a^{a+1}\sqrt{x^2}}{\sqrt{a^{a+1}}\sqrt{x}}} = \frac{1}{x^{a^2+3a+2}} = \frac{2}{x^{a^2+2a}}.$$

$$\frac{1}{x^{a^2+3a+2}} = \frac{2}{x^{a^2+2a}}.$$

$$\frac{1}{x^{a^2+3a+2}} + \frac{2a+2}{a^2+2a} = \frac{2}{x^{a^2+2a}} + \frac{1}{a^2+a}.$$

$$\frac{2a^2+5a+2}{x^{a^2+3a+2}} = \frac{2a+1}{x^{a^2+2a}}.$$

$$\frac{2a+1}{x^{a^2+3a+2}} = \frac{2a+1}{x^{a^2+2a}}.$$

1. $f(x) = 3x - 9$.

x	0	2	4
$f(x)$	-9	-3	3

2. Yes. At that point the function equals zero.

3. The graph of a linear function of x is a straight line which cuts the x -axis.

4. $f(x) = 2x^2 - x - 6$.

x	3	2	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	-2	-3
$f(x)$	9	0	-5	-6	-6	-5	-3	+4	15

5. Yes. At those points the function equals zero.

6. A parabola.

7. Transpose terms so that the second member of the equation is zero. Graph the function in the first member. Read from the graph the x -values of the points where the curve crosses the x -axis.

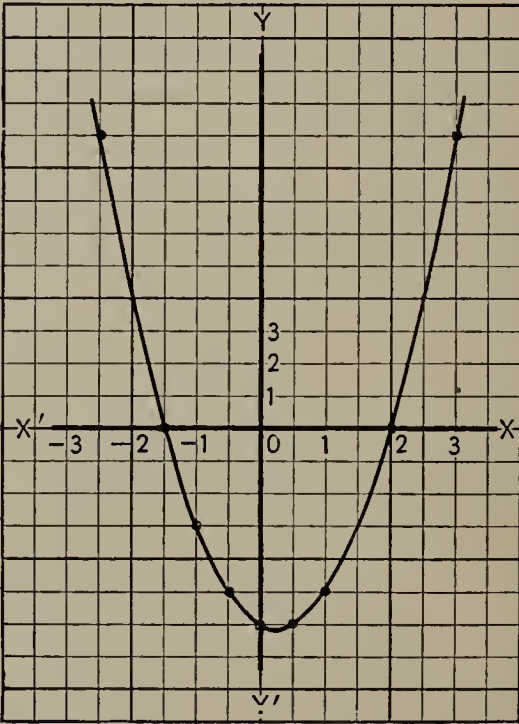
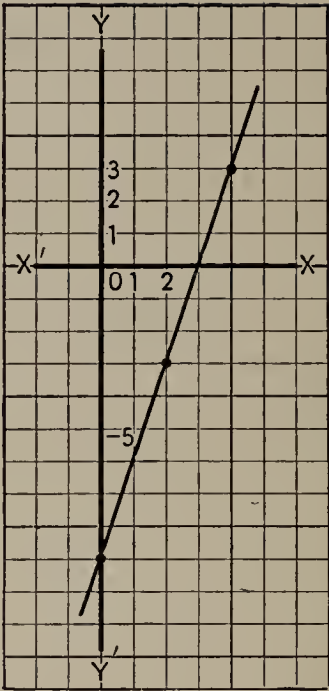
8. It raises every point on the graph, and consequently the entire graph, by the amount added to the constant term. The same effect is produced by lowering the x -axis.

9. One root is decreased and the other increased in such a manner that the two approach the same value.

10. One real root only is given when the curve touches but does not cross the x -axis.

11. The graphical solution of an equation in x fails when the curve does not touch or cut the x -axis.

12. Yes. 2.3, $-.2$, and -2.1 .



13. $-2.6, 1.3$. There is now but one point of intersection and one point of tangency.

14. $-2.7+$. There is now but one point of intersection. The other roots are not real.

15. 2.6 . The other roots are not real.

16. $2.7+$. The real root is increasing.

17. (a) No. (b) No.

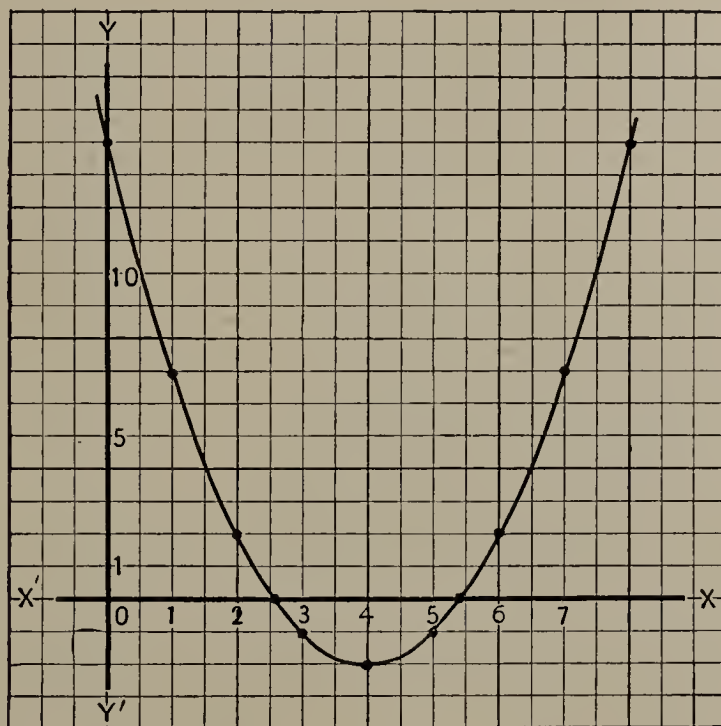
(c) $x + \frac{1}{x} + 5$ is rational in x but not integral.

$x - \sqrt{x} + 3$ is integral in x but not rational.

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1. $f(x) = x^2 - 8x + 14$.

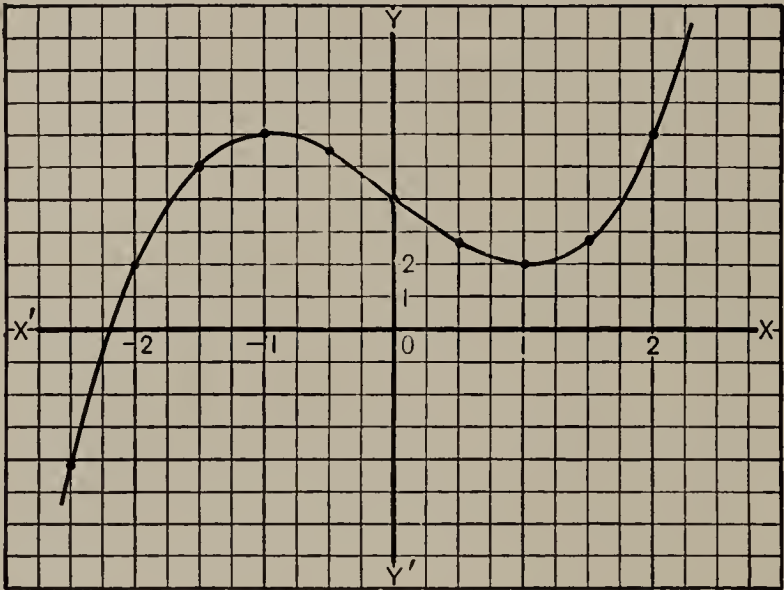
x	0	1	2	3	4	5	6	7	8
$f(x)$	14	7	2	-1	-2	-1	2	7	14



From the graph $x = 2.6$ and 5.4 .

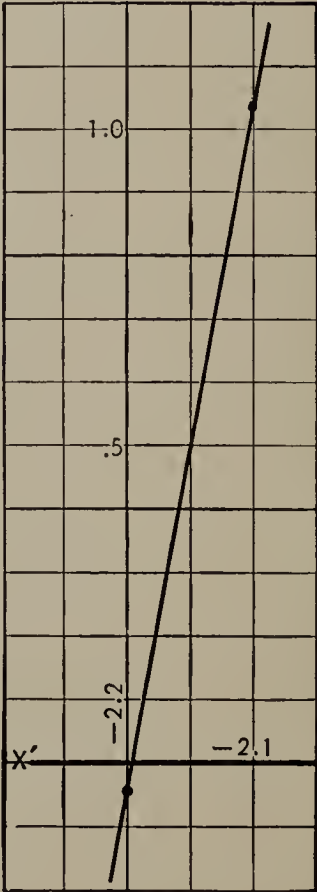
2. $f(x) = x^3 - 3x + 4$.

x	$-\frac{5}{2}$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-4.1	2	5.1	6	5.3	4	2.6	2	2.8	6



From the graph $x = -2.2$ approximately.

x	-2.1	-2.2
$f(x)$	1.04	$-.048$

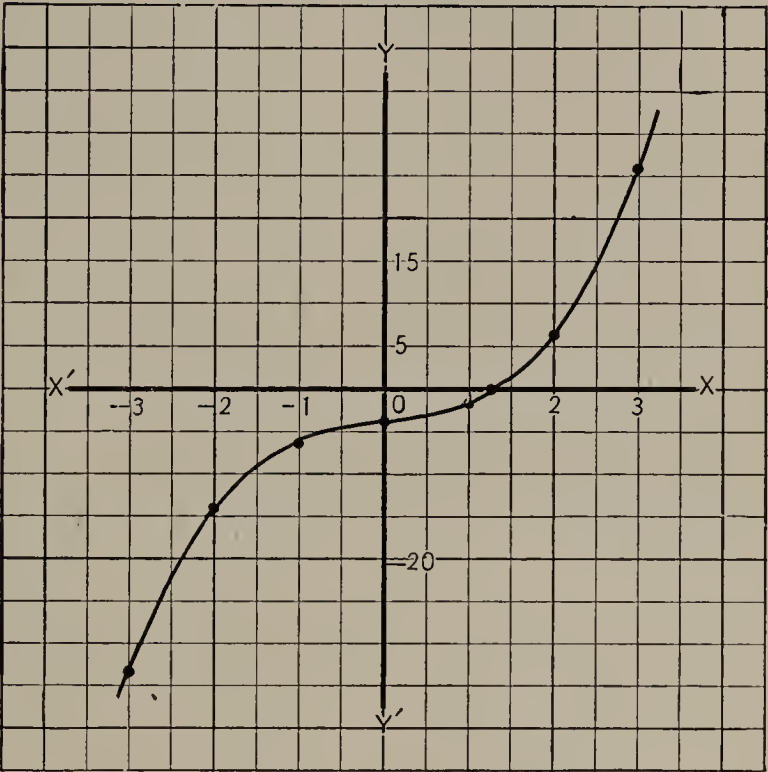


From the graph $x = -2.19$.

3. $f(x) = x^3 + x - 4$.

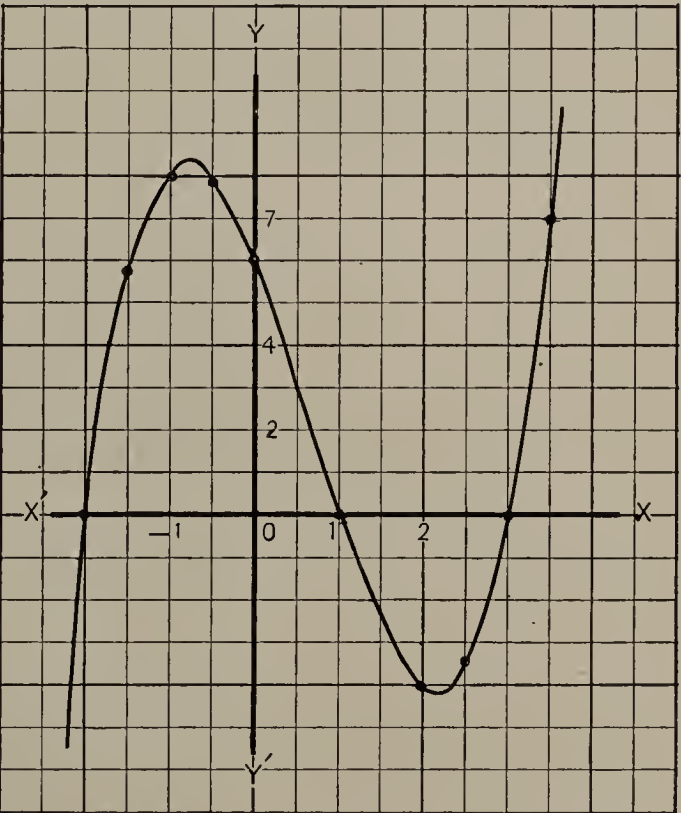
x	3	2	1	0	-1	-2	-3
$f(x)$	26	6	-2	-4	-6	-14	-34

From the graph $x = 1.3$ approximately.



4. $f(x) = x^3 - 2x^2 - 5x + 6$.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	1	2	$\frac{5}{2}$	3	$\frac{7}{2}$
$f(x)$	0	5.6	8	7.8	6	0	-4	-3.3	0	6.8

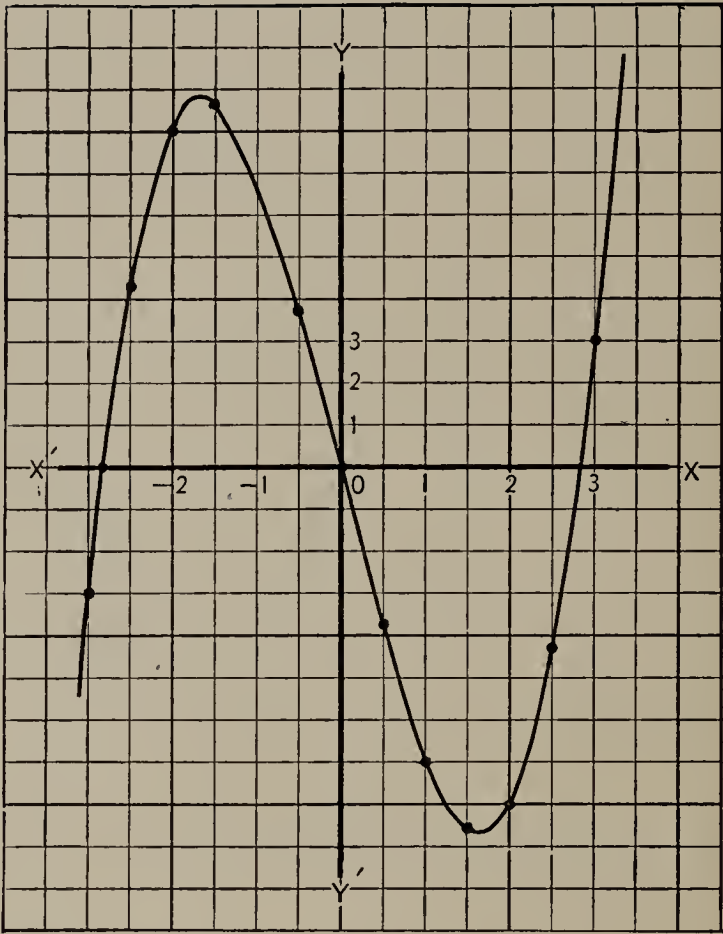


From the graph $x = -2$, 1 , and 3 .

5. $f(x) = x^3 - 8x$.

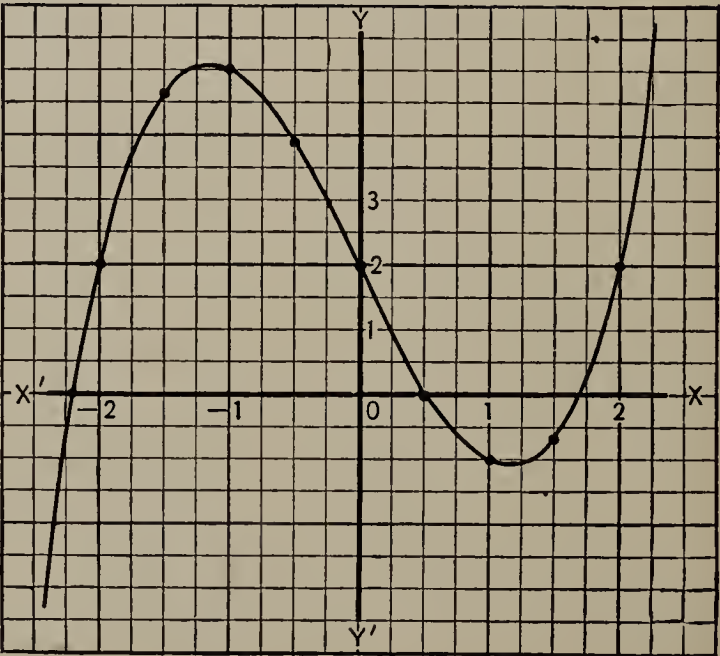
x	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2	$-\frac{5}{2}$	-3
$f(x)$	3	-4.3	-8	-8.6	-7	-3.8	0	3.8	7	8.6	8	4.3	-3

From the graph $x = 2.8$,
0, and -2.8 .



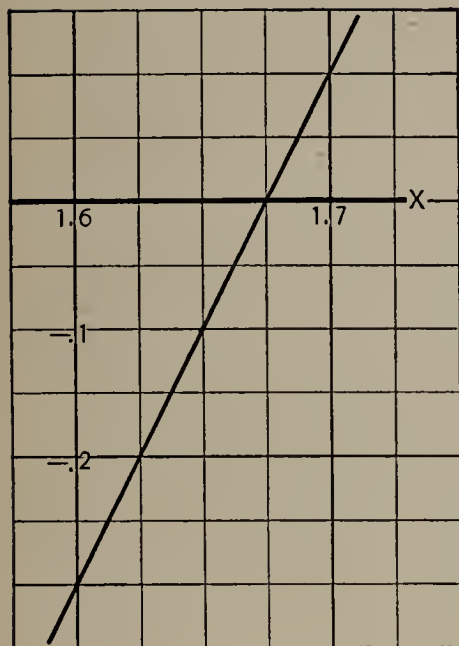
6. $f(x) = x^3 - 4x + 2$.

x	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2	$-\frac{5}{2}$
$f(x)$	2	$-\frac{5}{8}$	-1	$\frac{1}{8}$	2	$3\frac{7}{8}$	5	$4\frac{5}{8}$	2	$-3\frac{5}{8}$



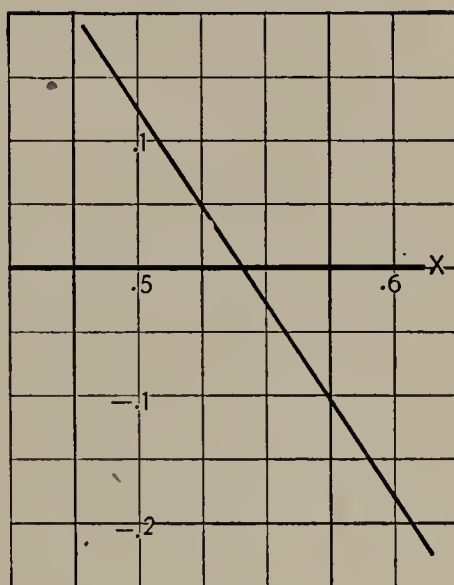
From the graph $x = 1.7$,
.5, and -2.2 .

x	1.6	1.7
$f(x)$	-.304	.113



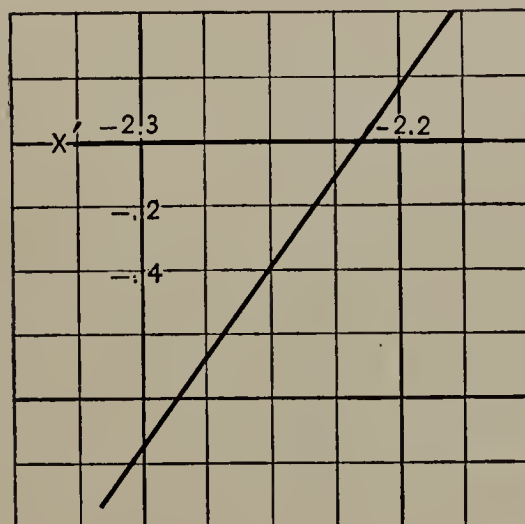
From the graph,
 $x = 1.67$.

x	.5	.6
$f(x)$	.125	-.184



From the graph,
 $x = .54$.

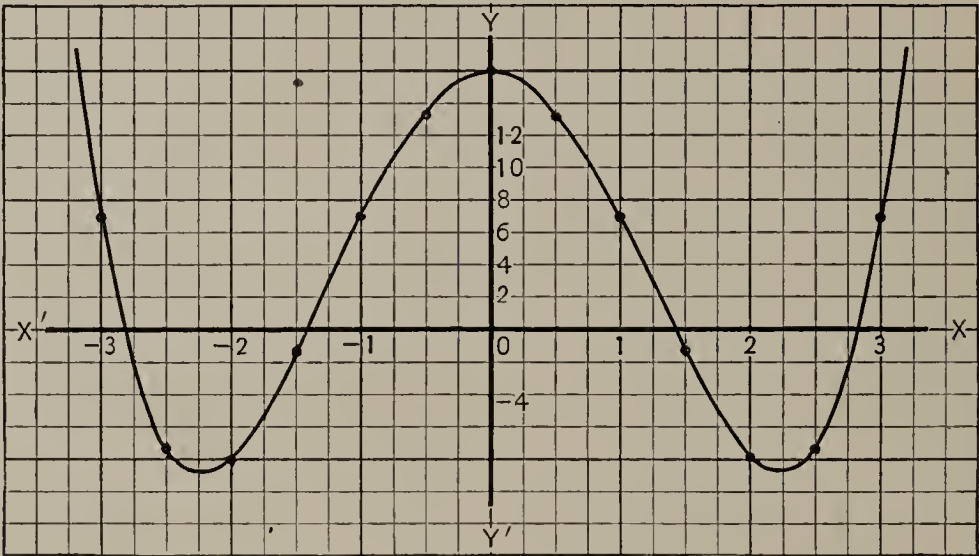
x	-2.2	-2.3
$f(x)$	.15	-.96



From the graph,
 $x = -2.22$.

7. $f(x) = x^4 - 10x^2 + 16$.

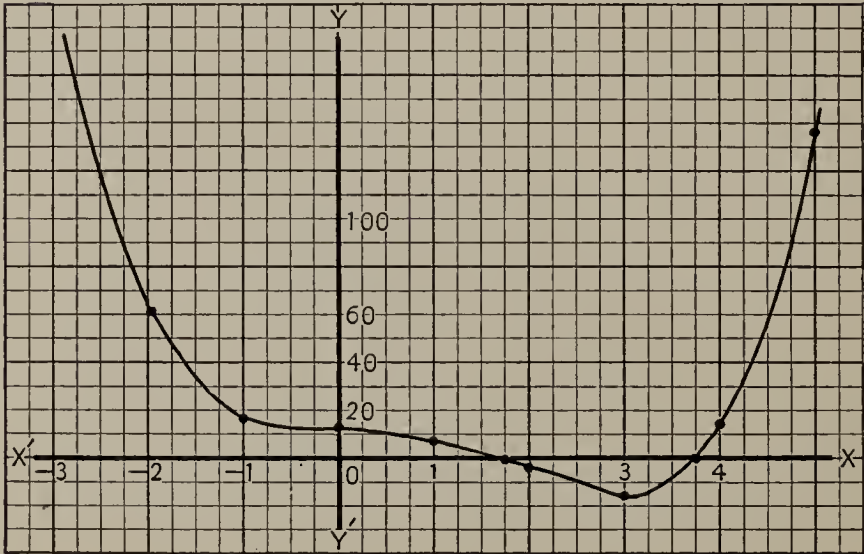
x	3	2.5	2	1.5	1	.5	0	-.5	-1	-1.5	-2	-2.5	-3
$f(x)$	7	-7.4	-8	-1.4	7	13.5	16	13.5	7	-1.4	-8	-7.4	7



From the graph $x = 2.8, 1.4, -1.4$, and -2.8 .

8. $f(x) = x^4 - 4x^3 + 12$.

x	5	4	3	2	1	0	-1	-2
$f(x)$	137	12	-15	-4	9	12	17	60



From the graph $x = 3.77$ and 1.74 .

9. (a) $x = -2.4$.
(b) By producing the graph upwards we have $x = 2.7$ approximately.
(c) $x = 2.2, -.5$, and -1.7 approximately.

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1. $\frac{V}{4}$ or $\frac{f(x)}{4} = x^3 - 11\frac{1}{2}x^2 + 30x$.

x	8	7	6	5	4	3	2	1	0	$-\frac{1}{2}$	$\frac{5}{3}$
$f(x)$	16	-10.5	-18	-12.5	0	13.5	22	19.5	0	-18	22.6
4											

From the graph the function is greatest at $x = \frac{5}{3}$.

Hence the side of the required square is $\frac{5}{3}$ inches.

2. (a) When

$$x = \frac{5}{3}, f(x) = 90.$$

(b) $f(x) = 90$

when $x = 8.1$ also.

(c) Any value of x greater than 8.1 makes $f(x)$ greater than 90.

(d) The board has dimensions too small to allow for turning up a side whose edge is greater than 4.

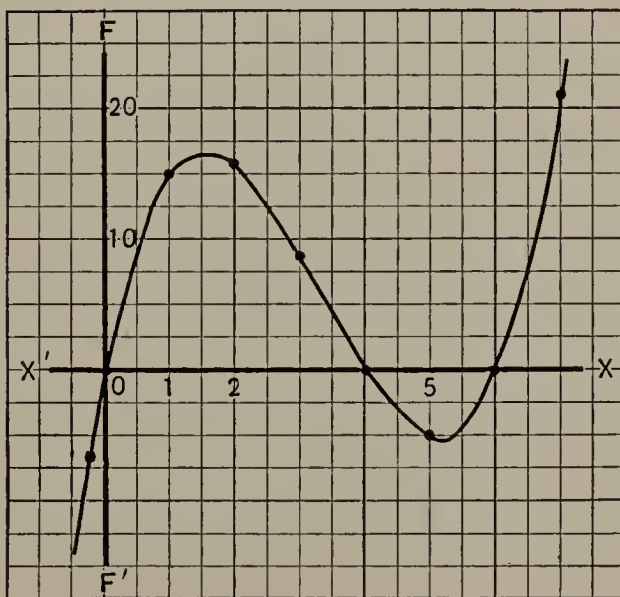
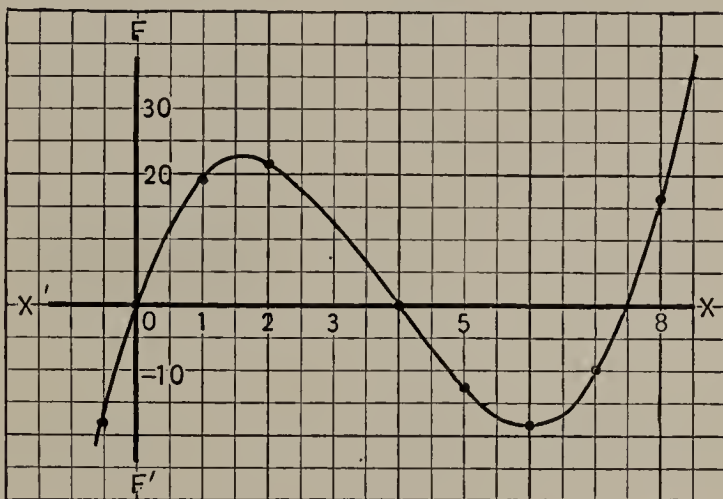
3. $v = 4x^3 - 40x^2 + 96x$.

$$\frac{f(x)}{4} = x^3 - 10x^2 + 24x.$$

x	0	1	2	3	4	5	6	7	$-\frac{1}{4}$
$f(x)$	0	15	16	9	0	-5	0	21	-6.6
4									

$\frac{f(x)}{4}$ and hence V is greatest for $x = 1.6$.

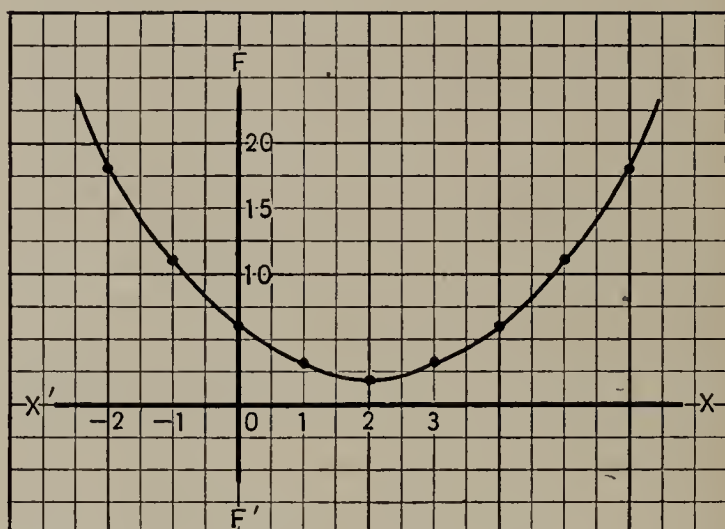
When $x = 1.6$, $V = 67$.



4. $f(x) = x^2 - 4x + 6$.

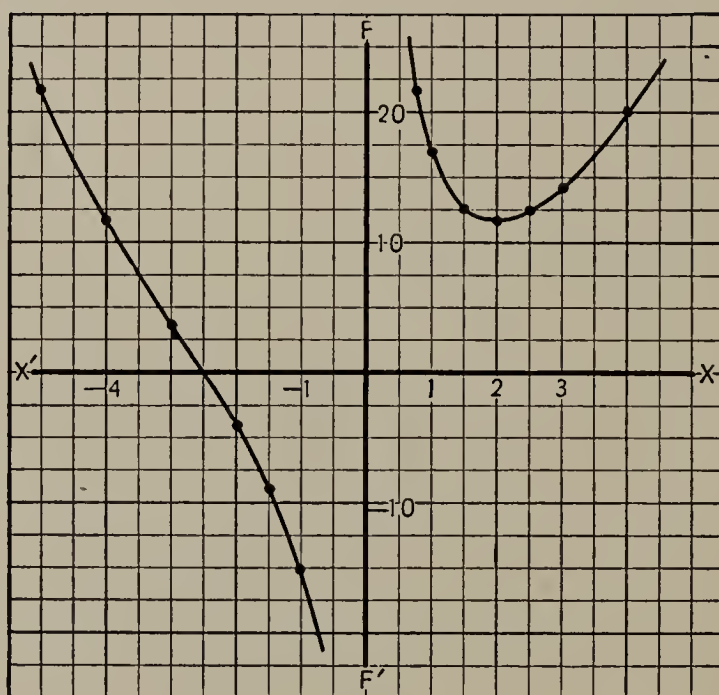
x	6	5	4	3	2	1	0	-1	-2
$f(x)$	18	11	6	3	2	3	6	11	18

The function is least
for $x = 2$.



5. $f(x) = x^2 + \frac{16}{x}$.

x	-5	-4	-3	-2	$-\frac{3}{2}$	-1	$\frac{3}{4}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4
$f(x)$	21.6	12	3.6	-4	-.84	-15	21.8	17	12.9	12	12.6	14.3	20



The function is least for positive values of x at $x = 2$.

Therefore the side of the base is 2 yards and the altitude is 1 yard.

6. Let x = radius of base and h = altitude in yards.

Then the volume $= \pi x^2 h$. Whence $h = \frac{4}{\pi x^2}$.

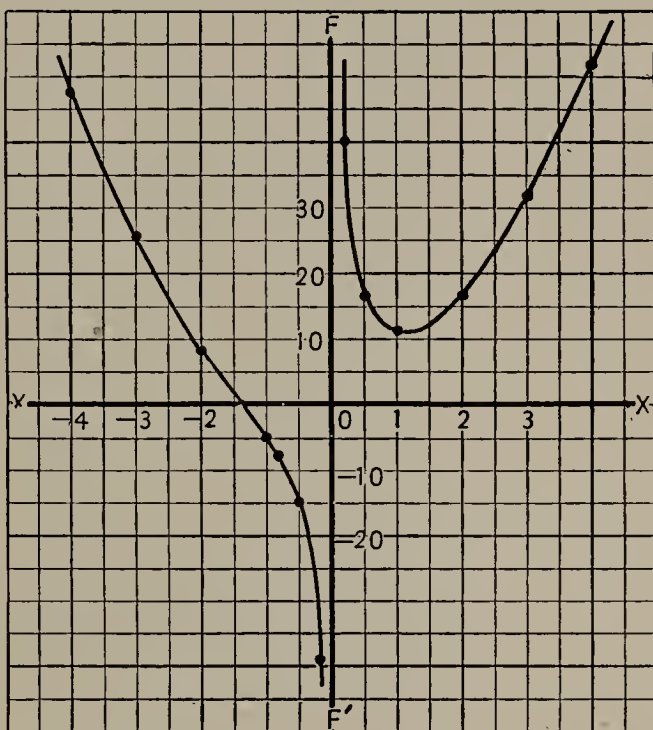
The inside surface = $\pi x^2 + 2\pi xh$. Substituting for h , $f(x) = \pi x^2 + \frac{8}{x}$.

x	-4	-3	-2	-1	-.8	-.5	-.2	.2	.5	.8	1	2	3	4
$f(x)$	48	25.6	8.5	-4.8	-7.8	-15.2	-39.8	40.1	16.7	12	11.1	16.5	30.9	52

It is evident that negative values for x have no meaning in the problem and need not have been considered.

From the graph the function is least for $x = 1$.

Hence $x = 1$ and $h = \frac{4}{\pi} = 1.26$

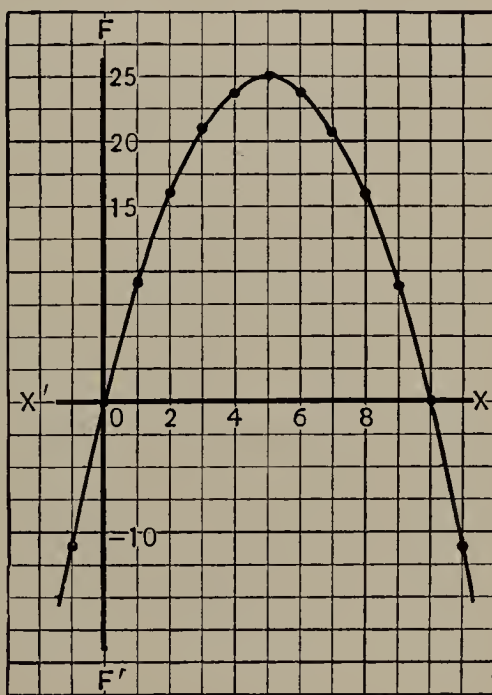


7. Let x = the width and $10 - x$ = the length in rods.

Then $x(10 - x)$ = the area = A .

Hence $A = f(x) = 10x - x^2$.

x	-1	0	1	2	3	4	5	6	7	8	9	10	11
$f(x)$	-11	0	9	16	21	24	25	24	21	16	9	0	-11

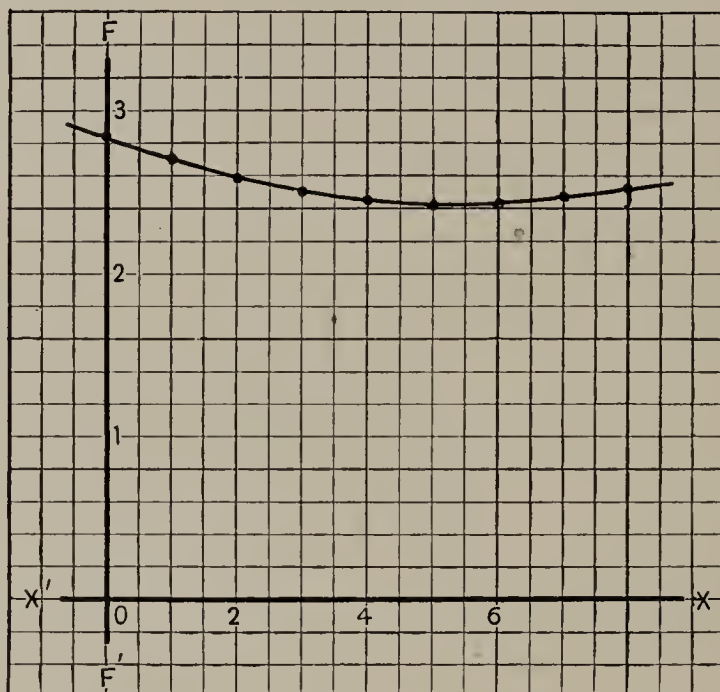


The function, and therefore the area A , is greatest for $x = 5$.

Hence length = width = 5.

8. $f(x) = \frac{8-x}{6} + \frac{\sqrt{x^2+36}}{4} = \text{time in hours.}$

x	0	1	2	3	4	5	6	7	8
$f(x)$	2.83	2.68	2.58	2.51	2.47	2.45	2.45	2.47	2.50



$f(x)$ is least for $x = 5.36$. Here $f(x) = \text{time} = 2.44$ hours.
Then $KA = 5.36$ miles.

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1. $x^2 - 4x - 32 = 0.$

$$x^2 - 4x + 4 = 32 + 4 = 36.$$

$$x - 2 = \pm 6.$$

$$x = 8, -4.$$

2. $2x^2 + 5x + 3 = 0.$

$$x^2 + \frac{5}{2}x + \frac{3}{2} = -\frac{3}{2} + \frac{25}{4} = \frac{17}{4}.$$

$$x + \frac{5}{4} = \pm \frac{\sqrt{17}}{2}.$$

$$x = -1, -\frac{3}{2}.$$

3. $\frac{x^2 + 3}{4} = \frac{5x^2 - 24}{7}.$

$$7x^2 + 21 = 20x^2 - 96.$$

$$13x^2 = 117.$$

$$x^2 = 9.$$

$$x = \pm 3.$$

4. $x^2 - 5x + 2 = 0.$

$$x^2 - 5x + \frac{25}{4} = -2 + \frac{25}{4} = \frac{17}{4}.$$

$$x - \frac{5}{2} = \pm \frac{\sqrt{17}}{2} = \pm \frac{4.12310}{2}.$$

$$x = 4.5615, .4384.$$

5. $7x^2 - 12x - 3 = 0.$

$$x^2 - \frac{12}{7}x + \frac{3}{7} = \frac{3}{7} + \frac{3}{7} = \frac{5}{7}.$$

$$x - \frac{6}{7} = \pm \frac{1}{7} \sqrt{57}.$$

$$x = \frac{6}{7} \pm \frac{7.54983}{7} = 1.9356, - .2214.$$

6. $\frac{3x^2}{2} - 3x\sqrt{2} = 9.$

$$x^2 - 2x\sqrt{2} + 2 = 6 + 2 = 8.$$

$$x - \sqrt{2} = \pm 2\sqrt{2}.$$

$$x = 3\sqrt{2}, -\sqrt{2}.$$

7. $(2x - 5)^2 - (x - 6)^2 = 80.$

$$4x^2 - 20x + 25 - x^2 + 12x - 36 = 80.$$

$$3x^2 - 8x = 91.$$

$$x^2 - \frac{8}{3}x + \frac{16}{9} = \frac{91}{3} + \frac{16}{9} = \frac{289}{9}.$$

$$x - \frac{4}{3} = \pm \frac{17}{3}; x = 7, -\frac{13}{3}.$$

8. $\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}.$

$$4x + 8 + 4x = 3x^2 + 6x.$$

$$3x^2 - 2x - 8 = 0.$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{8}{3} + \frac{1}{9} = \frac{25}{9}.$$

$$x - \frac{1}{3} = \pm \frac{5}{3}.$$

$$x = 2, -\frac{4}{3}.$$

9. $\frac{1}{w+7} - \frac{1}{w-3} = \frac{2}{5}.$

$$5w - 15 - 5w - 35 = 2w^2 + 8w - 42.$$

$$2w^2 + 8w = -8.$$

$$w^2 + 4w + 4 = -4 + 4 = 0.$$

$$w + 2 = 0.$$

$$w = -2.$$

10. $\frac{m}{m+1} - \frac{2m}{m+2} + \frac{9}{2} = 0.$

$$2m^2 + 4m - 4m^2 - 4m + 9m^2 + 27m + 18 = 0.$$

$$7m^2 + 27m = -18.$$

$$m^2 + \frac{27}{7}m + (\frac{27}{14})^2 = -\frac{18}{7} + \frac{729}{196}.$$

$$m + \frac{27}{14} = \pm \frac{15}{14}.$$

$$m = -3, -\frac{6}{7}.$$

$$11. \frac{10s^2 - 1 - s^3}{6s - s^2 - 9} - \frac{s^2 - 9}{s + 3} = 0.$$

$$\frac{s^3 - 10s^2 + 1}{s^2 - 6s + 9} = s - 3.$$

$$s^3 - 10s^2 + 1 = s^3 - 9s^2 + 27s - 27.$$

$$-s^2 - 27s + 28 = 0.$$

$$s^2 + 27s + \left(\frac{27}{2}\right)^2 = 28 + \frac{729}{4}.$$

$$s + \frac{27}{2} = \pm \frac{29}{2}.$$

$$s = 1, -28.$$

$$12. \frac{4}{x-4} - \frac{x+3}{x^2-5x+4} - 2 = 0.$$

$$4x - 4 - x - 3 - 2x^2 + 10x - 8 = 0.$$

$$-2x^2 + 13x - 15 = 0.$$

$$x^2 - \frac{13}{2}x + \left(\frac{-13}{4}\right)^2 = -\frac{15}{2} + \frac{169}{16} = \frac{49}{16}.$$

$$x - \frac{13}{4} = \pm \frac{7}{4}.$$

$$x = 5, \frac{5}{2}.$$

$$13. \frac{2(x^3 + 8)}{x+2} - \frac{x^3 - 1}{x-1} - \frac{19}{4} = 0.$$

$$2x^2 - 4x + 8 - x^2 - x - 1 - \frac{19}{4} = 0.$$

$$x^2 - 5x + \left(\frac{-5}{2}\right)^2 = -\frac{9}{4} + \frac{25}{4} = 4.$$

$$x - \frac{5}{2} = \pm 2.$$

$$x = 4\frac{1}{2}, \frac{1}{2}.$$

$$14. \frac{r + 2\sqrt{5}}{\sqrt{15}} - \frac{2}{r} = \frac{1}{\sqrt{5}}.$$

$$r^2 + 2r\sqrt{5} - 2\sqrt{15} = r\sqrt{3}.$$

$$r^2 + (2\sqrt{5} - \sqrt{3})r + \left(\frac{2\sqrt{5} - \sqrt{3}}{2}\right)^2 = 2\sqrt{15} + \frac{23 - 4\sqrt{15}}{4} = \frac{23 + 4\sqrt{15}}{4}.$$

$$r + \frac{2\sqrt{5} - \sqrt{3}}{2} = \pm \sqrt{\frac{23 + 4\sqrt{15}}{4}} = \pm \frac{2\sqrt{5} + \sqrt{3}}{2}.$$

$$r = -\frac{2\sqrt{5} - \sqrt{3}}{2} \pm \frac{2\sqrt{5} + \sqrt{3}}{2}$$

$$= \sqrt{3}, -2\sqrt{5}.$$

$$16. x^3 - 10x^{\frac{3}{2}} - 11 = 0.$$

$$x^3 - 10x^{\frac{3}{2}} + 25 = 11 + 25 = 36.$$

$$x^{\frac{3}{2}} - 5 = \pm 6.$$

$$x^{\frac{3}{2}} = 11, -1.$$

$$x^3 = 121, 1.$$

$$x = \sqrt[3]{121}, 1. \quad 1 \text{ is not a root.}$$

Every number has three cube roots. The cube roots of 1 are 1 and $-\frac{1}{2} \pm \frac{1}{2} \sqrt{-3}$; and the cube roots of 121 are $\sqrt[3]{121}$ and $\sqrt[3]{121}(-\frac{1}{2} \pm \frac{1}{2} \sqrt{-3})$ (see solution of Exercises 7-10, page 473). The student has not yet had sufficient practice in imaginaries to obtain such answers. Hence he cannot at this time solve completely equations like those of Exercises 16, 18, or 20. Therefore the imaginary answers to these will be omitted.

$$17. x^4 - 26x^2 + 25 = 0.$$

$$x^4 - 26x^2 + 169 = -25 + 169 = 144.$$

$$x^2 - 13 = \pm 12.$$

$$x^2 = 25, 1.$$

$$x = \pm 5, \pm 1.$$

$$18. x^6 - 7x^3 - 8 = 0.$$

$$x^6 - 7x^3 + \left(\frac{-7}{2}\right)^2 = 8 + \frac{49}{4} = \frac{81}{4}.$$

$$x^3 - \frac{7}{2} = \pm \frac{9}{2}.$$

$$x^3 = 8, -1.$$

$$x = 2, -1.$$

$$19. x + x^{\frac{1}{2}} - 6 = 0.$$

$$x + x^{\frac{1}{2}} + \frac{1}{4} = 6 + \frac{1}{4}.$$

$$x^{\frac{1}{2}} + \frac{1}{2} = \pm \frac{5}{2}.$$

$$x^{\frac{1}{2}} = 2, -3.$$

$$x = 4, 9. \quad 9 \text{ is not a root.}$$

$$20. 4x^6 - 7x^3 = 15.$$

$$x^6 - \frac{7}{4}x^3 + \left(\frac{-7}{8}\right)^2 = \frac{15}{4} + \frac{49}{64} = \frac{289}{64}.$$

$$x^3 - \frac{7}{8} = \pm \frac{17}{8}.$$

$$x^3 = 3, -\frac{5}{4}.$$

$$x = \sqrt[3]{3}, \frac{1}{2}\sqrt[3]{-10}.$$

$$21. 4x^2 + \frac{4}{x^2} - \frac{97}{9} = 0.$$

$$36x^4 + 36 - 97x^2 = 0.$$

$$x^4 - \frac{97}{36}x^2 + \left(\frac{-97}{72}\right)^2 = -1 + \frac{9409}{5184} = \frac{4225}{5184}.$$

$$x^2 - \frac{97}{72} = \pm \frac{65}{72}.$$

$$x = \pm \frac{8}{3}, \pm \frac{2}{3}.$$

$$22. 2x - 3x^{\frac{1}{2}} = 2.$$

$$x - \frac{3}{2}x^{\frac{1}{2}} + \left(-\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16}.$$

$$x^{\frac{1}{2}} - \frac{3}{4} = \pm \frac{5}{4}.$$

$$x^{\frac{1}{2}} = 2, -\frac{1}{2}.$$

$$x = 4, \frac{1}{4}. \quad \frac{1}{4} \text{ is not a root.}$$

23. $3x^4 - 11x^2 + 6 = 0.$

$$x^4 - \frac{11}{3}x^2 + \left(-\frac{11}{6}\right)^2 = -2 + \frac{121}{36} = \frac{49}{36}.$$

$$x^2 - \frac{11}{6} = \pm \frac{7}{6}.$$

$$x^2 = 3, \frac{2}{3}.$$

$$x = \pm \sqrt{3}, \pm \frac{1}{3}\sqrt{6}.$$

24. $9x^4 - 22x^2 + 8 = 0.$

$$x^4 - \frac{22}{9}x^2 + \left(-\frac{11}{9}\right)^2 = -\frac{8}{9} + \frac{121}{81} = \frac{49}{81}.$$

$$x^2 - \frac{11}{9} = \pm \frac{7}{9}.$$

$$x^2 = 2, \frac{4}{9}.$$

$$x = \pm \sqrt{2}, \pm \frac{2}{3}.$$

25. $3x^{\frac{3}{2}} + 5x^{\frac{3}{4}} + 2 = 0.$

$$x^{\frac{3}{2}} + \frac{5}{3}x^{\frac{3}{4}} + \left(\frac{5}{6}\right)^2 = -\frac{2}{3} + \frac{25}{36} = \frac{1}{36}.$$

$$x^{\frac{3}{4}} + \frac{5}{6} = \pm \frac{1}{6}.$$

$$x^{\frac{3}{4}} = -\frac{2}{3}, -1.$$

$$x^3 = \frac{16}{81}, 1.$$

$$x = \sqrt[3]{\frac{16}{81}}, 1. \text{ Neither is a root.}$$

26. $2x^{\frac{2}{3}} - 9\sqrt[3]{x} + 4 = 0.$

$$x^{\frac{2}{3}} - \frac{9}{2}x^{\frac{1}{3}} + \left(-\frac{9}{4}\right)^2 = -2 + \frac{81}{16} = \frac{49}{16}.$$

$$x^{\frac{1}{3}} - \frac{9}{4} = \pm \frac{7}{4}.$$

$$x^{\frac{1}{3}} = 4, \frac{1}{2}.$$

$$x = 64, \frac{1}{8}.$$

27. $3x - 11x^{\frac{1}{2}} - 20 = 0.$

$$x - \frac{11}{3}x^{\frac{1}{2}} + \left(-\frac{11}{6}\right)^2 = \frac{20}{3} + \frac{121}{36} = \frac{361}{36}.$$

$$x^{\frac{1}{2}} - \frac{11}{6} = \pm \frac{19}{6}.$$

$$x^{\frac{1}{2}} = 5, -\frac{4}{3}.$$

$$x = 25, \frac{16}{9}. \quad \frac{16}{9} \text{ is not a root.}$$

28. $6x^4 - 13x^2 + 6 = 0.$

$$x^4 - \frac{13}{6}x^2 + \left(\frac{-13}{12}\right)^2 = -1 + \frac{169}{144} = \frac{25}{144}.$$

$$x^2 - \frac{13}{12} = \pm \frac{5}{12}.$$

$$x^2 = \frac{3}{2}, \frac{2}{3}.$$

$$x = \pm \frac{1}{2}\sqrt{6}, \pm \frac{1}{3}\sqrt{6}.$$

29. $x^{-2} + 16x^{-1} - 17 = 0.$

$$x^{-2} + 16x^{-1} + 64 = 17 + 64 = 81.$$

$$x^{-1} + 8 = \pm 9.$$

$$x^{-1} = 1, -17.$$

$$x = 1, -\frac{1}{17}.$$

30. $y^{-4} - 10y^{-2} + 9 = 0.$

$$y^{-4} - 10y^{-2} + 25 = -9 + 25 = 16.$$

$$y^{-2} - 5 = \pm 4.$$

$$y^{-2} = 9, 1.$$

$$y^{-1} = \pm 3, \pm 1.$$

$$y = \pm \frac{1}{3}, \pm 1.$$

$$31. x^{-1} - 13x^{-\frac{1}{2}} = -36.$$

$$x^{-1} - 13x^{-\frac{1}{2}} + \left(\frac{-13}{2}\right)^2 = -36 + \frac{169}{4} = \frac{25}{4}.$$

$$x^{-\frac{1}{2}} - \frac{13}{2} = \pm \frac{5}{2}.$$

$$x^{-\frac{1}{2}} = 9, 4.$$

$$x^{-1} = 81, 16.$$

$$x = \frac{1}{81}, \frac{1}{16}.$$

$$32. x^{2m} + 4 - 5x^m = 0.$$

$$x^{2m} - 5x^m = -4.$$

$$x^{2m} - 5x^m + \left(\frac{-5}{2}\right)^2 = -4 + \frac{25}{4} = \frac{9}{4}.$$

$$x^m - \frac{5}{2} = \pm \frac{3}{2}.$$

$$x^m = 4, 1.$$

$$x = \sqrt[m]{4}, 1.$$

$$34. 3(x^2 + 3x)^2 - 7(x^2 + 3x) - 20 = 0.$$

$$(x^2 + 3x)^2 - \frac{7}{3}(x^2 + 3x) + \left(\frac{-7}{6}\right)^2 = \frac{20}{3} + \frac{49}{36} = \frac{289}{36}.$$

$$x^2 + 3x - \frac{7}{6} = \pm \frac{17}{6}.$$

$$x^2 + 3x = 4, -\frac{5}{3}.$$

$$x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4} = \frac{25}{4}.$$

$$x + \frac{3}{2} = \pm \frac{5}{2}.$$

$$x = 1, -4.$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{5}{3} + \frac{9}{4} = \frac{7}{12}.$$

$$x + \frac{3}{2} = \pm \sqrt{\frac{7}{12}}.$$

$$x = -\frac{3}{2} \pm \frac{1}{6}\sqrt{21}.$$

$$35. \left(x - \frac{1}{x}\right)^2 + 4\left(x - \frac{1}{x}\right) = 8\frac{1}{4}.$$

$$\left(x - \frac{1}{x}\right)^2 + 4\left(x - \frac{1}{x}\right) + 4 = 8\frac{1}{4} + 4 = 12\frac{1}{4}.$$

$$x - \frac{1}{x} + 2 = \pm \frac{7}{2}.$$

$$x - \frac{1}{x} = \frac{3}{2}, -\frac{11}{2}.$$

$$x^2 - \frac{3x}{2} + \frac{9}{16} = 1 + \frac{9}{16} = \frac{25}{16}.$$

$$x - \frac{3}{4} = \pm \frac{5}{4}.$$

$$x = 2, -\frac{1}{2}.$$

$$x^2 + \frac{11}{2}x + \frac{121}{16} = 1 + \frac{121}{16} = \frac{137}{16}.$$

$$x + \frac{11}{4} = \pm \frac{1}{4}\sqrt{137}.$$

$$x = \frac{-11 \pm \sqrt{137}}{4}.$$

$$36. (4y + 5) + 2(4y + 5)^{\frac{1}{2}} = 15.$$

$$(4y + 5) + 2(4y + 5)^{\frac{1}{2}} + 1 = 15 + 1 = 16.$$

$$(4y + 5)^{\frac{1}{2}} = 3, -5.$$

$$4y + 5 = 9, 25.$$

$$y = 1, 5. \quad 5 \text{ is not a root.}$$

$$37. x^2 + 5x + 3\sqrt{x^2 + 5x - 54} = 0.$$

$$x^2 + 5x + 3\sqrt{x^2 + 5x} + \frac{9}{4} = 54 + \frac{9}{4} = \frac{225}{4}.$$

$$\sqrt{x^2 + 5x + \frac{9}{4}} = \pm \frac{15}{2}.$$

$$\sqrt{x^2 + 5x} = 6, -9.$$

$$x^2 + 5x = 36, 81.$$

$$x^2 + 5x - 36 = 0.$$

$$(x - 4)(x + 9) = 0. \therefore x = 4, -9.$$

$$x^2 + 5x + \frac{25}{4} = 81 + \frac{25}{4} = \frac{349}{4}.$$

$$x + \frac{5}{2} = \pm \frac{1}{2}\sqrt{349}.$$

$$x = \frac{-5 \pm \sqrt{349}}{2} \text{ (not roots).}$$

$$38. x^2 - 2x - 5\sqrt{x^2 - 2x - 4} + 2 = 0.$$

$$x^2 - 2x - 4 - 5\sqrt{x^2 - 2x - 4} + \frac{25}{4} = -6 + \frac{25}{4} = \frac{1}{4}.$$

$$\sqrt{x^2 - 2x - 4 - \frac{5}{2}} = \pm \frac{1}{2}.$$

$$\sqrt{x^2 - 2x - 4} = 3, 2.$$

$$x^2 - 2x - 4 = 9, 4.$$

$$x^2 - 2x - 13 = 0, x = 1 \pm \sqrt{14}.$$

$$x^2 - 2x - 8 = 0.$$

$$(x - 4)(x + 2) = 0, x = 4, -2.$$

$$39. 2y(2y + 1) + 3\sqrt{8y^2 + 12y + 5} = 25 - 4y.$$

$$4y^2 + 4y + 6\sqrt{8y^2 + 12y + 5} = 50 - 8y.$$

$$8y^2 + 12y + 5 + 6\sqrt{8y^2 + 12y + 5} + 9 = 55 + 9 = 64.$$

$$\sqrt{8y^2 + 12y + 5} + 3 = \pm 8.$$

$$\sqrt{8y^2 + 12y + 5} = 5, -11.$$

$$8y^2 + 12y + 5 = 25, 121.$$

$$8y^2 + 12y - 20 = 0, 2y^2 + 3y - 5 = 0. \therefore y = -\frac{5}{2}, 1.$$

$$8y^2 + 12y - 116 = 0, 2y^2 + 3y - 29 = 0. \therefore y = \frac{-3 \pm \sqrt{241}}{4}.$$

$$40. 2x^2 + ax - 6a^2 = 0.$$

$$x^2 + \frac{a}{2}x + \frac{a^2}{16} = 3a^2 + \frac{a^2}{16} = \frac{49a^2}{16}.$$

$$x + \frac{a}{4} = \pm \frac{7a}{4}.$$

$$x = -2a, \frac{3a}{2}.$$

41. $2x^3 - 17bx^2 + 8b^2x = 0.$

$$x(x - 8b)(2x - b) = 0.$$

$$\therefore x = 0, 8b, \frac{b}{2}.$$

42. $7a^2x^2 - 4ax - 11 = 0.$

$$(7ax - 11)(ax + 1) = 0.$$

$$x = \frac{11}{7a}, -\frac{1}{a}.$$

43. $ax^2 + bx + c = 0.$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}.$$

$$x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a}.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

44. $12kx - 4k^2 - 5x^2 = 0.$

$$5x^2 - 12kx + 4k^2 = 0.$$

$$x = \frac{12k \pm \sqrt{144k^2 - 80k^2}}{10} = \frac{12k \pm 8k}{10} = 2k, \frac{2k}{5}.$$

45. $c^2(x - d)^2 - d^2(c - x)^2 = 0.$

$$c^2x^2 - 2c^2dx + c^2d^2 - c^2d^2 + 2d^2cx - d^2x^2 = 0.$$

$$(c^2 - d^2)x^2 - (2c^2d - 2cd^2)x = 0.$$

$$x[(c^2 - d^2)x - (2c^2d - 2cd^2)] = 0.$$

$$x = 0.$$

$$x = \frac{2c^2d - 2cd^2}{c^2 - d^2} = \frac{2cd}{c + d}.$$

46. $mx^2 - x(m^2 + 1) + m = 0.$

$$x = \frac{m^2 + 1 \pm \sqrt{m^4 + 2m^2 + 1 - 4m^2}}{2m} = \frac{m^2 + 1 \pm (m^2 - 1)}{2m} = m, \frac{1}{m}.$$

47. $(x - r)^2 + (S - x)^2 = r^2 + S^2.$

$$x^2 - 2rx + r^2 + S^2 - 2Sx + x^2 = r^2 + S^2.$$

$$2x^2 - (2r + 2S)x = 0.$$

$$x(2x - 2r - 2S) = 0.$$

$$x = 0, r + S.$$

48. $\frac{m}{m-x} - \frac{m-x}{m} = 2.$

$$m^2 - m^2 + 2mx - x^2 = 2m^2 - 2mx.$$

$$x^2 - 4mx + 2m^2 = 0.$$

$$x = \frac{4m \pm \sqrt{16m^2 - 8m^2}}{2} = 2m \pm m\sqrt{2}.$$

$$x = 2m + m\sqrt{2}, 2m - m\sqrt{2}.$$

$$49. \frac{x-c}{c} - \frac{c}{x-c} = \frac{3}{2}.$$

$$2x^2 - 4cx + 2c^2 - 2c^2 = 3cx - 3c^2.$$

$$2x^2 - 7cx + 3c^2 = 0.$$

$$x = \frac{7c \pm \sqrt{49c^2 - 24c^2}}{4} = \frac{7c \pm 5c}{4} = 3c, \frac{c}{2}.$$

$$50. \frac{x^2 - 3mx}{m-n} + 2m = \frac{nx}{n-m}.$$

$$x^2 - 3mx + nx + 2m^2 - 2mn = 0.$$

$$x = \frac{3m - n \pm \sqrt{9m^2 - 6mn + n^2 + 8mn - 8m^2}}{2}$$

$$= \frac{3m - n \pm (m + n)}{2} = 2m, m - n.$$

$$51. \frac{x^2 + 2ax}{a-b} - \frac{x}{2b} = \frac{a}{b}.$$

$$2bx^2 + 4abx - ax + bx = 2a^2 - 2ab.$$

$$2b(x^2) + x(4ab - a + b) - 2a^2 + 2ab = 0.$$

$$x = \frac{a-b-4ab \pm \sqrt{a^2 + 16a^2b^2 + b^2 - 2ab - 8a^2b + 8ab^2 + 16a^2b - 16ab^2}}{4b}.$$

$$x = \frac{a-b-4ab \pm (b-a-4ab)}{4b} = -2a, \frac{a-b}{2b}.$$

$$52. \frac{2s+x}{x+s} - \frac{5s-x}{x-s} - \frac{s(x+s)}{s^2-x^2} = 0.$$

$$x^2 + sx - 2s^2 + x^2 - 4sx - 5s^2 + sx + s^2 = 0.$$

$$2x^2 - 2sx - 6s^2 = 0, \text{ or } x^2 - sx - 3s^2 = 0.$$

$$x = \frac{s \pm \sqrt{s^2 + 12s^2}}{2} = \frac{s \pm s\sqrt{13}}{2}.$$

$$53. x = -\frac{5-3c}{4-2c} \pm \frac{7-3c}{4-2c} = -3, \frac{1}{2-c}.$$

$$54. ax^2 + 3x^2 + ax - 5x - a + 1 = a - 1.$$

$$(a+3)x^2 + (a-5)x - 2(a-1) = 0.$$

$$x = \frac{5-a \pm \sqrt{a^2 - 10a + 25 + 8a^2 + 16a - 24}}{2a+6}$$

$$= \frac{5-a \pm (3a+1)}{2a+6} = 1, \frac{-2a+2}{a+3}.$$

$$55. cx^2 - ax^2 + cx + ax - c = c + a - ax.$$

$$(c - a)x^2 + (c + 2a)x - 2c - a = 0.$$

$$x = \frac{-c - 2a \pm \sqrt{c^2 + 4ca + 4a^2 + 8c^2 - 4ca - 4a^2}}{2c - 2a}$$

$$= \frac{-c - 2a \pm 3c}{2c - 2a} = 1, \frac{-2c - a}{c - a}.$$

$$56. x^2 + 2s + s^2 = 2sx + 2x.$$

$$x^2 - 2sx - 2x = -2s - s^2.$$

$$x^2 - 2x(s + 1) + (s + 1)^2 = -2s - s^2 + s^2 + 2s + 1.$$

$$x - s - 1 = \pm 1.$$

$$x = s + 2, s.$$

$$57. x^2 - 2x + 1 = ax - ax^2.$$

$$(1 + a)x^2 - (2 + a)x + 1 = 0.$$

$$x = \frac{2 + a \pm \sqrt{4 + 4a + a^2 - 4 - 4a}}{2 + 2a} = \frac{2 + a \pm a}{2 + 2a} = 1, \frac{1}{1 + a}.$$

$$58. x^2 + 2x + 1 = hx^2 + hx.$$

$$(1 - h)x^2 + (2 - h)x + 1 = 0.$$

$$x = \frac{h - 2 \pm \sqrt{4 - 4h + h^2 - 4 + 4h}}{2 - 2h} = \frac{h - 2 \pm h}{2 - 2h} = -1, \frac{1}{h - 1}.$$

$$59. cx^2 + 3x = 2x^2 + 2cx - 2.$$

$$(c - 2)x^2 + (3 - 2c)x + 2 = 0.$$

$$x = \frac{2c - 3 \pm \sqrt{9 - 12c + 4c^2 - 8c + 16}}{2c - 4} = \frac{2c - 3 \pm (5 - 2c)}{2c - 4}.$$

$$x = \frac{1}{c - 2}, 2.$$

$$60. \frac{a}{x + a} + \frac{b}{x + b} - \frac{2c}{x + c} = 0.$$

$$ax^2 + abx + acx + abc + bx^2 + abx + cbx$$

$$+ abc - 2cx^2 - 2acx - 2cbx - 2abc = 0.$$

$$(a + b - 2c)x^2 + (2ab - ac - cb)x = 0.$$

$$x[(a + b - 2c)x + (2ab - ac - bc)] = 0.$$

$$x = 0, \frac{ac + bc - 2ab}{a + b - 2c}.$$

$$61. \frac{a + x}{b + x} + \frac{b + x}{a + x} = \frac{5}{2}.$$

$$2a^2 + 4ax + 2x^2 + 2b^2 + 4bx + 2x^2 = 5ab + 5ax + 5bx + 5x^2.$$

$$x^2 + (a + b)x - 2a^2 - 2b^2 + 5ab = 0.$$

$$x = \frac{-a - b \pm \sqrt{a^2 + 2ab + b^2 + 8a^2 - 20ab + 8b^2}}{2}$$

$$= \frac{-a - b \pm (3a - 3b)}{2}.$$

$$x = a - 2b, -2a + b.$$

$$62. \frac{ax + b}{bx + a} = \frac{mx - n}{nx - m}.$$

$$anx^2 + bnx - amx - bm = mbx^2 - nbx + amx - an.$$

$$(an - bm)x^2 + (2bn - 2am)x + an - bm = 0.$$

$$x = \frac{2am - 2bn \pm \sqrt{4b^2n^2 - 8abmn + 4a^2m^2 - 4a^2n^2 + 8abmn - 4b^2m^2}}{2an - 2bm}.$$

$$x = \frac{am - bn \pm \sqrt{(a^2 - b^2)(m^2 - n^2)}}{an - bm}.$$

$$63. \frac{ax^2 + bx + c}{bx^2 - mx + n} = \frac{c}{n}.$$

$$anx^2 + bnx + cn = bcx^2 - cmx + cn.$$

$$(an - bc)x^2 + (bn + cm)x = 0.$$

$$x = 0, -\frac{bn + cm}{an - bc}.$$

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$$1. 2x^2 - 7x + 3 = 0.$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{4} = 3, \frac{1}{2}.$$

$$2. 3x^2 - x - 2 = 0.$$

$$x = \frac{1 \pm \sqrt{1 + 24}}{6} = 1, -\frac{2}{3}.$$

$$3. 3x^2 - 11hx - 20h^2 = 0.$$

$$x = \frac{11h \pm \sqrt{121h^2 + 240h^2}}{6} = \frac{11h \pm 19h}{6} = 5h, -\frac{4h}{3}.$$

$$4. 5x^2 + 2cx - 16c^2 = 0.$$

$$x = \frac{-2c \pm \sqrt{4c^2 + 320c^2}}{10} = \frac{-2c \pm 18c}{10} = -2c, \frac{8c}{5}.$$

$$5. 22x^2 - 3mx - 7m^2 = 0.$$

$$x = \frac{3m \pm \sqrt{9m^2 + 616m^2}}{44} = \frac{3m \pm 25m}{44} = \frac{-m}{2}, \frac{7m}{11}.$$

$$6. 6x^2 - 7rx + 2r^2 = 0.$$

$$x = \frac{7r \pm \sqrt{49r^2 - 48r^2}}{12} = \frac{7r \pm r}{12} = \frac{2r}{3}, \frac{r}{2}.$$

$$7. 3x^2 - 6ax + 2a^2 = 0.$$

$$x = \frac{6a \pm \sqrt{36a^2 - 24a^2}}{6} = \frac{6a \pm 2a\sqrt{3}}{6} = \frac{3a \pm a\sqrt{3}}{3}.$$

$$8. 3m^2 + 4mx - 7x^2 = 0.$$

$$x = \frac{-4m \pm \sqrt{16m^2 + 84m^2}}{-14} = \frac{-4m \pm 10m}{-14} = m, -\frac{3m}{7}.$$

$$9. 4ax - 10a^2x^2 + 3 = 0.$$

$$x = \frac{-4a \pm \sqrt{16a^2 + 120a^2}}{-20a^2} = \frac{-4a \pm 2a\sqrt{34}}{-20a^2} = \frac{2 \pm \sqrt{34}}{10a}.$$

10. $12v^4 + 7v^2x - 10x^2 = 0.$

$$x = \frac{-7v^2 \pm \sqrt{49v^4 + 480v^4}}{-20} = \frac{-7v^2 \pm 23v^2}{-20} = \frac{3v^2}{2}, \frac{-4v^2}{5}.$$

11. $x = \frac{-(3-m) \pm \sqrt{9-6m+m^2+12m}}{2}$

$$= \frac{-3+m \pm (3+m)}{2} = m, -3.$$

12. $x^2 + nx = cx + cn.$

$$x^2 + (n-c)x - cn = 0.$$

$$x = \frac{c-n \pm \sqrt{n^2-2cn+c^2+4cn}}{2} = \frac{c-n \pm (n+c)}{2} = c, -n.$$

13. $3x^2 - 6cx + 2c = x.$

$$3x^2 - (6c+1)x + 2c = 0.$$

$$x = \frac{6c+1 \pm \sqrt{36c^2+12c+1-24c}}{6}$$

$$= \frac{6c+1 \pm (6c-1)}{6} = 2c, \frac{1}{3}.$$

14. $mx^2 + mkx = kc + cx.$

$$mx^2 + (mk-c)x - kc = 0.$$

$$x = \frac{c-mk \pm \sqrt{m^2k^2+c^2-2mkc+4mkc}}{2m}$$

$$= \frac{c-mk \pm (mk+c)}{2m} = \frac{c}{m}, -k.$$

15. $a^2x^2 - 2ax = b^2x^2 - 1.$

$$x^2(a^2-b^2) - 2ax + 1 = 0.$$

$$x = \frac{2a \pm \sqrt{4a^2-4a^2+4b^2}}{2a^2-2b^2} = \frac{2a \pm 2b}{2a^2-2b^2} = \frac{1}{a-b}, \frac{1}{a+b}.$$

16. $n^3x^2 + 2nx = 5n^2x + 10.$

$$n^3x^2 + (2n-5n^2)x - 10 = 0.$$

$$x = \frac{5n^2-2n \pm \sqrt{25n^4-20n^3+4n^2+40n^3}}{2n^3}$$

$$= \frac{5n^2-2n \pm (5n^2+2n)}{2n^3} = \frac{5}{n}, -\frac{2}{n^2}.$$

17. $h k x^2 - h k = h^2 x + h^2 x^2.$

$$(h k - h^2)x^2 - h^2x - h k = 0.$$

$$x = \frac{h^2 \pm \sqrt{h^4 + 4h^2k^2 - 4h^3k}}{2hk - 2h^2}$$

$$= \frac{h^2 \pm (h^2 - 2hk)}{2hk - 2h^2} = -1, \frac{k}{k-h}.$$

$$18. cx^2 + cmx + 5 = cx + 5(x + m).$$

$$cx^2 + (cm - c - 5)x + 5 - 5m = 0.$$

$$x = \frac{-cm + c + 5 \pm \sqrt{c^2m^2 + c^2 + 25 - 2c^2m - 10cm + 10c - 20c + 20cm}}{2c} = \frac{-cm + c + 5 \pm (cm - c + 5)}{2c} = \frac{5}{c}, 1 - m.$$

$$19. n^2x + 3nx + 2x = nx^2 + 2n + 3n^2.$$

$$nx^2 - (n^2 + 3n + 2)x + 2n + 3n^2 = 0.$$

$$x = \frac{n^2 + 3n + 2 \pm \sqrt{n^4 + 9n^2 + 4 + 6n^3 + 4n^2 + 12n - 8n^2 - 12n^3}}{2n} = \frac{n^2 + 3n + 2 \pm \sqrt{n^4 - 6n^3 + 5n^2 + 12n + 4}}{2n}$$

$$= \frac{n^2 + 3n + 2 \pm (n^2 - 3n - 2)}{2n} = n, \frac{2}{n} + 3.$$

$$20. m^2x^2 + 4mx + hmx + 3hx = 9x^2 + 12x - 4h.$$

$$(m^2 - 9)x^2 + (4m + hm + 3h - 12)x + 4h = 0.$$

$$x = \frac{-4m - hm - 3h + 12 \pm \sqrt{16m^2 + h^2m^2 + 9h^2 + 144 + 8hm^2 + 24mh - 96m + 6mh^2 - 24hm - 72h - 16hm^2 + 144h}}{2(m^2 - 9)}$$

$$x = \frac{-4m - hm - 3h + 12 \pm (4m - hm - 3h - 12)}{2(m^2 - 9)}.$$

$$x = -\frac{4}{m + 3}, -\frac{h}{m - 3}.$$

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1. Let $x =$ the first part.
 Then $20 - x =$ the second part.
 Then $20 - x = x^2$.
 Solving, $x = 4$ or -5 . -5 is impossible.
 Therefore 4 and 16 are the required parts.
2. Let $h =$ the hypotenuse in feet.
 Then $h - 2 =$ one leg in feet.
 Then $h^2 = (h - 2)^2 + 8^2$.
 Whence $h = 17$, and $h - 2 = 15$.
 $\therefore \text{area} = 8 \cdot 15 \div 2 = 60$.
3. Let $t =$ the time of trip in hours.
 Then $\frac{112}{t} =$ rate in miles per hour.
 $\therefore \frac{112}{t} + 6 = t$.
 Whence $t = 14$ or -8 . -8 is impossible.
 $\frac{112}{14} = 8$, the rate.
4. Let $n =$ the less integer.
 Then $n + 1 =$ the next greater integer.
 $\therefore \frac{1}{n} + \frac{1}{n+1} = \frac{5}{6}$.
 Whence $n = 2$ or $-\frac{3}{5}$. $-\frac{3}{5}$ is impossible.
 $n + 1 = 3$.
5. Let $x =$ altitude in feet.
 Then $x + 4 =$ base in feet.
 $\therefore \frac{x^2 + 4x}{2} = 48$.
 Whence $x = 8$ or -12 . -12 is impossible.
 $8 =$ altitude; $12 =$ base.
6. Let $x =$ shorter leg in feet.
 Then $x + 7 =$ longer leg in feet.
 $\therefore \frac{x^2 + 7x}{2} = 30$.
 Whence $x = 5$, -12 . -12 is impossible.
 Hypotenuse $= \sqrt{25 + 144} = 13$.
 Therefore the sides are 5, 12, and 13.
7. Let $x =$ altitude in feet.
 Then $7x + 2 =$ base in feet.
 $\therefore \frac{7x^2 + 2x}{2} = 360$.
 $7x^2 + 2x = 720$.
 Whence $x = 10$ or $-10\frac{2}{7}$. $-10\frac{2}{7}$ is impossible.
 $\therefore 10 =$ altitude; $7x + 2 = 72 =$ base.

8. Let $x =$ altitude in feet.

Then $x + 2$ and $2x =$ bases.

$$\therefore \frac{3x^2 + 2x}{2} = 60.$$

Whence $x = 6, -\frac{20}{3}$. $-\frac{20}{3}$ is impossible.

$\therefore 6 =$ altitude ; 8 and 12 are the bases.

9. Let $x =$ number of days B takes.

Then $x + 4 =$ number of days A takes.

$$\therefore \frac{1}{x} + \frac{1}{x+4} = \frac{9}{80}.$$

$$160x + 320 = 9x^2 + 36x.$$

$$9x^2 - 124x - 320 = 0.$$

Whence $x = 16$ or $-\frac{20}{9}$. $-\frac{20}{9}$ is impossible.

$\therefore 16 =$ number of days B takes,

and $20 =$ number of days A takes.

10. Let $x =$ the amount in inches the radius is shortened.

Then $(21 - x)^2 \cdot \frac{2}{7} = \frac{2}{7} (441) - 770.$

$$x^2 - 42x + 245 = 0.$$

Whence $x = 7$ or 35 . 35 is impossible.

$\therefore 7 =$ the amount the radius is shortened.

11. Let $x =$ number of gallons of wine in the cask.

$$\text{Then } \frac{(x-5) - 5 \cdot \frac{x-5}{x}}{x} = \frac{64}{100},$$

$$\text{or } 36x^2 - 1000x + 2500 = 0.$$

Whence $x = 25$ or $2\frac{7}{9}$. $2\frac{7}{9}$ is impossible.

12. Let $x =$ number of letters per line.

Then $x + 15 =$ number of lines.

$$\therefore x(x+15) = (x-10)(x+30).$$

Whence $x = 60,$

and $x + 15 = 75.$

Therefore $60 =$ number of letters per line,

$75 =$ number of lines,

and $60 \cdot 75 = 4500,$ letters on a page.

13. Let $x =$ rate of interest.

$$\text{Then } (50x + 300) + \frac{x}{100} (5000 + 50x + 300) + 300 = 1236.$$

$$\frac{x^2}{2} + 103x - 636 = 0.$$

Whence $x = 6$ or -212 . -212 is impossible.

Therefore $6\% =$ rate of interest.

14. Let $x =$ number of seconds.

$$\text{Then } (123 + 239x)^2 + (239 - 123x)^2 = \overline{850}^2.$$

$$72,250x^2 + 72,250 = 722,500.$$

Whence $x = +3$ or -3 .

Ans. 3 seconds before, and 3 seconds after the stated time.

15. Let $x =$ one dimension in inches.

Then $x + 1$ and $x + 2 =$ other dimensions.

$$\therefore 2[x(x + 1) + x(x + 2) + (x + 1)(x + 2)] = 292,$$

or $x^2 + 2x - 48 = 0.$

Whence $x = 6$ or -8 . -8 is impossible.

The dimensions in inches are 6, 7, and 8.

16. Let $x =$ side of square in inches.

Then $x - 6 =$ length of the square box in inches.

$$\therefore 3(x - 6)^2 = 300.$$

$$x^2 - 12x - 64 = 0.$$

Whence $x = 16$ or -4 . -4 is impossible.

17. Let $BC = x$.

Then $144 = x(x + 20).$

$$\therefore x^2 + 20x - 144 = 0.$$

Whence $x = \frac{-20 \pm 31.24}{2} = 5.62$ or -25.62 .

-25.62 is impossible.

18. If BC in the figure on page 374 is very small with respect to CD , $\overline{AB}^2 = BC \cdot BD$ becomes $\overline{AB}^2 = BC(BC + CD)$ or approximately $BC \cdot CD$.

Referring to the figure, $\overline{80}^2 = BC \cdot 7920$.

Whence $BC = \frac{640}{792} = \frac{80}{99}$ miles, or $4266\frac{2}{3}$ feet.

19. Referring to the figure, let $AB =$ the distance in miles.

$$\overline{AB}^2 = \frac{6}{5280} \cdot 7920.$$

$$\overline{AB}^2 = 9.$$

$AB = 3$ or -3 . -3 is impossible.

20. The distance between them consists of the sum of two tangents. Let AB represent one tangent in miles and $A'B'$ the other.

$$AB = \pm \sqrt{\frac{250}{5280} \cdot 7920} = \pm 5\sqrt{15} = \pm 19.364. \quad -19.364 \text{ is impossible.}$$

$$A'B' = \pm \sqrt{\frac{150}{5280} \cdot 7920} = \pm 15. \quad -15 \text{ is impossible.}$$

$$AB + A'B' = 19.364 + 15 = 34.364.$$

The distance = 34.364 miles.

$$21. S = \frac{gt^2}{2} = \frac{32 \cdot 144}{2} = 2304 \text{ feet.}$$

22. Let $x =$ time in seconds for falling.

Then $13 - x =$ time in seconds for sound to return; and, from Exercise 21, $16x^2 =$ distance fallen in feet.

$(13 - x) 1120 =$ distance the sound returns in feet.

$$16x^2 = 14560 - 1120x.$$

$$x^2 + 70x - 910 = 0.$$

$$x = 11.206. \quad -81.206 \text{ is impossible.}$$

The height $= 16x^2 = 2009.28$ feet.

23. Let $x =$ the number of miles the head of army moves while the messenger travels from its rear to its head.

$$\text{Then} \quad \frac{x}{28 + x} = \frac{28 - x}{x}.$$

$$x^2 = -x^2 + 784 \text{ or } 2x^2 = 784.$$

Whence $x^2 = 392$, or $x = \pm 14\sqrt{2}$. $-14\sqrt{2}$ is impossible.

Altogether he traveled $28 + 2x$ or $28 + 28\sqrt{2} = 67.5979$ miles.

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$$1. \sqrt{x + 3} = 8.$$

$$x + 3 = 64.$$

$$\therefore x = 61.$$

$$2. \sqrt{2x - 6} + 4 = 7.$$

$$\sqrt{2x - 6} = 3.$$

$$2x - 6 = 9.$$

$$\therefore x = 7\frac{1}{2}.$$

$$3. 3\sqrt{2x - 8} - 7 = 17.$$

$$3\sqrt{2x - 8} = 24.$$

$$\sqrt{2x - 8} = 8.$$

$$2x - 8 = 64.$$

$$\therefore x = 36.$$

$$4. \sqrt[3]{3x - 4} = 2.$$

$$3x - 4 = 8.$$

$$\therefore x = 4.$$

$$5. (7x + 15)^{\frac{1}{3}} + 18 = 17.$$

$$\sqrt[3]{7x + 15} = -1.$$

$$7x + 15 = -1.$$

$$\therefore x = -\frac{16}{7}.$$

$$6. 2\sqrt[3]{3n - 25} + 3 = 7.$$

$$2\sqrt[3]{3n - 25} = 4.$$

$$\sqrt[3]{3n - 25} = 2.$$

$$3n - 25 = 8.$$

$$\therefore n = 11.$$

$$7. 3x\sqrt{3x} = 18\sqrt{27x}.$$

$$x\sqrt{3x} = 18\sqrt{3x}.$$

$$x^3 = 324x.$$

$$x(x^2 - 324) = 0.$$

$$\therefore x = 0, +18, -18.$$

-18 is extraneous.

$$8. (9x)^{\frac{1}{2}} = x^{\frac{1}{2}} + 4.$$

$$3\sqrt{x} = \sqrt{x} + 4.$$

$$2\sqrt{x} = 4.$$

$$\sqrt{x} = 2.$$

$$\therefore x = 4.$$

$$9. \sqrt{5n^2 + 19} + n = 7.$$

$$\sqrt{5n^2 + 19} = 7 - n.$$

$$5n^2 + 19 = n^2 - 14n + 49.$$

$$4n^2 + 14n - 30 = 0.$$

$$\therefore n = -5, \frac{3}{2}.$$

$$10. (3x - 4)^{\frac{1}{3}} + (4x + 3)^{\frac{1}{3}} = 0.$$

$$(3x - 4)^{\frac{1}{3}} = -(4x + 3)^{\frac{1}{3}}.$$

$$3x - 4 = -4x - 3.$$

$$\therefore x = -\frac{1}{7}.$$

$$11. 4\sqrt{v^2 - v - 4} + 3 = 15.$$

$$4\sqrt{v^2 - v - 4} = 12.$$

$$v^2 - v - 4 = 9.$$

$$v^2 - v - 13 = 0.$$

$$\therefore v = \frac{1 \pm \sqrt{53}}{2}.$$

$$12. \sqrt{x + 4} = \sqrt[4]{x^2 - 5x + 6}.$$

$$x + 4 = \sqrt[4]{(x^2 - 5x + 6)^2}.$$

$$x^2 + 8x + 16 = x^2 - 5x + 6.$$

$$13x = -10.$$

$$\therefore x = -\frac{10}{13}.$$

13. $\sqrt[3]{x+1} = \sqrt{x+1}$.
 $\sqrt[3]{x^2+2x+1} = x+1$.
 $x^2+2x+1 = x^3+3x^2+3x+1$.
 $x^3+2x^2+x=0$.
 $x(x^2+2x+1)=0$.
 $\therefore x=0, -1$.
14. $(s-2)^{\frac{1}{2}} = s^{\frac{1}{2}} + 2^{\frac{1}{2}}$.
 $s-2 = s+2+2\sqrt{2s}$.
 $-4 = 2\sqrt{2s}$.
 $16 = 8s$.
 $\therefore s=2$. 2 is extraneous.
15. $3\sqrt{r+1} - 2\sqrt{r+3} = \sqrt{2r+4} - \sqrt{r+3} + 2\sqrt{r+1}$.
 $\sqrt{r+1} - \sqrt{r+3} = \sqrt{2r+4}$.
 $r+1 - 2\sqrt{r^2+4r+3} + r+3 = 2r+4$.
 $r^2+4r+3=0$.
 $\therefore r=-1, -3$. -1 is extraneous.
16. $\sqrt{2x-3} + 3\sqrt{x-5} = \sqrt{3x-8} + 2\sqrt{x-5}$.
 $\sqrt{2x-3} + \sqrt{x-5} = \sqrt{3x-8}$.
 $2x-3 + 2\sqrt{2x^2-13x+15} + x-5 = x-8$.
 $2x^2-13x+15=0$.
 $\therefore x=5, \frac{3}{2}$.
17. $\sqrt{4x-12} + \sqrt{5x-2} + \sqrt{9x-14} = 0$.
 $\sqrt{4x-12} + \sqrt{5x-2} = -\sqrt{9x-14}$.
 $4x-12+5x-2+2\sqrt{(4x-12)(5x-2)} = 9x-14$.
 $\sqrt{(4x-12)(5x-2)} = 0$.
 $\therefore x=3, \frac{2}{5}$. Both are extraneous.
18. $\sqrt{4x-12} - \sqrt{5x-2} - \sqrt{9x-14} = 0$.
 $\sqrt{4x-12} - \sqrt{5x-2} = \sqrt{9x-14}$.
 $4x-12+5x-2-2\sqrt{(4x-12)(5x-2)} = 9x-14$.
 $-\sqrt{(4x-12)(5x-2)} = 0$.
 $\therefore x=3, \frac{2}{5}$. 3 is extraneous.
19. $\sqrt{4x-12} + \sqrt{5x-2} - \sqrt{9x-14} = 0$.
 $\sqrt{4x-12} + \sqrt{5x-2} = \sqrt{9x-14}$.
 $4x-12+5x-2+2\sqrt{(4x-12)(5x-2)} = 9x-14$.
 $\sqrt{(4x-12)(5x-2)} = 0$.
 $x=3, \frac{2}{5}$.
20. $\sqrt{4x-12} - \sqrt{5x-2} + \sqrt{9x-14} = 0$.
 $\sqrt{4x-12} - \sqrt{5x-2} = -\sqrt{9x-14}$.
 $4x-12+5x-2-2\sqrt{(4x-12)(5x-2)} = 9x-14$.
 $\sqrt{(4x-12)(5x-2)} = 0$.
 $x=3, \frac{2}{5}$. $\frac{2}{5}$ is extraneous.

$$21. \frac{7\sqrt{n} + 10}{\sqrt{4n} - 2} = 3.$$

$$7\sqrt{n} + 10 = 6\sqrt{n} - 6.$$

$$\sqrt{n} = -16.$$

$$n = 256. \quad 256 \text{ is extraneous.}$$

$$22. \frac{2\sqrt{a}}{\sqrt{2x} - a} = \frac{\sqrt{2x + 4a}}{3\sqrt{a}}.$$

$$6a = \sqrt{4x^2 + 6ax - 4a^2}.$$

$$36a^2 = 4x^2 + 6ax - 4a^2.$$

$$2x^2 + 3ax - 20a^2 = 0.$$

$$x = \frac{5a}{2}, -4a. \quad -4a \text{ is extraneous.}$$

$$23. \frac{n^{\frac{1}{2}} - 3}{n^{\frac{1}{2}}} - \frac{5 - n^{\frac{1}{2}}}{4} = 0.$$

$$4\sqrt{n} - 12 = 5\sqrt{n} - n.$$

$$n - \sqrt{n} - 12 = 0.$$

$$(\sqrt{n} - 4)(\sqrt{n} + 3) = 0.$$

$$\therefore n = 16, \text{ or } n = 9. \quad 9 \text{ is extraneous.}$$

$$24. \frac{\sqrt{x + 16}}{\sqrt{4 - x}} + \frac{\sqrt{4 - x}}{\sqrt{x + 16}} = \frac{5}{2}.$$

$$2x + 32 + 8 - 2x = 5\sqrt{-x^2 - 12x + 64}.$$

$$-x^2 - 12x + 64 = 64.$$

$$x = 0, -12.$$

$$25. 5r - 13r^{\frac{1}{2}} + 6 = 0.$$

$$(5r^{\frac{1}{2}} - 3)(r^{\frac{1}{2}} - 2) = 0.$$

$$r = \frac{9}{25}, r = 4.$$

$$26. 7x - 3x^{\frac{1}{2}} - 2x^{\frac{3}{2}} = 0.$$

$$x^{\frac{1}{2}}(2x - 7x^{\frac{1}{2}} + 3) = 0.$$

$$x^{\frac{1}{2}}(2x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 3) = 0.$$

$$x = 0, \frac{1}{4}, 9.$$

$$27. \frac{(r + 5)^{\frac{1}{2}}}{(r + 3)^{\frac{1}{2}}} - \frac{(r + 3)^{\frac{1}{2}}}{(r + 5)^{\frac{1}{2}}} = \frac{2}{\sqrt{3}}.$$

$$(r + 5)\sqrt{3} - (r + 3)\sqrt{3} = 2\sqrt{r^2 + 8r + 15}.$$

$$r\sqrt{3} + 5\sqrt{3} - r\sqrt{3} - 3\sqrt{3} = 2\sqrt{r^2 + 8r + 15}.$$

$$\sqrt{3} = \sqrt{r^2 + 8r + 15}.$$

$$r^2 + 8r + 12 = 0.$$

$$r = -6, -2.$$

$$28. \sqrt{7 + 4x + 3\sqrt{2x^2 + 5x + 7}} - 3 = 0.$$

$$7 + 4x + 3\sqrt{2x^2 + 5x + 7} = 9.$$

$$3\sqrt{2x^2 + 5x + 7} = 2 - 4x.$$

$$18x^2 + 45x + 63 = 4 - 16x + 16x^2.$$

$$2x^2 + 61x + 59 = 0.$$

$$x = -\frac{59}{2}, -1.$$

$$29. \sqrt{17 + 2\sqrt{3 + s + \sqrt{s + 7}}} - 5 = 0.$$

$$17 + 2\sqrt{3 + s + \sqrt{s + 7}} = 25.$$

$$3 + s + \sqrt{s + 7} = 16.$$

$$s + 7 = 169 - 26s + s^2.$$

$$s^2 - 27s + 162 = 0.$$

$$s = 9, 18. \quad 18 \text{ is extraneous.}$$

$$30. 4x^2 = 10x + 10 - 2\sqrt{4x^2 - 10x - 2}.$$

$$4x^2 - 10x - 2 + 2\sqrt{4x^2 - 10x - 2} + 1 = 8 + 1.$$

$$\sqrt{4x^2 - 10x - 2} + 1 = \pm 3.$$

$$\sqrt{4x^2 - 10x - 2} = 2, -4.$$

$$4x^2 - 10x - 6 = 0.$$

$$x = 3, -\frac{1}{2}.$$

$$\sqrt{4x^2 - 10x - 2} = -4.$$

$$4x^2 - 10x - 18 = 0.$$

$$\therefore x = \frac{5 \pm \sqrt{97}}{4}. \quad \text{Both are extraneous.}$$

$$31. 3m^2 = 6\sqrt{3m^2 - m - 6} + m + 22.$$

$$3m^2 - m - 6 - 6\sqrt{3m^2 - m - 6} + 9 = 16 + 9.$$

$$\sqrt{3m^2 - m - 6} - 3 = \pm 5.$$

$$\sqrt{3m^2 - m - 6} = 8, -2.$$

$$3m^2 - m - 6 = 64, 4.$$

$$3m^2 - m - 70 = 0.$$

$$m = 5, -4\frac{2}{3}.$$

$$3m^2 - m - 6 = 4.$$

$$3m^2 - m - 10 = 0.$$

$$m = 2, -\frac{5}{3}. \quad \text{Both are extraneous.}$$

The teacher should note that the roots resulting in the solutions of Exercises 30 and 31 from the equations formed by setting the radical expression equal to the negative number are extraneous. This step, then, need not be performed.

$$32. \sqrt{x + 15} + \sqrt{x - 24} - \sqrt{x - 13} = \sqrt{x}.$$

$$\sqrt{x + 15} - \sqrt{x - 13} = \sqrt{x} - \sqrt{x - 24}.$$

$$x + 15 + x - 13 - 2\sqrt{x^2 + 2x - 195} = x + x - 24 - 2\sqrt{x^2 - 24x}.$$

$$\sqrt{x^2 + 2x - 195} = \sqrt{x^2 - 24x + 13}.$$

$$\begin{aligned}
 x^2 + 2x - 195 &= x^2 - 24x + 169 - 26\sqrt{x^2 - 24x}. \\
 26x - 364 &= -26\sqrt{x^2 - 24x}. \\
 x - 14 &= -\sqrt{x^2 - 24x}. \\
 x^2 - 28x + 196 &= x^2 - 24x. \\
 -4x &= -196. \\
 x &= 49.
 \end{aligned}$$

$$33. t = \pi \sqrt{\frac{l}{g}}.$$

$$t^2 = \pi^2 \frac{l}{g}.$$

$$gt^2 = \pi^2 l.$$

$$\therefore l = \frac{gt^2}{\pi^2},$$

$$\text{and} \quad g = \frac{\pi^2 l}{t^2}.$$

$$34. s = \frac{gt^2}{2}.$$

$$2s = gt^2.$$

$$t^2 = \frac{2s}{g}.$$

$$t = \sqrt{\frac{2s}{g}}.$$

$$t = \frac{\sqrt{2sg}}{g}.$$

$$37. K = 2r^2\sqrt{2} \text{ and } a = \frac{r}{2}\sqrt{2 + \sqrt{2}}.$$

$$2a = r\sqrt{2 + \sqrt{2}}.$$

$$r = \frac{2a}{\sqrt{2 + \sqrt{2}}}.$$

$$K = 2r^2\sqrt{2}.$$

$$K = 2\left(\frac{2a}{\sqrt{2 + \sqrt{2}}}\right)^2\sqrt{2} = \frac{2 \cdot 4a^2}{2 + \sqrt{2}}\sqrt{2} = \frac{8a^2}{\sqrt{2} + 1} = 8a^2(\sqrt{2} - 1).$$

$$38. K = 3r^2 \text{ and } a = \frac{r}{2}\sqrt{2 + \sqrt{3}}.$$

$$2a = r\sqrt{2 + \sqrt{3}}.$$

$$r = \frac{2a}{\sqrt{2 + \sqrt{3}}}.$$

$$K = 3r^2 = 3\left(\frac{2a}{\sqrt{2 + \sqrt{3}}}\right)^2 = \frac{3 \cdot 4a^2}{2 + \sqrt{3}} = \frac{12a^2}{2 + \sqrt{3}} = 12a^2(2 - \sqrt{3}).$$

$$35. a = \frac{R}{2}\sqrt{2} \text{ and } K = 2R^2.$$

$$2a = R\sqrt{2}.$$

$$R = a\sqrt{2}.$$

$$K = 2(a\sqrt{2})^2.$$

$$K = 4a^2.$$

$$36. K = \frac{3R^2}{2}\sqrt{3} \text{ and } a = \frac{R}{2}\sqrt{3}.$$

$$2a = R\sqrt{3}.$$

$$R = \frac{2a}{\sqrt{3}}.$$

$$K = \frac{3R^2}{2}\sqrt{3}.$$

$$\begin{aligned}
 K &= \frac{3}{2}\left(\frac{2a}{\sqrt{3}}\right)^2\sqrt{3} \\
 &= 2a^2\sqrt{3}.
 \end{aligned}$$

39. Let $x =$ the side of the first square.

Then $\frac{4x - 72}{4} =$ the side of the second square.

$$x^2 - (x - 18)^2 = 900.$$

$$36x - 324 = 900.$$

$$36x = 1224.$$

$$x = 34,$$

and $x - 18 = 16.$

40. $V = \sqrt{\frac{64}{5280} \cdot 4000}$ miles.

$$V = \sqrt{1\frac{600}{33}}.$$

$$V = \frac{40}{33} \sqrt{33}.$$

$$V = 7.$$

41. $.976v^2x - gx^2 = 0.$

$$.976 \cdot 10,000x - 32x^2 = 0.$$

$$9760x - 32x^2 = 0.$$

$$x(9760 - 32x) = 0.$$

$$x = 0, 305. \quad 0 \text{ is impossible.}$$

42. Substituting in the formula given in Problem 41,

$$.976v^2 - 32 \cdot 426.52 = 0.$$

$$.976v^2 - 13,648.6 = 0.$$

$$.976v^2 = 13,648.6.$$

$$v = \sqrt{\frac{13,648.6}{.976}} = 118.2.$$

43. Substituting in the formula of Problem 41:

(a) $(.976v^2 - 32 \cdot 497.63)497.63 = 0,$

$$.976v^2 = 15,924.76.$$

$$v^2 = 16,315.$$

$$v = 127.7.$$

(b) $51(.976v^2 - 32 \cdot 51) = 0.$

$$.976v^2 = 1632.$$

$$v^2 = 1672.$$

$$v = 40.8.$$

(c) $184.3(.976v^2 - 32 \cdot 184.3) = 0.$

$$.976v^2 = 5897.6.$$

$$v^2 = 6042.6.$$

$$v = 77.7.$$

(d) $200(.976v^2 - 32 \cdot 200) = 0.$

$$.976v^2 - 32 \cdot 200 = 0.$$

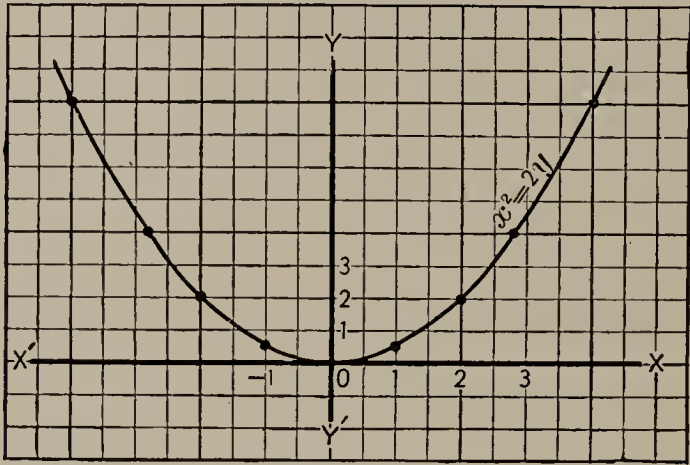
$$.976v^2 = 6400.$$

$$v^2 = 6557.3.$$

$$v = 80.9.$$

1. $x^2 = 2y.$ $\therefore x = \pm \sqrt{2y}.$

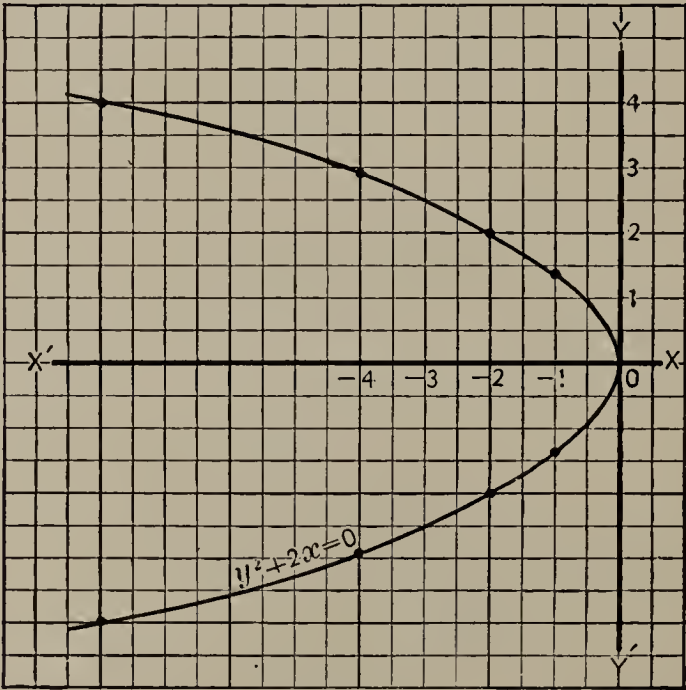
x	± 4	± 3.46	± 2.82	± 2	± 1.41	0	Imaginary
y	8	6	4	2	1	0	Negative values



Parabola

2. $y^2 + 2x = 0,$ $\therefore y = \pm \sqrt{-2x}.$

x	0	-1	-2	-4	-6	-8	Positive values
y	0	± 1.41	± 2	± 2.82	± 3.46	± 4	Imaginary

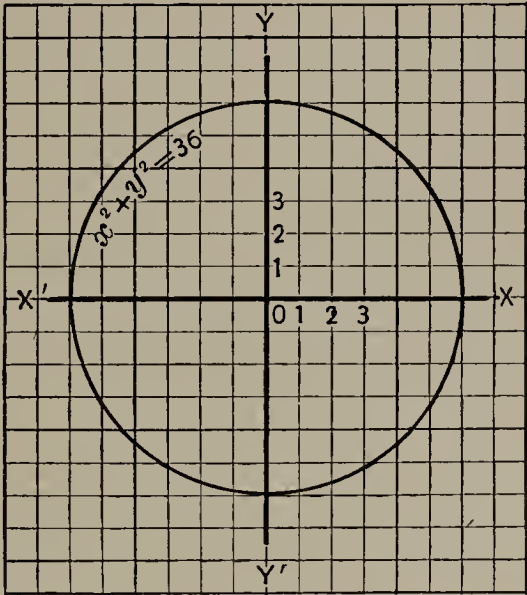


Parabola

3. $x^2 + y^2 = 36$. $\therefore y = \pm \sqrt{36 - x^2}$.

x	± 6	± 5	± 4	± 3	± 2	± 1	0
y	0	± 3.31	± 4.47	± 5.19	± 5.65	± 5.91	± 6

or $r = 6$.

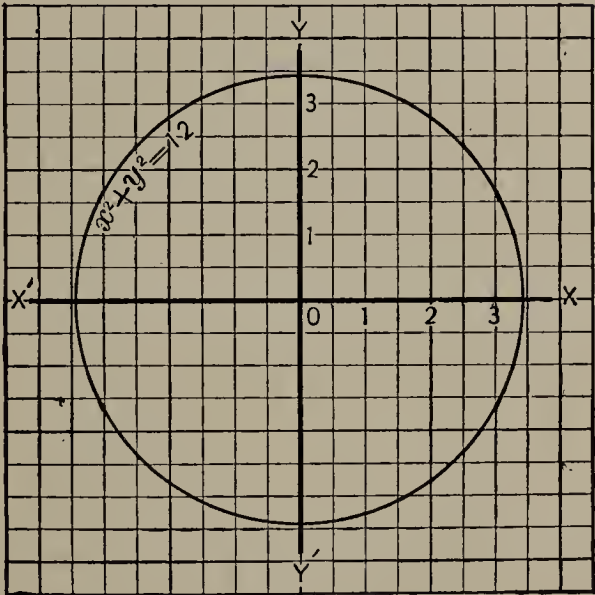


Circle

4. $x^2 + y^2 = 12$. $\therefore y = \pm \sqrt{12 - x^2}$.

x	0	± 1	± 2	± 3	± 3.46
y	± 3.46	± 3.31	± 2.82	± 1.73	0

or $r = 3.46$.

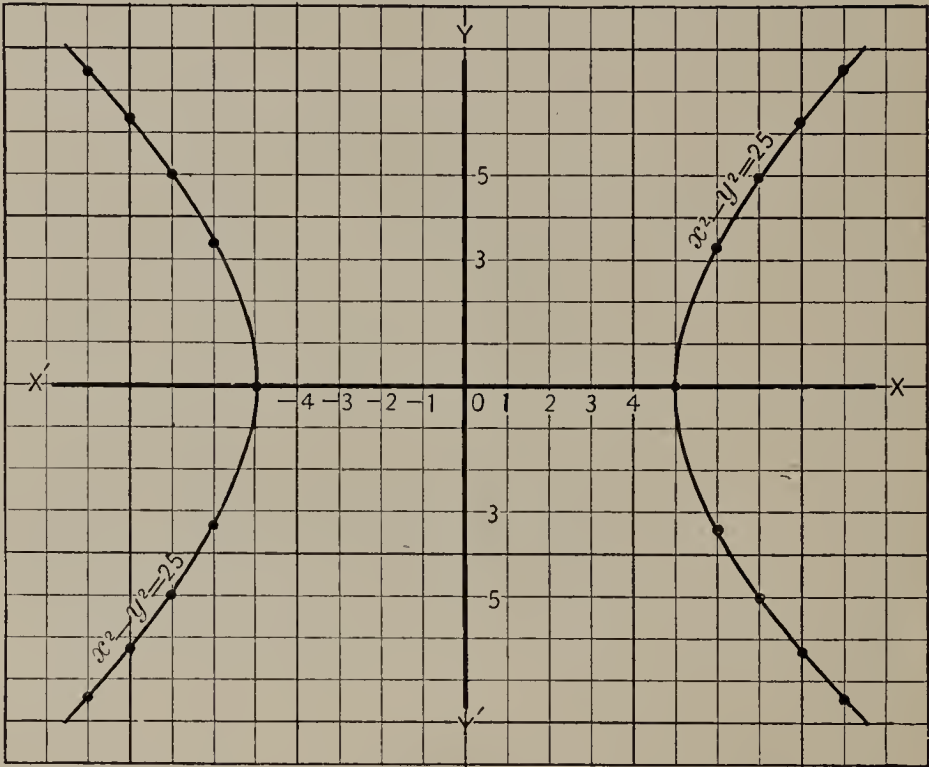


Circle

5. $x^2 - y^2 = 25.$ $\therefore y = \pm \sqrt{x^2 - 25}.$

x	Values between 5 and -5	+5	+6	+7	+8	+9	-5	-6	-7	-8	-9
y	Imaginary	0	± 3.31	± 4.89	± 6.24	± 7.48	0	± 3.31	± 4.89	± 6.24	± 7.48

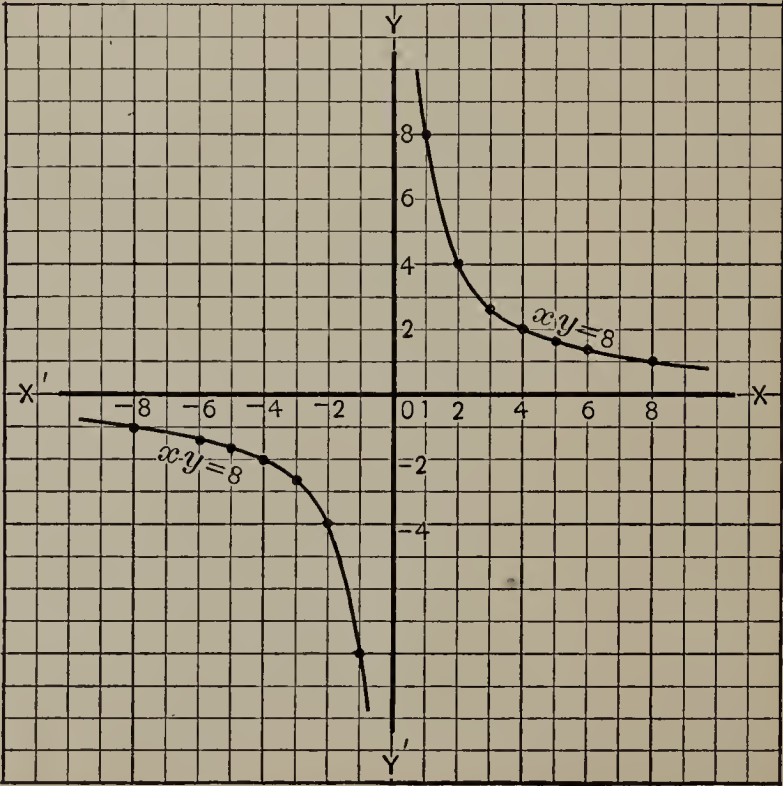
Hyperbola



6. $xy = 8.$ $\therefore y = \frac{8}{x}.$

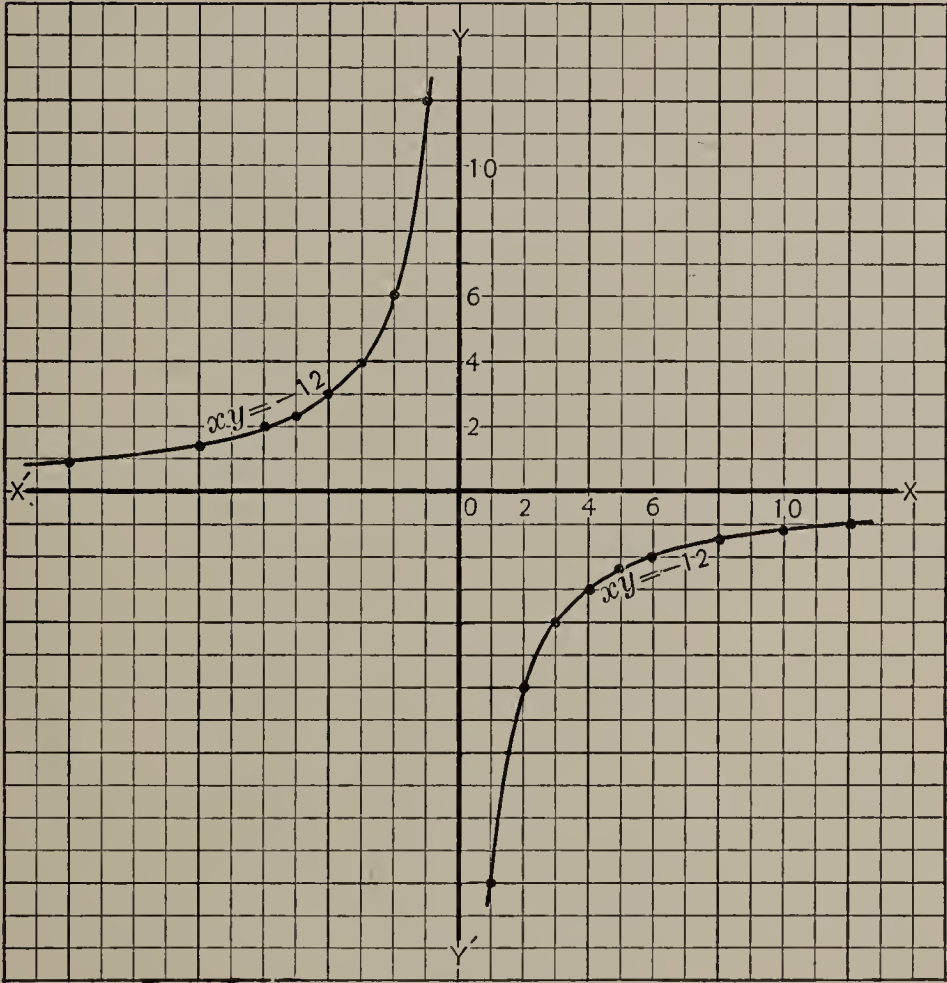
x	± 8	± 6	± 5	± 4	± 3	± 2	± 1	$\pm \frac{1}{2}$
y	± 1	$\pm \frac{4}{3}$	$\pm \frac{8}{5}$	± 2	$\pm \frac{8}{3}$	± 4	± 8	± 16

Hyperbola



7. $xy = -12$. $\therefore y = \frac{-12}{x}$.

x	± 12	± 10	± 8	± 6	± 5	± 4	± 3	± 2	± 1
y	∓ 1	$\mp \frac{6}{5}$	$\mp \frac{3}{2}$	∓ 2	$\mp \frac{12}{5}$	∓ 3	∓ 4	∓ 6	∓ 12

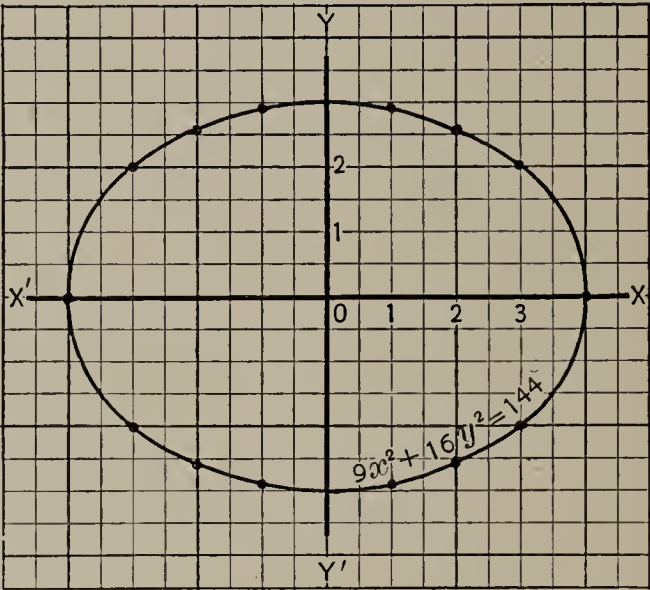


Hyperbola

8. $9x^2 + 16y^2 = 144.$ $\therefore y = \pm \frac{3}{4} \sqrt{16 - x^2}.$

x	4	3	2	1	0	-1	-2	-3	-4
y	0	± 1.98	± 2.59	± 2.90	± 3	± 2.90	± 2.59	± 1.98	0

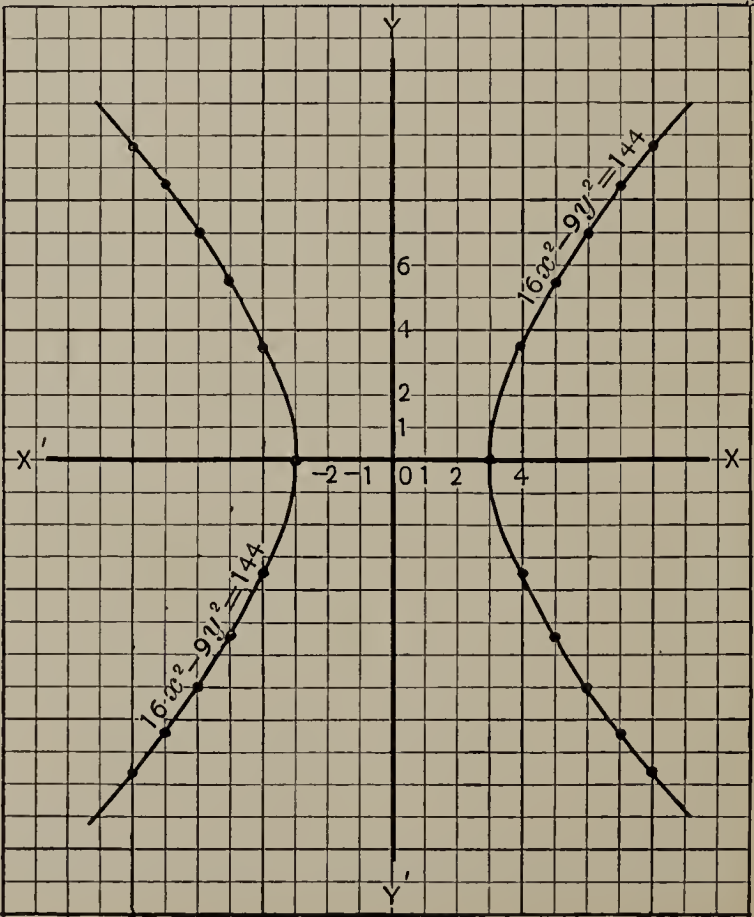
Ellipse



9. $16x^2 - 9y^2 = 144.$
 $\therefore y = \pm \frac{4}{3} \sqrt{x^2 - 9}.$

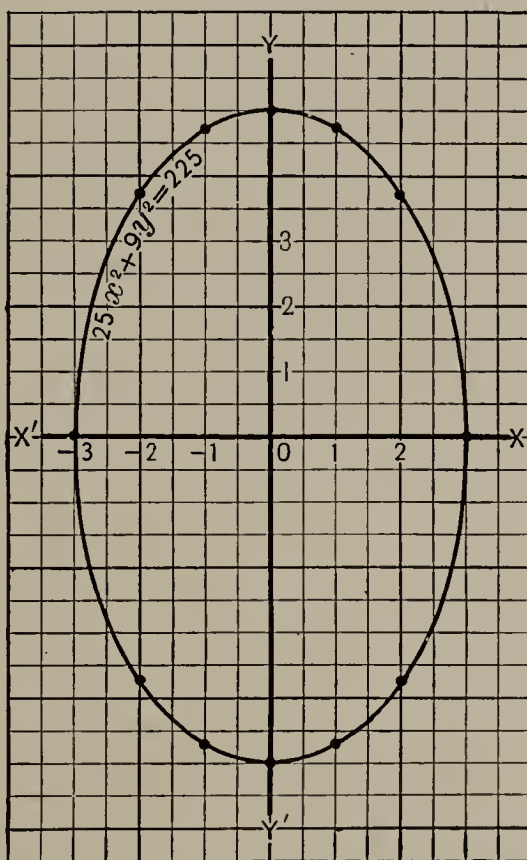
x	8	7	6	5	4	± 3	-4	-5	-6	-7	-8
y	± 9.88	± 8.43	± 6.92	± 5.33	± 3.52	0	± 3.52	± 5.33	± 6.92	± 8.43	± 9.88

Hyperbola



10. $25x^2 + 9y^2 = 225$. $\therefore y = \pm \frac{5}{3} \sqrt{9 - x^2}$.

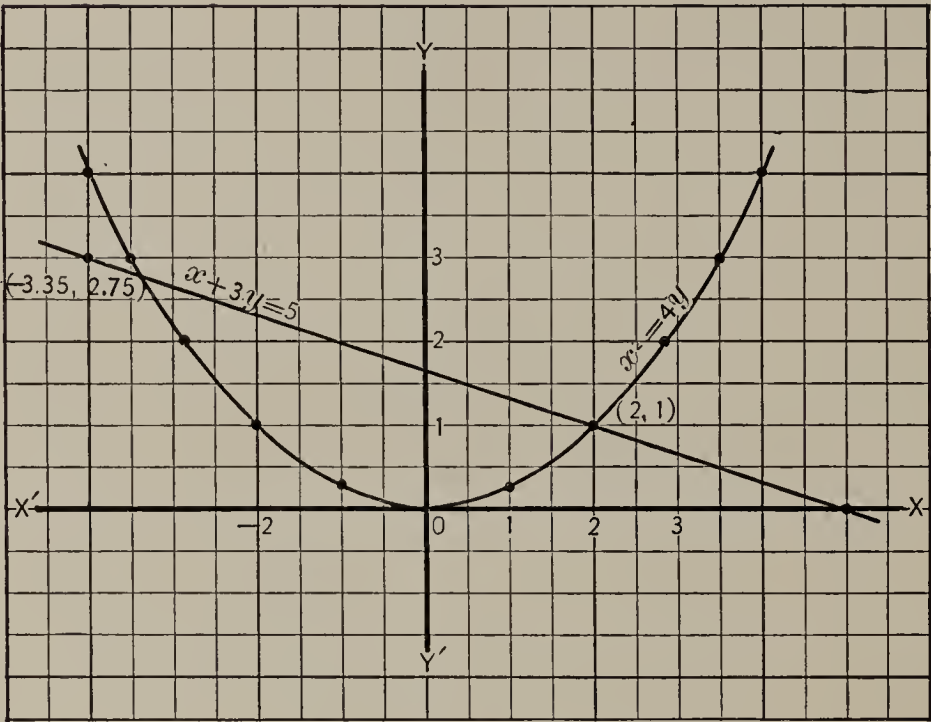
x	3	2	1	0	-1	-2	-3
y	0	± 3.72	± 4.71	± 5	± 4.71	± 3.72	0



Ellipse

1. $x^2 = 4y.$ $\therefore x = \pm 2\sqrt{y}.$

x	0	± 2	± 2.82	± 3.46	± 4
y	0	1	2	3	4



$x + 3y = 5.$

x	0	5
y	$\frac{5}{3}$	0

From the graphs

x	2	-3.35
y	1	2.75

2. $x^2 + y^2 = 25.$

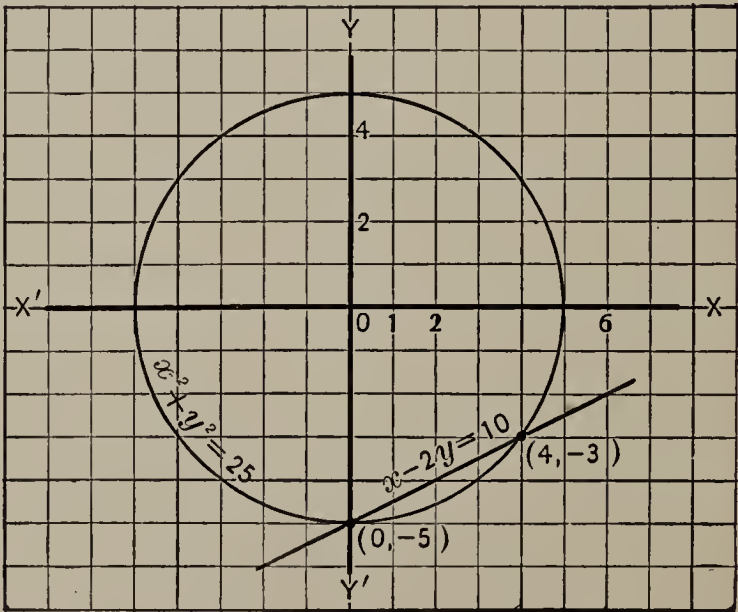
Circle, center (0,0)
and $r = 5.$

$x - 2y = 10.$

x	0	6
y	-5	-2

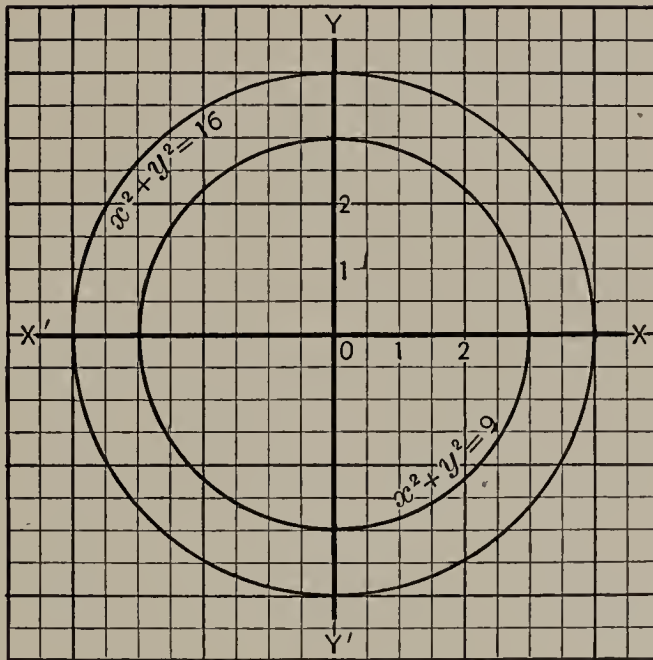
From the graphs

x	0	4
y	-5	-3



3. $x^2 + y^2 = 16$.

Circle, center at (0,0) and $r = 4$.



$x^2 + y^2 = 9$.

Circle, center at (0,0) and $r = 3$.

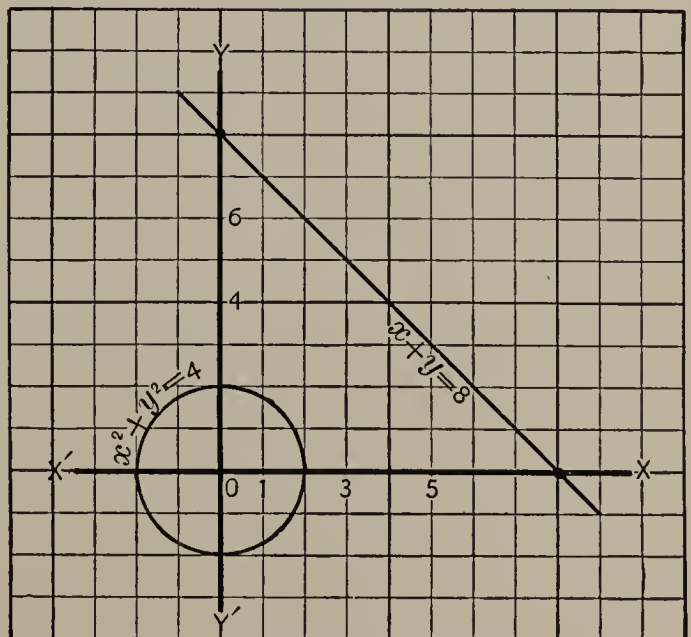
A graphical solution is impossible, for the graphs do not intersect.

4. $x^2 + y^2 = 4$.

Circle, center at (0,0) and $r = 2$.

$x + y = 8$.

x	0	8
y	8	0



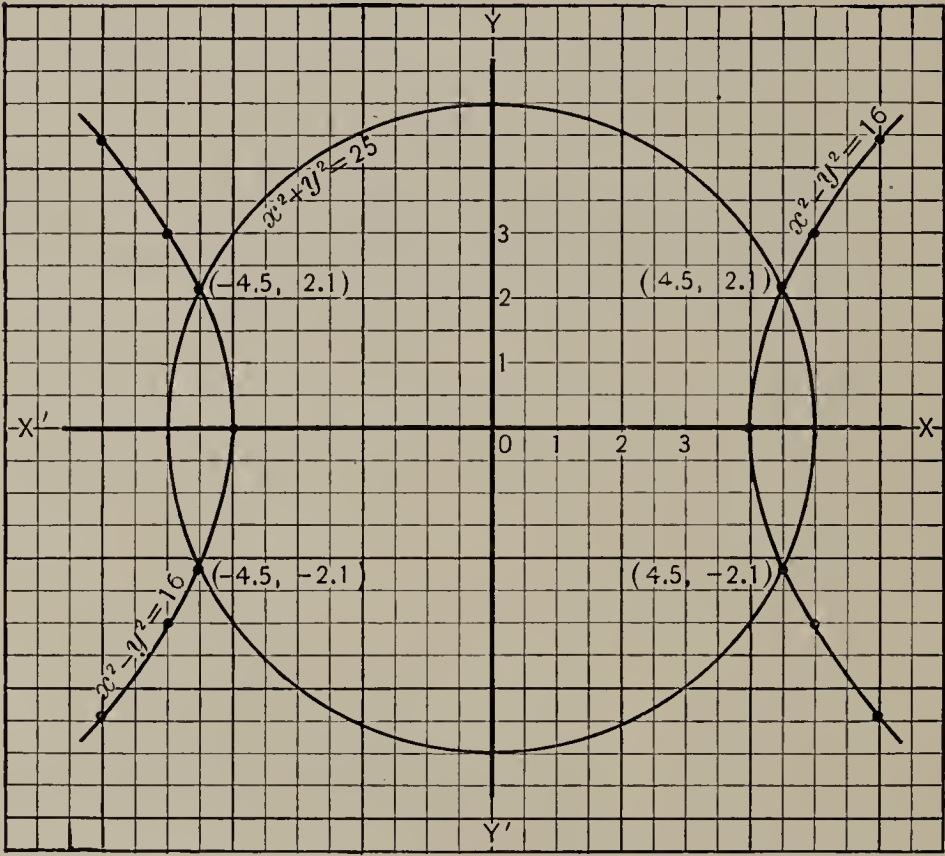
A graphical solution is impossible, for the graphs do not intersect.

5. $x^2 + y^2 = 25$.

Circle, center (0,0) and $r = 5$.

$x^2 - y^2 = 16$. $\therefore y = \pm \sqrt{x^2 - 16}$.

x	6	5	4	-4	-5	-6
y	± 4.47	± 3	0	0	± 3	± 4.47



From the graphs

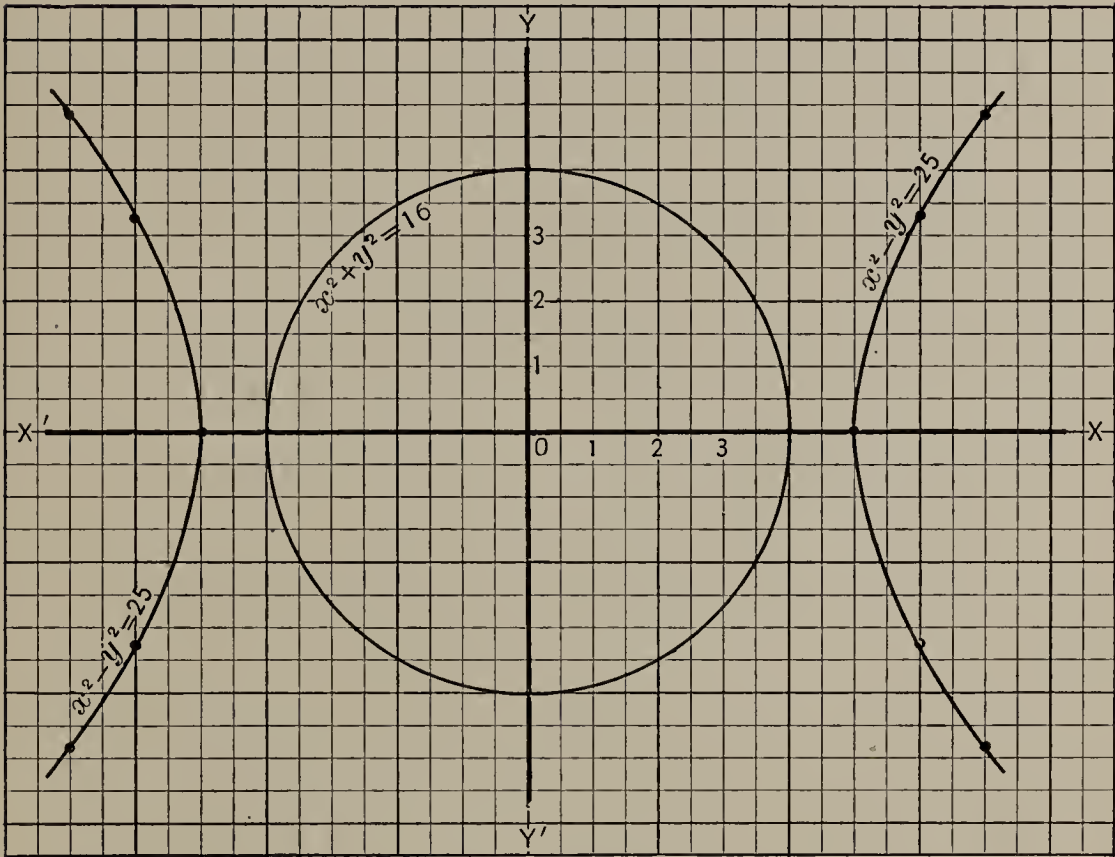
x	4.5	-4.5
y	± 2.1	± 2.1

6. $x^2 + y^2 = 16$.

Circle, center at (0,0) and $r = 4$.

$x^2 - y^2 = 25$. $\therefore y = \pm \sqrt{x^2 - 25}$.

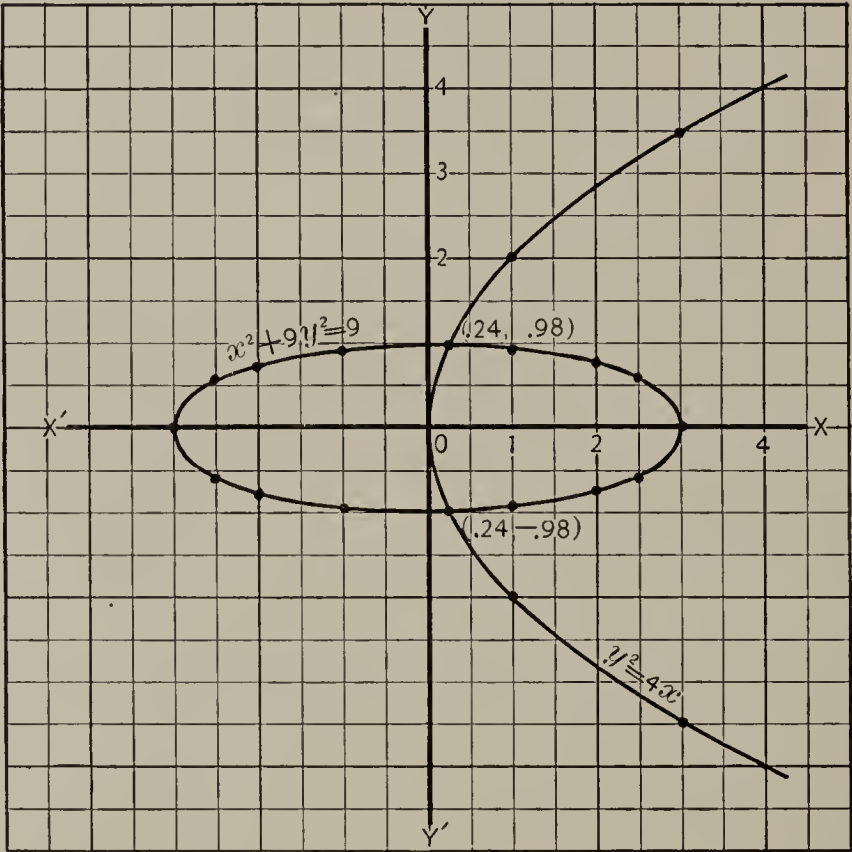
x	7	6	5	-5	-6	-7
y	± 4.89	± 3.31	0	0	± 3.31	± 4.89



The system has no graphical solution, for the curves do not intersect.

7. $y^2 = 4x \quad \therefore y = \pm 2\sqrt{x}.$

x	4	3	2	1	0
y	± 4	± 3.46	± 2.82	± 2	0



$x^2 + 9y^2 = 9. \quad \therefore y = \pm \frac{1}{3}\sqrt{9 - x^2}.$

x	3	2	1	0	-1	-2	-3	$\frac{5}{2}$	$-\frac{5}{2}$
y	0	$\pm .74$	$\pm .94$	± 1	$\pm .94$	$\pm .74$	0	$\pm .55$	$\pm .55$

From the graphs

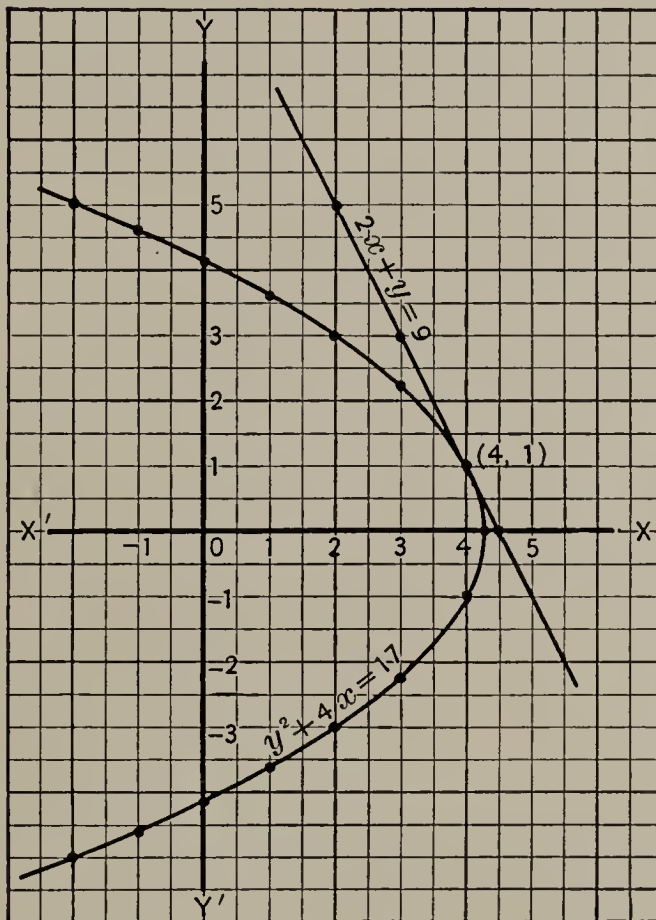
x	.24	.24
y	.98	-.98

8. $y^2 + 4x = 17$. $\therefore y = \pm \sqrt{17 - 4x}$.

x	4	3	2	1	0	-1	-2	$4\frac{1}{4}$
y	± 1	± 2.23	± 3	± 3.60	± 4.12	± 4.58	± 5	0

$$2x + y = 9.$$

x	$\frac{9}{2}$	3	2
y	0	3	5



From the graphs

x	4
y	1

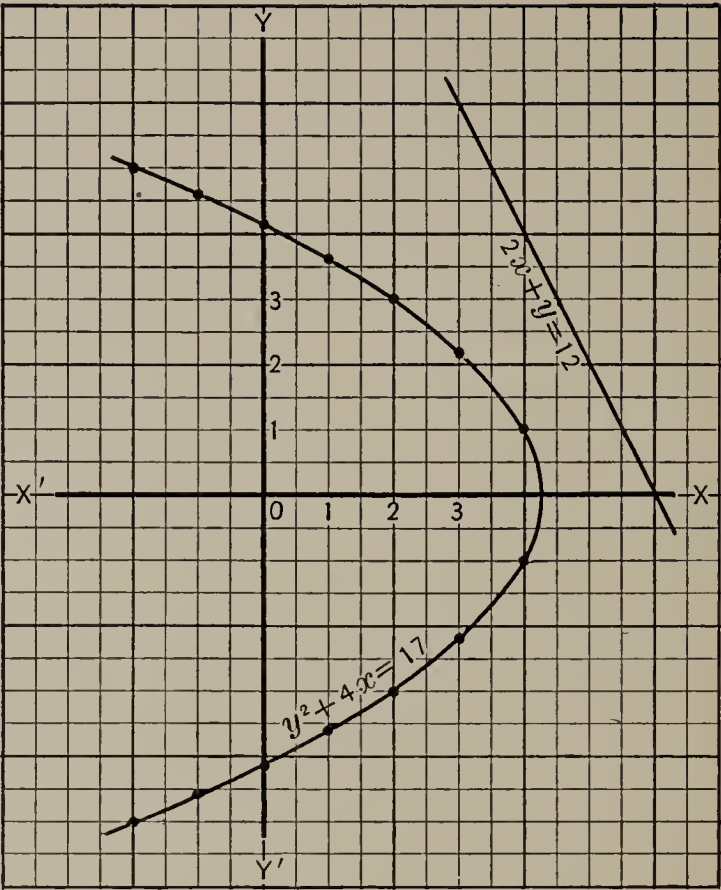
9. $y^2 + 4x = 17$.

For values of x and y ,
see Exercise 8.

$2x + y = 12$.

x	6	4	3
y	0	4	6

The system has no
graphical solution, for the
curves do not intersect.



10. $x^2 + y^2 = 16$.

Circle, center at $(0,0)$
and $r = 4$.

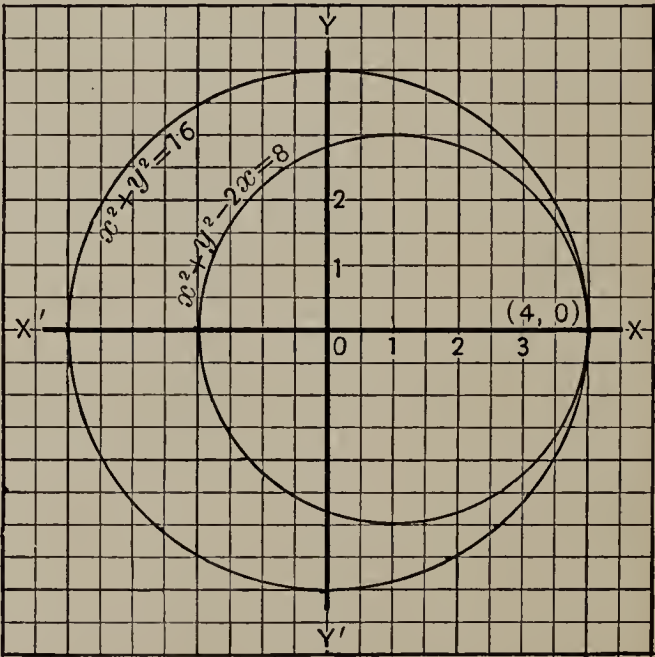
$x^2 + y^2 - 2x = 8$. $\therefore y = \pm \sqrt{8 + 2x - x^2}$.

x	4	3	2	1	0	-1	-2
y	0	± 2.23	± 2.82	± 3	± 2.82	± 2.23	0

or circle, center at $(0,1)$ and $r = 3$.

From the graphs

x	4
y	0

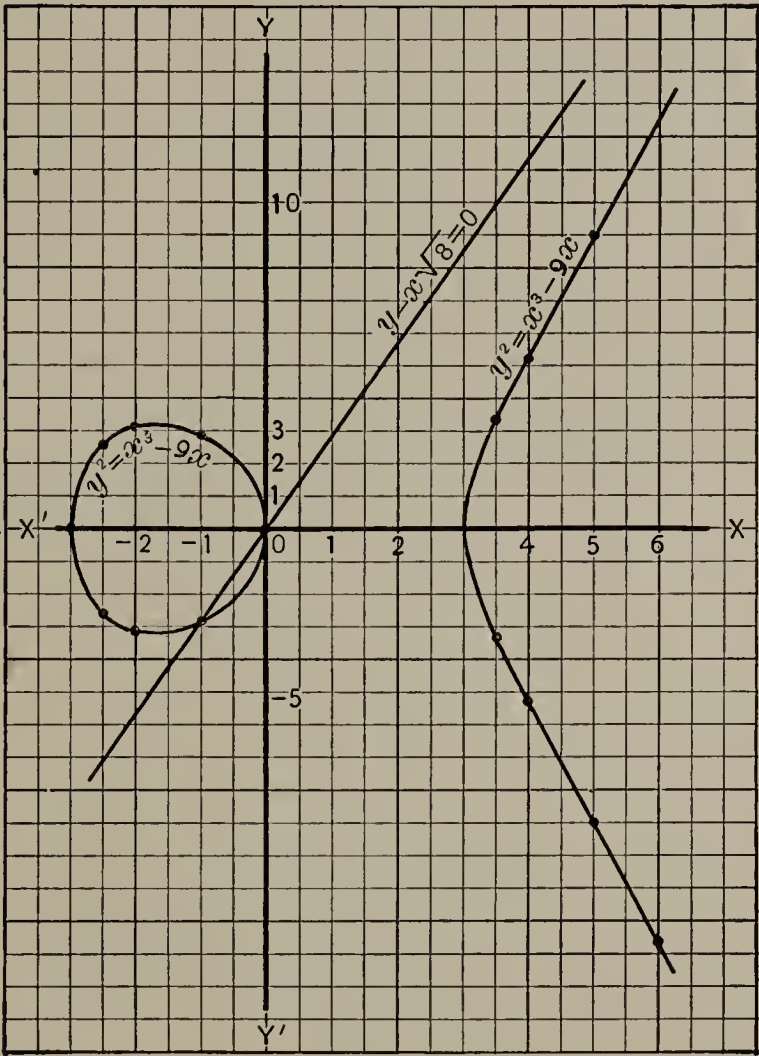


11. $y^2 = x^3 - 9x$. $\therefore y = \pm \sqrt{x^3 - 9x}$.

x	0	3	3.5	4	5	6	-1	-2	-3
y	0	0	± 3.37	± 5.29	± 8.94	± 12.72	± 2.82	± 3.16	0

$y - x\sqrt{8} = 0$.

x	0	1	-1
y	0	2.82	-2.82



From the graphs

x	-1	0
y	-2.82	0

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$$1. \quad x + y = 6, \quad (1)$$

$$x^2 + y^2 = 20. \quad (2)$$

$$x = 6 - y. \quad (3)$$

Substituting $6 - y$ for x in (2),

$$36 - 12y + y^2 + y^2 = 20. \quad (4)$$

$$2y^2 - 12y + 16 = 0. \quad (5)$$

$$y = \frac{12 \pm \sqrt{144 - 128}}{4}.$$

$$y = \frac{12 \pm 4}{4}.$$

$$y = 4, 2.$$

Substituting 4 for y in (3), $x = 6 - 4 = 2.$

Substituting 2 for y in (3), $x = 6 - 2 = 4.$

$$x = 2, 4.$$

$$y = 4, 2.$$

$$2. \quad 4m + n = 28, \quad (1)$$

$$2m^2 + 3mn = 98. \quad (2)$$

$$n = 28 - 4m. \quad (3)$$

From (3) and (2), $2m^2 + 84m - 12m^2 = 98. \quad (4)$

$$-10m^2 + 84m - 98 = 0. \quad (5)$$

$$m = \frac{-84 \pm \sqrt{7056 - 3920}}{-20}.$$

$$m = \frac{-84 \pm 56}{-20}.$$

$$m = 7, \frac{7}{5}.$$

Substituting 7 for m in (3), $n = 28 - 28.$

$$n = 0.$$

Substituting $\frac{7}{5}$ for m in (3), $n = 28 - \frac{28}{5}.$

$$n = \frac{11}{5}.$$

$$m = 7, \frac{7}{5}.$$

$$n = 0, \frac{11}{5}.$$

$$3. \quad m^2 + 2n^2 = 44, \quad (1)$$

$$m - 2n\sqrt{5} = 0. \quad (2)$$

$$m = 2n\sqrt{5}. \quad (3)$$

From (3) and (1), $20n^2 + 2n^2 = 44.$

$$n^2 = 2.$$

$$n = \pm \sqrt{2}.$$

Substituting $\sqrt{2}$ for n in (3), $m = 2\sqrt{2}\sqrt{5} = 2\sqrt{10}.$

Substituting $-\sqrt{2}$ for n in (3), $m = 2(-\sqrt{2})\sqrt{5} = -2\sqrt{10}$.
 $m = 2\sqrt{10}, -2\sqrt{10}$.
 $n = \sqrt{2}, -\sqrt{2}$.

$$4. \quad 4s + t = 6, \quad (1)$$

$$st = -10. \quad (2)$$

$$t = 6 - 4s. \quad (3)$$

$$\text{From (2) and (3),} \quad 6s - 4s^2 = -10. \quad (4)$$

$$2s^2 - 3s - 5 = 0. \quad (5)$$

$$\text{Whence} \quad s = \frac{5}{2}, -1.$$

$$\text{Substituting } \frac{5}{2} \text{ for } s \text{ in (3),} \quad t = 6 - 10, -4.$$

$$\text{Substituting } -1 \text{ for } s \text{ in (3),} \quad t = 6 + 4, 10.$$

$$s = \frac{5}{2}, -1.$$

$$t = -4, 10.$$

$$5. \quad xy + 36 = 0, \quad (1)$$

$$4x - y = 30. \quad (2)$$

$$y = 4x - 30. \quad (3)$$

$$\text{From (3) and (1),} \quad 4x^2 - 30x + 36 = 0. \quad (4)$$

$$(x - 6)(2x - 3) = 0. \quad (5)$$

$$\text{Whence} \quad x = \frac{3}{2}, 6.$$

$$\text{Substituting } \frac{3}{2} \text{ for } x \text{ in (3),} \quad y = 6 - 30 = -24.$$

$$\text{Substituting } 6 \text{ for } x \text{ in (3),} \quad y = 24 - 30 = -6.$$

$$x = \frac{3}{2}, 6.$$

$$y = -24, -6.$$

$$6. \quad x\sqrt{3} + 5y = -72, \quad (1)$$

$$xy = -15\sqrt{3}. \quad (2)$$

$$x = \frac{-15\sqrt{3}}{y}. \quad (3)$$

$$\text{Substituting } \frac{-15\sqrt{3}}{y} \text{ for } x \text{ in (1),}$$

$$\frac{-45}{y} + 5y = -72. \quad (4)$$

$$5y^2 + 72y - 45 = 0. \quad (5)$$

$$(5y - 3)(y + 15) = 0. \quad (6)$$

$$y = -15.$$

$$y = \frac{3}{5}.$$

$$\text{Substituting } \frac{3}{5} \text{ for } y \text{ in (3),} \quad x = \frac{-15\sqrt{3}}{\frac{3}{5}} = -25\sqrt{3}.$$

$$\text{Substituting } -15 \text{ for } y \text{ in (3),} \quad x = \frac{-15\sqrt{3}}{-15} = \sqrt{3}.$$

$$x = \sqrt{3}, -25\sqrt{3}.$$

$$y = -15, \frac{3}{5}.$$

$$7. \quad 3R_1 + 4R_2 = 5. \quad (1)$$

$$2R_1R_2 - 6R_1 = -3. \quad (2)$$

$$R_2 = \frac{5 - 3R_1}{4}. \quad (3)$$

Substituting $\frac{5 - 3R_1}{4}$ for R_2 in (2),

$$\frac{5R_1 - 3R_1^2}{2} - 6R_1 = -3. \quad (4)$$

$$-3R_1^2 - 7R_1 + 6 = 0. \quad (5)$$

$$R_1 = \frac{7 \pm \sqrt{49 + 72}}{-6} = \frac{7 \pm 11}{-6} = -3, \frac{2}{3}.$$

Substituting -3 for R_1 in (3),

$$R_2 = \frac{5 + 9}{4} = \frac{7}{2}.$$

Substituting $\frac{2}{3}$ for R_1 in (3), $R_2 = \frac{5 - 2}{4} = \frac{3}{4}.$

$$R_1 = -3, \frac{2}{3}.$$

$$R_2 = \frac{7}{2}, \frac{3}{4}.$$

$$8. \quad 2xy + y^2 - 20 = 0, \quad (1)$$

$$xy + 40 = 0. \quad (2)$$

$$2 \cdot (2), \quad 2xy + 80 = 0. \quad (3)$$

$$(1) - (3), \quad y^2 - 100 = 0. \quad (4)$$

$$y = \pm 10.$$

Substituting 10 for y in (2),

$$10x + 40 = 0. \quad (5)$$

$$x = -4.$$

Substituting -10 for y in (2),

$$-10x + 40 = 0. \quad (6)$$

$$x = 4.$$

$$x = 4, -4.$$

$$y = -10, 10.$$

$$9. \quad h^2 + k^2 + 2k = 40, \quad (1)$$

$$2 + h + k = 0. \quad (2)$$

$$h = -k - 2. \quad (3)$$

From (3) and (1),

$$k^2 + 4k + 4 + k^2 + 2k = 40. \quad (4)$$

$$2k^2 + 6k - 36 = 0. \quad (5)$$

$$(k - 3)(k + 6) = 0. \quad (6)$$

$$k = -6, 3.$$

Substituting -6 for k in (3), $h = 6 - 2 = 4.$

Substituting 3 for k in (3), $h = -3 - 2 = -5.$

$$h = 4, -5.$$

$$k = -6, 3.$$

$$10. \quad m^2 + 3mn + n^2 = 88, \quad (1)$$

$$2m = n. \quad (2)$$

$$m = \frac{n}{2}. \quad (3)$$

$$\text{From (3) and (1),} \quad \frac{n^2}{4} + \frac{3n^2}{2} + n^2 = 88. \quad (4)$$

$$\text{Whence} \quad n = \pm 4\sqrt{2}.$$

$$\text{Substituting } 4\sqrt{2} \text{ for } n \text{ in (3),} \quad m = 2\sqrt{2}.$$

$$\text{Substituting } -4\sqrt{2} \text{ for } n \text{ in (3),} \quad m = -2\sqrt{2}.$$

$$m = 2\sqrt{2}, -2\sqrt{2}.$$

$$n = 4\sqrt{2}, -4\sqrt{2}.$$

$$11. \quad x^2 + y^2 + 4x + 6y = 40, \quad (1)$$

$$x - 10 = y. \quad (2)$$

$$x = y + 10. \quad (3)$$

$$\text{From (3) and (1), } y^2 + 20y + 100 + y^2 + 4y + 40 + 6y = 40. \quad (4)$$

$$y^2 + 15y + 50 = 0. \quad (5)$$

$$\text{Whence} \quad y = -10, -5.$$

$$\text{Substituting } -10 \text{ for } y \text{ in (3),} \quad x = -10 + 10 = 0.$$

$$\text{Substituting } -5 \text{ for } y \text{ in (3),} \quad x = -5 + 10 = 5.$$

$$x = 0, 5.$$

$$y = -10, -5.$$

$$12. \quad y + x\sqrt{15} = 0. \quad (1)$$

$$y^2 + x^3 = 16x. \quad (2)$$

$$y = -x\sqrt{15}. \quad (3)$$

$$\text{From (3) and (2),} \quad 15x^2 + x^3 = 16x. \quad (4)$$

$$x(x + 16)(x - 1) = 0. \quad (5)$$

$$\text{Whence} \quad x = 0, -16, 1.$$

$$\text{Substituting } 0 \text{ for } x \text{ in (3),} \quad y = -0\sqrt{15} = 0.$$

$$\text{Substituting } -16 \text{ for } x \text{ in (3),} \quad y = 16\sqrt{15}.$$

$$\text{Substituting } 1 \text{ for } x \text{ in (3),} \quad y = -\sqrt{15}.$$

$$x = 0, -16, 1.$$

$$y = 0, 16\sqrt{15}, -\sqrt{15}.$$

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$$1. \quad x^2 + xy = 3, \quad (1)$$

$$y^2 - xy = 10. \quad (2)$$

$$(1) \cdot 10, \quad 10x^2 + 10xy = 30. \quad (3)$$

$$(2) \cdot 3, \quad 3y^2 - 3xy = 30. \quad (4)$$

$$(3) - (4), \quad 10x^2 + 13xy - 3y^2 = 0. \quad (5)$$

$$(5x - y)(2x + 3y) = 0. \quad (6)$$

$$\therefore x = \frac{y}{5}, \quad \frac{-3y}{2}. \quad (7)$$

$$x = \frac{-3y}{2}. \quad (8)$$

Substituting $\frac{y}{5}$ for x in (2), $y^2 - \frac{y^2}{5} = 10.$

$$\therefore y = \pm \frac{5}{2} \sqrt{2}.$$

Substituting $\frac{5}{2} \sqrt{2}$ for y in (7), $x = \frac{\frac{5}{2} \sqrt{2}}{5} = \frac{1}{2} \sqrt{2}.$

Substituting $-\frac{5}{2} \sqrt{2}$ for y in (7), $x = \frac{-\frac{5}{2} \sqrt{2}}{5} = -\frac{1}{2} \sqrt{2}.$

Substituting $\frac{-3y}{2}$ for x in (2),

$$y^2 + \frac{3y^2}{2} = 10.$$

$$y = \pm 2.$$

Substituting 2 for y in (8), $x = -3.$

Substituting -2 for y in (8), $x = 3.$

$$x = \frac{1}{2} \sqrt{2}, -\frac{1}{2} \sqrt{2}, -3, 3.$$

$$y = \frac{5}{2} \sqrt{2}, -\frac{5}{2} \sqrt{2}, 2, -2.$$

2. $x^2 + y^2 = 10, \quad (1)$

$$3y^2 + xy = 6. \quad (2)$$

$$6x^2 + 6y^2 = 60. \quad (3)$$

$$30y^2 + 10xy = 60. \quad (4)$$

$$6x^2 - 10xy - 24y^2 = 0. \quad (5)$$

$$(3x + 4y)(x - 3y) = 0. \quad (6)$$

$$x = -\frac{4y}{3}. \quad (7)$$

$$x = 3y. \quad (8)$$

From (7) and (2), $3y^2 - \frac{4y^2}{3} = 6.$

$$y = \pm \frac{3}{5} \sqrt{10}.$$

Substituting $\frac{3}{5} \sqrt{10}$ for y in (7), $x = -\frac{4}{5} \cdot \frac{3}{5} \sqrt{10} = -\frac{4}{5} \sqrt{10}.$

Substituting $-\frac{3}{5} \sqrt{10}$ for y in (7), $x = -\frac{4}{5}, -\frac{3}{5} \sqrt{10} = \frac{4}{5} \sqrt{10}.$

From (8) and (2), $3y^2 + 3y^2 = 6.$

$$y = \pm 1.$$

Substituting 1 for y in (8), $x = 3.$

Substituting -1 for y in (8), $x = -3.$

$$x = \pm \frac{4}{5} \sqrt{10}, \pm 3.$$

$$y = \mp \frac{3}{5} \sqrt{10}, \pm 1.$$

$$3. \quad u^2 + 2uv = 0, \quad (1)$$

$$2v^2 + 3uv = -16. \quad (2)$$

$$u(u + 2v) = 0. \quad (3)$$

$$u = 0. \quad (4)$$

$$u = -2v. \quad (5)$$

$$\text{From (4) and (2),} \quad 2v^2 + 0 = -16.$$

$$v = \pm 2\sqrt{-2}.$$

$$\text{From (5) and (2),} \quad 2v^2 - 6v^2 = -16.$$

$$v = \pm 2.$$

$$\text{Substituting 2 for } v \text{ in (5),} \quad u = -4.$$

$$\text{Substituting } -2 \text{ for } v \text{ in (5),} \quad u = 4.$$

$$u = 0, \pm 4.$$

$$v = \pm 2\sqrt{-2}, \mp 2.$$

$$4. \quad s^2 - 3st = 4, \quad (1)$$

$$3t^2 + 3s^2 = 12. \quad (2)$$

$$3s^2 - 9st = 12. \quad (3)$$

$$(3) - (2), \quad -9st - 3t^2 = 0. \quad (4)$$

$$\text{From (4),} \quad t = 0, \quad (5)$$

$$\text{and} \quad t = -3s. \quad (6)$$

$$\text{From (6) and (1),} \quad s^2 + 9s^2 = 4, \quad s = \pm \frac{1}{3}\sqrt{10}.$$

$$\text{Substituting } \frac{1}{3}\sqrt{10} \text{ for } s \text{ in (6),} \quad t = -\frac{2}{3}\sqrt{10}.$$

$$\text{Substituting } -\frac{1}{3}\sqrt{10} \text{ for } s \text{ in (6),} \quad t = \frac{2}{3}\sqrt{10}.$$

$$\text{Substituting 0 for } t \text{ in (1),} \quad s^2 = 4.$$

$$s = \pm 2.$$

$$s = \pm \frac{1}{3}\sqrt{10}, \pm 2.$$

$$t = \mp \frac{2}{3}\sqrt{10}, 0.$$

$$5. \quad x(x + 2y) = 16, \quad (1)$$

$$y(y - x) = 3. \quad (2)$$

$$x^2 + 2xy = 16. \quad (3)$$

$$y^2 - xy = 3. \quad (4)$$

$$3x^2 + 6xy = 48. \quad (5)$$

$$16y^2 - 16xy = 48. \quad (6)$$

$$3x^2 + 22xy - 16y^2 = 0. \quad (7)$$

$$(3x - 2y)(x + 8y) = 0. \quad (8)$$

$$x = \frac{2y}{3}. \quad (9)$$

$$x = -8y. \quad (10)$$

$$\text{From (9) and (4),} \quad y^2 - \frac{2y^2}{3} = 3.$$

$$y = \pm 3.$$

Substituting 3 for y in (9), $x = 2$.

Substituting -3 for y in (9), $x = -2$.

From (10) and (4), $y^2 + 8y^2 = 3$.

$$y = \pm \frac{1}{3} \sqrt{3}.$$

Substituting $\frac{1}{3} \sqrt{3}$ for y in (10), $x = -\frac{8}{3} \sqrt{3}$.

Substituting $-\frac{1}{3} \sqrt{3}$ for y in (10), $x = \frac{8}{3} \sqrt{3}$.

$$x = \pm \frac{8}{3} \sqrt{3}, \pm 2.$$

$$y = \mp \frac{1}{3} \sqrt{3}, \pm 3.$$

$$6. \quad x^2 + xy + y^2 = 4, \quad (1)$$

$$x^2 - 2xy = 12. \quad (2)$$

$$3x^2 + 3xy + 3y^2 = 12. \quad (3)$$

$$2x^2 + 5xy + 3y^2 = 0. \quad (4)$$

$$(x + y)(2x + 3y) = 0.$$

$$x = -y. \quad (5)$$

$$x = -\frac{3y}{2}. \quad (6)$$

From (5) and (2), $y^2 + 2y^2 = 12, y = \pm 2$.

Substituting 2 for y in (5), $x = -2$.

Substituting -2 for y in (5), $x = 2$.

From (6) and (2), $\frac{9y^2}{4} + 3y^2 = 12$.

$$y^2 = \frac{4}{3}.$$

$$y = \pm \frac{2}{3} \sqrt{3}.$$

Substituting $\frac{2}{3} \sqrt{3}$ for y in (6), $x = -\frac{2}{3} \sqrt{3}$.

Substituting $-\frac{2}{3} \sqrt{3}$ for y in (6), $x = \frac{2}{3} \sqrt{3}$.

$$x = \pm 2, \pm \frac{2}{3} \sqrt{3}.$$

$$y = \mp 2, \pm \frac{2}{3} \sqrt{3}.$$

$$7. \quad x^2 + xy = 2y^2, \quad (1)$$

$$2x^2 + x = 2 + y^2. \quad (2)$$

$$\text{From (1), } (x + 2y)(x - y) = 0. \quad (3)$$

$$\text{Then } x = y. \quad (4)$$

$$x = -2y. \quad (5)$$

$$\text{From (4) and (2), } 2y^2 + y = 2 + y^2. \quad (6)$$

$$\text{From (6), } y = -2, 1. \quad (7)$$

$$\text{From (7) and (4), } x = -2, 1. \quad (8)$$

$$\text{From (5) and (2), } 8y^2 - 2y = 2 + y^2. \quad (9)$$

$$\text{From (9), } y = \frac{1 \pm \sqrt{15}}{7}. \quad (10)$$

$$\text{From (10) and (5), } x = \frac{-2 \mp 2\sqrt{15}}{7}. \quad (11)$$

$$8. \quad x^2 + 2xy - y^2 = 32, \quad (1)$$

$$2x^2 - 3xy + y^2 = 0. \quad (2)$$

$$\text{From (2),} \quad (2x - y)(x - y) = 0. \quad (3)$$

$$\text{Whence} \quad x = \frac{y}{2}, \quad (4)$$

$$\text{and} \quad x = y. \quad (5)$$

$$\text{From (5) and (1),} \quad y^2 + 2y^2 - y^2 = 32.$$

$$y = \pm 4.$$

$$\text{Substituting 4 for } y \text{ in (5),} \quad x = 4.$$

$$\text{Substituting } -4 \text{ for } y \text{ in (5),} \quad x = -4.$$

$$\text{From (4) and (1),} \quad \frac{y^2}{4} + y^2 - y^2 = 32.$$

$$y = \pm 8\sqrt{2}.$$

$$\text{Substituting } 8\sqrt{2} \text{ for } y \text{ in (4),} \quad x = \frac{8\sqrt{2}}{2} = 4\sqrt{2}.$$

$$\text{Substituting } -8\sqrt{2} \text{ for } y \text{ in (4),} \quad x = -4\sqrt{2}.$$

$$x = \pm 4, \pm 4\sqrt{2}.$$

$$y = \pm 4, \pm 8\sqrt{2}.$$

$$9. \quad 2x^2 - xy + 2y^2 = 12, \quad (1)$$

$$2x^2 + xy + 2y^2 = 8. \quad (2)$$

$$(1) \cdot 2, \quad 4x^2 - 2xy + 4y^2 = 24. \quad (3)$$

$$(2) \cdot 3, \quad 6x^2 + 3xy + 6y^2 = 24. \quad (4)$$

$$(3) - (4), \quad -2x^2 - 5xy - 2y^2 = 0. \quad (5)$$

$$(2x + y)(x + 2y) = 0. \quad (6)$$

$$\text{From (6),} \quad x = -\frac{y}{2}, \quad (7)$$

$$\text{and} \quad x = -2y. \quad (8)$$

$$\text{From (7) and (1),} \quad \frac{y^2}{2} + \frac{y^2}{2} + 2y^2 = 12.$$

$$y = \pm 2.$$

$$\text{Substituting 2 for } y \text{ in (7),} \quad x = -1.$$

$$\text{Substituting } -2 \text{ for } y \text{ in (7),} \quad x = 1.$$

$$\text{From (8) and (1),} \quad 8y^2 + 2y^2 + 2y^2 = 12.$$

$$y = \pm 1.$$

$$\text{Substituting 1 for } y \text{ in (8),} \quad x = -2.$$

$$\text{Substituting } -1 \text{ for } y \text{ in (8),} \quad x = 2.$$

$$x = \pm 1, \pm 2.$$

$$y = \mp 2, \mp 1.$$

$$10. \quad x^2 - xy - 5y^2 = 15, \quad (1)$$

$$x^2 - 6y^2 = 1. \quad (2)$$

$$(2) \cdot 15, \quad 15x^2 - 90y^2 = 15. \quad (3)$$

$$(1) - (3), \quad -14x^2 - xy + 85y^2 = 0. \quad (4)$$

$$(7x - 17y)(2x + 5y) = 0. \quad (5)$$

$$x = \frac{17y}{7}. \quad (6)$$

$$x = -\frac{5y}{2}. \quad (7)$$

$$\text{From (7) and (2),} \quad \frac{25y^2}{4} - 6y^2 = 1. \quad (8)$$

$$\text{From (8),} \quad y = \pm 2.$$

$$\text{Substituting 2 for } y \text{ in (7),} \quad x = -5.$$

$$\text{Substituting } -2 \text{ for } y \text{ in (7),} \quad x = 5.$$

$$\text{From (6) and (2),} \quad \frac{289}{49}y^2 - 6y^2 = 1.$$

$$\text{Whence} \quad y = \pm \frac{7}{5}\sqrt{-5}.$$

$$\text{Substituting } \frac{7}{5}\sqrt{-5} \text{ for } y \text{ in (6),} \quad x = \frac{17}{5}\sqrt{-5}.$$

$$\text{Substituting } -\frac{7}{5}\sqrt{-5} \text{ for } y \text{ in (6),} \quad x = -\frac{17}{5}\sqrt{-5}.$$

$$x = \pm 5, \pm \frac{17}{5}\sqrt{-5},$$

$$y = \mp 2, \pm \frac{7}{5}\sqrt{-5}.$$

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$$1. \quad x - y = 4, \quad (1)$$

$$xy = 5. \quad (2)$$

$$\text{Squaring (1),} \quad x^2 - 2xy + y^2 = 16. \quad (3)$$

$$(2) \cdot 4, \quad 4xy = 20. \quad (4)$$

$$(3) + (4), \quad x^2 + 2xy + y^2 = 36. \quad (5)$$

$$\text{From (5),} \quad x + y = \pm 6. \quad (6)$$

$$\text{From (6) and (1),} \quad \begin{cases} x - y = 4, \\ x + y = 6. \end{cases} \quad \begin{cases} x - y = 4, \\ x + y = -6. \end{cases}$$

$$\text{Whence,} \quad \begin{aligned} x &= 5, -1, \\ y &= 1, -5. \end{aligned}$$

$$2. \quad x + 2y = 8, \quad (1)$$

$$xy + 6 = 0. \quad (2)$$

$$\text{From (2),} \quad xy = -6. \quad (3)$$

$$\text{Squaring (1),} \quad x^2 + 4xy + 4y^2 = 64. \quad (4)$$

$$(3) \cdot 8, \quad 8xy = -48. \quad (5)$$

$$(4) - (5), \quad x^2 - 4xy + 4y^2 = 112. \quad (6)$$

$$x - 2y = \pm 4\sqrt{7}. \quad (7)$$

$$\text{From (1) and (7),} \quad \begin{cases} x - 2y = 4\sqrt{7}, \\ x + 2y = 8. \end{cases} \quad \begin{cases} x - 2y = -4\sqrt{7}, \\ x + 2y = 8. \end{cases}$$

$$\text{Whence} \quad \begin{aligned} x &= 4 + 2\sqrt{7}, & x &= 4 - 2\sqrt{7}, \\ y &= 2 - \sqrt{7}, & y &= 2 + \sqrt{7}. \end{aligned}$$

$$x = 4 + 2\sqrt{7}, 4 - 2\sqrt{7}.$$

$$y = 2 - \sqrt{7}, 2 + \sqrt{7}.$$

$$\begin{aligned}
 3. \quad & x^2 + 4y^2 = 101, & (1) \\
 & xy + 5 = 0. & (2) \\
 & xy = -5. & (3) \\
 & 4xy = -20. & (4) \\
 & x^2 - 4xy + 4y^2 = 121. & (5) \\
 & x - 2y = \pm 11. & (6) \\
 & x^2 + 4xy + 4y^2 = 81. & (7) \\
 & x + 2y = \pm 9. & (8)
 \end{aligned}$$

From (6) and (8), $\begin{cases} x - 2y = 11, \\ x + 2y = 9. \end{cases}$ $\begin{cases} x - 2y = 11, \\ x + 2y = -9. \end{cases}$

$\begin{cases} x - 2y = -11, \\ x + 2y = 9. \end{cases}$ $\begin{cases} x - 2y = -11, \\ x + 2y = -9. \end{cases}$

Whence $x = \pm 10, \pm 1.$
 $y = \mp \frac{1}{2}, \mp 5.$

$$\begin{aligned}
 4. \quad & 6x - y = 24, & (1) & \quad 5. \quad 4x^2 - 6xy + y^2 = 24, & (1) \\
 & 36x^2 + y^2 = 288. & (2) & & xy = 20. & (2) \\
 & 36x^2 - 12xy + y^2 = 576. & (3) & & 6xy = 120. \\
 & 12xy = -288. & (4) & & 4x^2 = 144. \\
 & 36x^2 + 12xy + y^2 = 0. & (5) & & x = \pm 6. \\
 & 6x + y = 0. & & \text{Substituting 6 for } x \text{ in (2),} \\
 & 6x - y = 24. & & y = \frac{10}{3}. \\
 & x = 2. & & \text{Substituting } -6 \text{ for } x \text{ in (2);} \\
 & y = -12. & & y = -\frac{10}{3}. \\
 & & & x = \pm 6. \\
 & & & y = \pm \frac{10}{3}.
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & 4x^2 + y^2 = 25, & (1) \\
 & 4x^2 + 4xy + y^2 = 49. & (2) \\
 \text{From (2),} & 2x + y = \pm 7. & (3) \\
 (1) - (2), & -4xy = -24. & (4) \\
 (1) + (4), & 4x^2 - 4xy + y^2 = 1. & (5) \\
 \text{From (5),} & 2x - y = \pm 1. & (6) \\
 \text{Combining (3) and (6),}
 \end{aligned}$$

$$\begin{aligned}
 & \begin{cases} 2x + y = 7. \\ 2x - y = 1. \end{cases} & \begin{cases} 2x + y = 7. \\ 2x - y = -1. \end{cases} \\
 & \begin{cases} 2x + y = -7. \\ 2x - y = 1. \end{cases} & \begin{cases} 2x + y = -7. \\ 2x - y = -1. \end{cases}
 \end{aligned}$$

Whence $x = \pm 2, \pm \frac{3}{2}.$
 $y = \pm 3, \pm 4.$

$$\begin{aligned}
 7. \quad & x^2 + 4y^2 = 15, & (1) \\
 & x + 2y = 3\sqrt{3}. & (2) \\
 \text{Squaring (2),} & x^2 + 4xy + 4y^2 = 27. & (3)
 \end{aligned}$$

$$(1) - (3), \quad -4xy = -12. \quad (4)$$

$$(1) + (4), \quad x^2 - 4xy + 4y^2 = 3. \quad (5)$$

$$\text{From (5),} \quad x - 2y = \pm \sqrt{3}. \quad (6)$$

$$\text{From (2) and (6),} \quad \begin{cases} x + 2y = 3\sqrt{3}, \\ x - 2y = \sqrt{3}. \end{cases} \quad \begin{cases} x + 2y = 3\sqrt{3}, \\ x - 2y = -\sqrt{3}. \end{cases}$$

$$\text{Whence} \quad x = 2\sqrt{3}, \sqrt{3}, \text{ and } y = \frac{1}{2}\sqrt{3}, \sqrt{3}.$$

$$8. \quad x^2 - 2xy = 16. \quad (1)$$

$$2y^2 - xy = -6. \quad (2)$$

$$(1) + 2(2), \quad x^2 - 4xy + 4y^2 = 4. \quad (3)$$

$$x - 2y = \pm 2. \quad (4)$$

$$\text{From (1),} \quad x(x - 2y) = 16. \quad (5)$$

$$\text{From (5) and (4),} \quad x(\pm 2) = 16, \quad x = \pm 8. \quad (6)$$

$$\text{From (1) and (6),} \quad 64 - 2(\pm 8)y = 16. \quad (7)$$

$$\text{From (7),} \quad y = \pm 3.$$

$$x = 8, -8.$$

$$y = 3, -3.$$

$$9. \quad \frac{1}{x^2} + \frac{1}{y^2} = 13. \quad (1)$$

$$\frac{1}{xy} = 6. \quad (2)$$

$$(2) \cdot 2, \quad \frac{2}{xy} = 12. \quad (3)$$

$$(1) + (3), \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (4)$$

$$\text{From (4),} \quad \frac{1}{x} + \frac{1}{y} = \pm 5. \quad (5)$$

$$(1) - (3), \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$

$$\text{From (6),} \quad \frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7)$$

Combining (5) and (7),

$$\begin{array}{ll} A \begin{cases} \frac{1}{x} + \frac{1}{y} = 5, \\ \frac{1}{x} - \frac{1}{y} = 1. \end{cases} & B \begin{cases} \frac{1}{x} + \frac{1}{y} = 5, \\ \frac{1}{x} - \frac{1}{y} = -1. \end{cases} \\ C \begin{cases} \frac{1}{x} + \frac{1}{y} = -5, \\ \frac{1}{x} - \frac{1}{y} = 1. \end{cases} & D \begin{cases} \frac{1}{x} + \frac{1}{y} = -5, \\ \frac{1}{x} - \frac{1}{y} = -1. \end{cases} \end{array}$$

Whence
and

$$\begin{aligned} x &= \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, \\ y &= \frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{2}. \end{aligned}$$

$$10. \quad \frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 7. \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

$$\text{Squaring (2),} \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (3)$$

$$(1) - (3), \quad \frac{1}{xy} = 6. \quad (4)$$

$$(4) \cdot 3, \quad \frac{3}{xy} = 18. \quad (5)$$

$$(1) + (5), \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (6)$$

$$\text{From (6),} \quad \frac{1}{x} + \frac{1}{y} = \pm 5. \quad (7)$$

$$\text{From (2) and (7),} \quad A \begin{cases} \frac{1}{x} - \frac{1}{y} = 1, \\ \frac{1}{x} + \frac{1}{y} = 5. \end{cases} \quad B \begin{cases} \frac{1}{x} - \frac{1}{y} = 1, \\ \frac{1}{x} + \frac{1}{y} = -5. \end{cases}$$

$$\begin{array}{l} \text{Whence} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{3}, -\frac{1}{2}, \\ y = \frac{1}{2}, -\frac{1}{3}. \end{array}$$

$$11. \quad 3x - 3y = 7. \quad (1)$$

$$9x^2 + 18xy + 9y^2 = 1. \quad (2)$$

$$\text{From (2),} \quad 3x + 3y = \pm 1. \quad (3)$$

$$\text{From (1) and (3),} \quad A \begin{cases} 3x - 3y = 7, \\ 3x + 3y = 1. \end{cases} \quad B \begin{cases} 3x - 3y = 7, \\ 3x + 3y = -1. \end{cases}$$

$$\begin{array}{l} \text{Whence} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{4}{3}, 1, \\ y = -1, -\frac{4}{3}. \end{array}$$

$$12. \quad \frac{1}{x^3} + \frac{1}{y^3} = 35. \quad (1)$$

$$\frac{1}{x} + \frac{1}{y} = 5. \quad (2)$$

$$(1) \div (2), \quad \frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 7. \quad (3)$$

$$\text{Squaring (2),} \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (4)$$

$$(3) - (4), \quad -\frac{3}{xy} = -18. \quad (5)$$

$$\text{From (5),} \quad -\frac{1}{xy} = -6. \quad (6)$$

$$(6) + (3), \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (7)$$

$$\text{From (7),} \quad \frac{1}{x} - \frac{1}{y} = \pm 1.$$

$$\text{From (2) and (8),} \quad A \begin{cases} \frac{1}{x} + \frac{1}{y} = 5, \\ \frac{1}{x} - \frac{1}{y} = 1. \end{cases} \quad B \begin{cases} \frac{1}{x} + \frac{1}{y} = 5, \\ \frac{1}{x} - \frac{1}{y} = -1. \end{cases}$$

Whence
and

$$x = \frac{1}{3}, \frac{1}{2}, \\ y = \frac{1}{2}, \frac{1}{3}.$$

$$13. \quad x^2 + 4y^2 = c. \quad (1)$$

$$4xy = d. \quad (2)$$

$$(1) + (2), \quad x^2 + 4xy + 4y^2 = c + d. \quad (3)$$

$$\text{From (3),} \quad x + 2y = \pm \sqrt{c + d}. \quad (4)$$

$$(1) - (2), \quad x^2 - 4xy + 4y^2 = c - d. \quad (5)$$

$$\text{From (5),} \quad x - 2y = \pm \sqrt{c - d}. \quad (6)$$

From (4) and (6),

$$A \begin{cases} x + 2y = \sqrt{c + d}, \\ x - 2y = \sqrt{c - d}. \end{cases} \quad B \begin{cases} x + 2y = \sqrt{c + d}, \\ x - 2y = -\sqrt{c - d}. \end{cases}$$

$$C \begin{cases} x + 2y = -\sqrt{c + d}, \\ x - 2y = \sqrt{c - d}. \end{cases} \quad D \begin{cases} x + 2y = -\sqrt{c + d}, \\ x - 2y = -\sqrt{c - d}. \end{cases}$$

Solving A, B, C, and D,

$$x = \frac{\sqrt{c + d} + \sqrt{c - d}}{2}, \quad \frac{\sqrt{c + d} - \sqrt{c - d}}{2},$$

$$\frac{-\sqrt{c + d} + \sqrt{c - d}}{2}, \quad \frac{-\sqrt{c + d} - \sqrt{c - d}}{2}.$$

$$y = \frac{\sqrt{c + d} - \sqrt{c - d}}{4}, \quad \frac{\sqrt{c + d} + \sqrt{c - d}}{4},$$

$$\frac{-\sqrt{c + d} - \sqrt{c - d}}{4}, \quad \frac{-\sqrt{c + d} + \sqrt{c - d}}{4}.$$

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$$1. \quad 4x^2 - y^2 = 16. \quad (1)$$

$$2x + y = 8. \quad (2)$$

$$(1) \div (2), \quad 2x - y = 2. \quad (3)$$

$$(2) + (3), \quad 4x = 10.$$

$$x = \frac{5}{2}.$$

$$5 + y = 8.$$

$$y = 3.$$

$$x = \frac{5}{2}.$$

$$y = 3.$$

$$2. \quad R^2h - 75 = 0. \quad (1)$$

$$Rh = 15. \quad (2)$$

$$R^2h = 75. \quad (3)$$

$$(3) \div (2), \quad R = 5. \quad (4)$$

$$\text{From (2) and (4),} \\ h = 3.$$

$$3. \quad \frac{1}{x^2} - \frac{1}{y^2} = 15. \quad (1)$$

$$\frac{1}{x} + \frac{1}{y} = 3. \quad (2)$$

$$(1) \div (2), \quad \frac{1}{x} - \frac{1}{y} = 5. \quad (3)$$

$$(2) - (3), \quad \frac{2}{x} = 8. \quad (4)$$

$$\text{Whence} \quad x = \frac{1}{4}, \\ \text{and} \quad y = -1.$$

$$4. \quad 9h^2k - 100 = 0. \quad (1)$$

$$3k^2h + 80 = 0. \quad (2)$$

$$9h^2k = 100. \quad (3)$$

$$3hk^2 = -80. \quad (4)$$

$$(3) \div (4), \quad \frac{3h}{k} = -\frac{5}{4}. \quad (5)$$

$$\text{From (5),} \quad h = \frac{-5k}{12}. \quad (6)$$

$$\text{From (6) and (4),}$$

$$3 \cdot \frac{-5k}{12} \cdot k^2 + 80 = 0. \quad (7)$$

$$\text{Whence} \quad k = 4. \quad (8)$$

$$\text{From (8) and (2),}$$

$$3 \cdot 4^2 \cdot h + 80 = 0. \\ 48h = -80. \\ h = -\frac{5}{3}.$$

$$5. \quad P(1+r)^2 = 224.72. \quad (1)$$

$$P + Pr = 212. \quad (2)$$

$$(1) \div (2), \quad 1 + r = 1.06. \quad (3)$$

$$r = .06. \quad (4)$$

$$\text{From (2) and (4),}$$

$$1.06P = 212. \\ P = 200.$$

$$6. \quad 9x^2y^2 + 6 = 15xy. \quad (1)$$

$$3xy + 4 = 6. \quad (2)$$

From (1),

$$(3xy - 2)(3xy - 3) = 0.$$

$$\text{From (2),} \quad 3xy - 3 = 0.$$

$\therefore$ any set of roots for

$$3xy - 3 = 0,$$

will satisfy the system, $\frac{1}{2}$ and 2, for example.

$$7. \quad x^4 = 9y^4 + 48. \quad (1)$$

$$x^2 = 3y^2 + 2. \quad (2)$$

$$x^4 - 9y^4 = 48. \quad (3)$$

$$x^2 - 3y^2 = 2. \quad (4)$$

$$(3) \div (4), \quad x^2 + 3y^2 = 24. \quad (5)$$

$$(4) + (5), \quad 2x^2 = 26.$$

$$x = \pm \sqrt{13}.$$

$$(4) - (5), \quad -6y^2 = -22.$$

$$y = \pm \frac{1}{3} \sqrt{33}.$$

$$x = \sqrt{13}, \quad \sqrt{13}, \quad -\sqrt{13}, \quad -\sqrt{13}.$$

$$y = \frac{1}{3} \sqrt{33}, \quad -\frac{1}{3} \sqrt{33},$$

$$-\frac{1}{3} \sqrt{33}, \quad -\frac{1}{3} \sqrt{33}.$$

$$8. \quad \frac{gt^2}{2} = .16. \quad (1)$$

$$gt = 3.2. \quad (2)$$

$$(1) \div (2), \quad \frac{t}{2} = \frac{1}{20}. \quad (3)$$

$$t = \frac{1}{10} = .1. \quad (4)$$

From (4) and (2),

$$g = 32.$$

$$9. \quad 1 - x = y. \quad (1)$$

$$1 - x^3 = y^2. \quad (2)$$

$$(1 - x)(1 + x + x^2) = y \cdot y. \quad (3)$$

$$\therefore \begin{cases} 1 + x + x^2 = y. \end{cases} \quad (4)$$

$$1 - x = y. \quad (5)$$

$$\text{and by § 81,} \begin{cases} 1 - x = 0, \\ y = 0. \end{cases} \quad (6) \quad (7)$$

From (4) and (5),

$$\begin{aligned} 1 - x &= 1 + x + x^2. \\ x^2 + 2x &= 0. \end{aligned}$$

$$x = 0, -2.$$

$$y = 1, 3.$$

From (6) and (7), $x = 1, y = 0$.

$$\therefore x = 0, -2, 1.$$

$$y = 1, 3, 0.$$

$$10. \quad x^2 - 2xy - 24y^2 = 32. \quad (1)$$

$$x - 6y = 2. \quad (2)$$

$$(1) \div (2), \quad x + 4y = 16.$$

$$10y = 14.$$

$$y = \frac{7}{5}.$$

$$x = \frac{52}{5}.$$

$$12. \quad x^3 + y^3 = 4x - 6y - 8, \quad (1)$$

$$x + y = 2x - 3y - 4. \quad (2)$$

$$(x + y)(x^2 - xy + y^2) = (2x - 3y - 4)2. \quad (3)$$

$\therefore$ by § 81 we obtain the two systems (4), (5) and (6), (7).

$$\text{From (2) and (3),} \quad \begin{cases} x^2 - xy + y^2 = 2, \\ x - 4y = 4. \end{cases} \quad (4)$$

$$\text{From (2),} \quad \begin{cases} x + y = 0, \\ 2x - 3y - 4 = 0. \end{cases} \quad (6)$$

$$\text{Substituting, from (5), } 4y + 4 \text{ for } x \text{ in (4),}$$

$$16x^2 + 32y + 16 - 4y^2 - 4y + y^2 = 2. \quad (8)$$

$$13y^2 + 28y + 14 = 0. \quad (9)$$

Whence

$$y = \frac{-14 \pm \sqrt{14}}{13}. \quad (10)$$

From (10) and (5),

$$x = \frac{-4 \pm 4\sqrt{14}}{13}. \quad (11)$$

Solving (6), (7),

$$x = \frac{4}{5}, \text{ and } y = -\frac{4}{5}.$$

From (4) and (5),

$$x^2 - 2x^2 + 4x^2 = 2.$$

$$3x^2 = 2.$$

$$x = \pm \frac{1}{3}\sqrt{6}.$$

$$y = \mp \frac{2}{3}\sqrt{6}.$$

From (6) and (7), $x = 0$.

$$y = 0.$$

$$x = \frac{1}{3}\sqrt{6}, -\frac{1}{3}\sqrt{6}, 0.$$

$$y = -\frac{2}{3}\sqrt{6}, \frac{2}{3}\sqrt{6}, 0.$$

$$13. \quad x^3 - y^3 = 6x. \quad (1)$$

$$x - y = 3x. \quad (2)$$

$$(x - y)(x^2 + xy + y^2) = 2 \cdot 3x. \quad (3)$$

By § 81,

$$\begin{cases} x^2 + xy + y^2 = 2, \\ x - y = 3x, \end{cases} \quad (4)$$

$$\begin{cases} x - y = 3x, \\ 3x = 0. \end{cases} \quad (5)$$

$$\begin{cases} x - y = 0, \\ 3x = 0. \end{cases} \quad (6)$$

and

$$\begin{cases} x - y = 0, \\ 3x = 0. \end{cases} \quad (7)$$

$$14. \quad x^3 + y^3 = 28. \quad (1)$$

$$x + y = 4. \quad (2)$$

$$(1) \div (2), \quad x^2 - xy + y^2 = 7. \quad (3)$$

From (2) and (4),

$$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$$

$$3y^2 - 12y + 9 = 0.$$

$$(y - 3)(y - 1) = 0.$$

$$y = 3, 1.$$

$$x = 1, 3.$$

$$15. \quad x^2 - xy + y^2 = 7. \quad (1)$$

$$x^3 + y^3 = 28. \quad (2)$$

$$(1) \div (2), \quad x + y = 4. \quad (3)$$

From (3) and (1),

$$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$$

$$3y^2 - 12y + 9 = 0.$$

$$y^2 - 4y + 3 = 0.$$

$$(y - 3)(y - 1) = 0.$$

$$y = 3, 1.$$

$$x = 1, 3.$$

$$16. \quad x + y = 4. \quad (1)$$

$$x^2 - xy + y^2 = 7. \quad (2)$$

Substituting,

$$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$$

$$\text{Collecting,} \quad y^2 - 4y + 3 = 0.$$

$$y = 1, 3.$$

$$x = 3, 1.$$

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$$1. \quad 2x^2 + y^2 = 33, \quad (1)$$

$$x^2 + 2y^2 = 54. \quad (2)$$

$$4x^2 + 2y^2 = 66. \quad (3)$$

$$x^2 + 2y^2 = 54. \quad (4)$$

$$3x^2 = 12. \quad (5)$$

$$x^2 = 4.$$

$$x = \pm 2,$$

$$y = \pm 5.$$

$$A \begin{cases} x = 2, \\ y = \pm 5. \end{cases} \quad B \begin{cases} x = -2, \\ y = \pm 5. \end{cases}$$

$$2. \quad 3h^2 - 8k^2 = 40, \quad (1)$$

$$5h^2 + k^2 = 81. \quad (2)$$

$$40h^2 + 8k^2 = 648. \quad (3)$$

$$(3) + (1), \quad 43h^2 = 688.$$

$$h^2 = 16.$$

$$h = \pm 4.$$

$$k = \pm 1.$$

$$A \begin{cases} h = 4, \\ k = \pm 1. \end{cases} \quad B \begin{cases} h = -4, \\ k = \pm 1. \end{cases}$$

$$3. \quad 4R_1^2 + 3 = 9R_2^2, \quad (1)$$

$$12R_1^2 + R_2^2 = \frac{31}{9}. \quad (2)$$

$$4R_1^2 - 9R_2^2 = -3. \quad (3)$$

$$108R_1^2 + 9R_2^2 = 31. \quad (4)$$

$$(3) + (4), \quad 112R_1^2 = 28. \quad (5)$$

$$4R_1^2 = 1.$$

$$R_1 = \pm \frac{1}{2}.$$

$$R_2 = \pm \frac{2}{3}.$$

$$R_1 = \frac{1}{2}, -\frac{1}{2}.$$

$$R_2 = \pm \frac{2}{3}, \pm \frac{2}{3}.$$

$$4. \quad xy + x = 18, \quad (1)$$

$$xy + y = 20. \quad (2)$$

$$(1) - (2), \quad x - y = -2. \quad (3)$$

$$x = y - 2. \quad (4)$$

From (4) and (2),

$$y^2 - 2y + y = 20.$$

$$y^2 - y - 20 = 0.$$

$$y = -4, 5.$$

$$\text{Then} \quad x = -6, 3.$$

$$5. \quad x^2 = y,$$

$$xy = 8.$$

$$x^2 \cdot x = 8.$$

$$x^3 - 8 = 0.$$

$$(x - 2)(x^2 + 2x + 4) = 0.$$

$$x = 2, -1 \pm \sqrt{-3}.$$

$$y = 4, -2 \mp 2\sqrt{-3}.$$

$$x = 2, -1 + \sqrt{-3}, -1 - \sqrt{-3}.$$

$$y = 4, -2 - 2\sqrt{-3}, -2 + \sqrt{-3}.$$

$$\begin{aligned}
 6. \quad & x - xy = 5, & (1) \\
 & 2y + xy = 6. & (2) \\
 (1) + (2), & x + 2y = 11. & (3) \\
 & x = 11 - 2y. & (4)
 \end{aligned}$$

From (4) and (1),

$$\begin{aligned}
 11 - 2y - 11y + 2y^2 &= 5. & (5) \\
 2y^2 - 13y + 6 &= 0.
 \end{aligned}$$

$$y = \frac{13 \pm \sqrt{169 - 48}}{4}$$

$$= \frac{13 \pm 11}{4} = 6, \frac{1}{2}.$$

$$x = -1, 10.$$

$$y = 6, \frac{1}{2}.$$

$$\begin{aligned}
 7. \quad & x^3 - y^3 = 19, & (1) \\
 & x - y = 1. & (2) \\
 (1) \div (2), & x^2 + xy + y^2 = 19. & (3)
 \end{aligned}$$

From (2) and (3),

$$y^2 + 2y + 1 + y^2 + y + y^2 = 19.$$

$$3y^2 + 3y - 18 = 0.$$

Whence,

$$y = -3, +2.$$

$$x = y + 1 = -3 + 1 = -2.$$

$$x = y + 1 = 2 + 1 = 3.$$

$$x = -2, 3.$$

$$y = -3, 2.$$

$$\begin{aligned}
 8. \quad & x^3 - y^3 = 19, & (1) \\
 & x^2 + xy + y^2 = 19. & (2) \\
 (1) \div (2), & x - y = 1. & (3)
 \end{aligned}$$

From (1) and (3),

$$y^2 + 2y + 1 + y^2 + y + y^2 = 19. \quad (4)$$

$$\text{Factoring, } (y + 3)(y - 2) = 0.$$

$$\text{Whence } y = +2, -3.$$

$$\text{Then } x = 3, -2.$$

$$\begin{aligned}
 9. \quad & x^2 + xy + y^2 = 19, & (1) \\
 & x - y = 1. & (2)
 \end{aligned}$$

From (2) and (1),

$$y^2 + 2y + 1 + y^2 + y + y^2 = 19.$$

$$(y + 3)(y - 2) = 0.$$

$$\text{Whence } y = -3 \text{ or } +2.$$

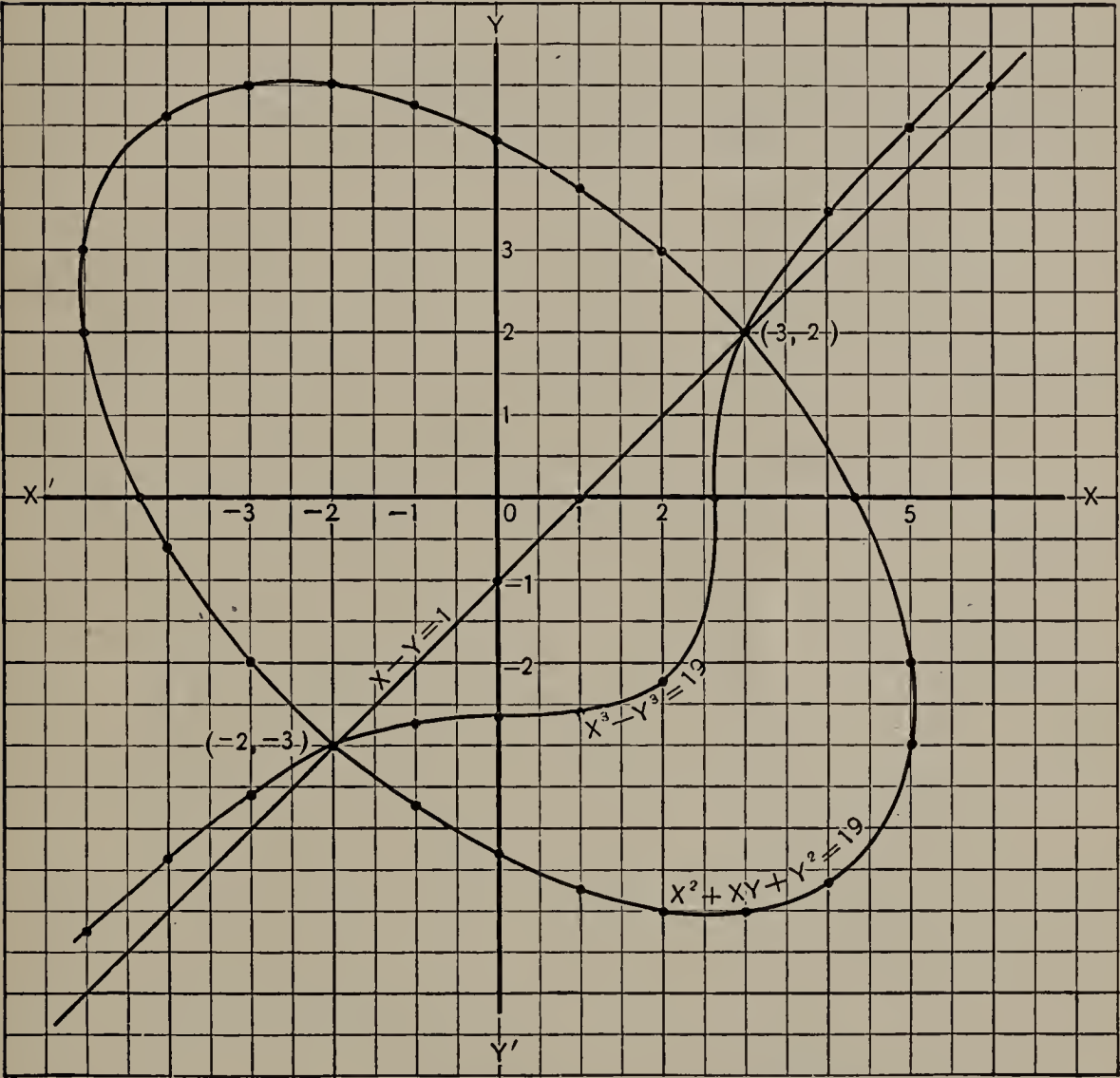
$$\text{Then } x = -2, +3.$$

10. $x^3 - y^3 = 19$. Then $y = \sqrt[3]{x^3 - 19}$. (1)

x	0	1	2	3	4	5	-1	-2	-3	-4	-5	2.62
y	-2.66	-2.62	-2.22	2	3.55	4.73	-2.71	-3	-3.58	-4.36	-5.24	0

$x - y = 1$. (2)

x	0	1	6
y	-1	0	5



$x^2 + xy + y^2 = 19$. Then $y = \frac{-x \pm \sqrt{76 - 3x^2}}{2}$. (3)

x	5	4.35	4	3	2	1	0	-1	-2	-3	-4	-4.35	-5
y	-2	0	-4.63	-5	-5	3.77	± 4.35	4.77	5	+5	4.63	0	2
	-3		.63	2	+3	-4.77		-3.77	-3	-2	-.63		3

For (1), (2),

z	3	-2
y	2	-3

For (1), (3),

x	3	-2
y	2	-3

For (3), (2),

x	3	-2
y	2	-3

The systems (7), (8), and (9) are equivalent. The graph shows for all the systems but two points of intersection: 3, 2 and -2, -3.

$$11. \quad 3s^2 - 2t^2 = 0, \quad (1)$$

$$5s^2 - 3t^2 = 1. \quad (2)$$

$$15s^2 - 10t^2 = 0. \quad (3)$$

$$15s^2 - 9t^2 = 3. \quad (4)$$

$$(3) - (4), \quad -t^2 = -3. \quad (5)$$

$$t = \pm \sqrt{3}. \quad (6)$$

$$9s^2 - 6t^2 = 0. \quad (7)$$

$$10s^2 - 6t^2 = 2. \quad (8)$$

$$(7) - (8), \quad -s^2 = -2, s^2 = 2.$$

$$s = \pm \sqrt{2}.$$

$$\therefore t = \sqrt{3}, \quad \sqrt{3}, -\sqrt{3}, -\sqrt{3}.$$

$$s = \sqrt{2}, -\sqrt{2}, \quad \sqrt{2}, -\sqrt{2}.$$

$$12. \quad 4n^2 + 7m^2 = 9, \quad (1)$$

$$2n^2 - \frac{9}{2} = m^2. \quad (2)$$

$$4n^2 + 7m^2 = 9. \quad (3)$$

$$4n^2 - 2m^2 = 9. \quad (4)$$

$$(3) - (4), \quad 9m^2 = 0, m = 0. \quad (5)$$

From (5) and (1),

$$4n^2 = 9.$$

$$\text{Whence,} \quad n = \pm \frac{3}{2}.$$

$$\therefore m = 0, 0.$$

$$n = \frac{3}{2}, -\frac{3}{2}.$$

$$13. \quad 5W_1^2 - 6.8W_2^2 = 99.55. \quad (1)$$

$$W_1^2 - W_2^2 = 20. \quad (2)$$

$$5 \cdot (2), \quad 5W_1^2 - 5W_2^2 = 100.$$

$$-1.8W_2^2 = -45.$$

$$W_2^2 = \frac{1}{4}.$$

$$W_2 = \pm \frac{1}{2}.$$

$$5W_1^2 - 6.8W_2^2 = 99.55.$$

$$6.8W_1^2 - 6.8W_2^2 = 136.0.$$

$$-1.8W_1^2 = -36.45.$$

$$W_1^2 = \frac{81}{4}. \text{ Whence } W_1 = \pm \frac{9}{2}.$$

$$\therefore W_2 = \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}.$$

$$W_1 = \frac{9}{2}, -\frac{9}{2}, \frac{9}{2}, -\frac{9}{2}.$$

$$14. \quad xy + 2y^2 = 2, \quad (1)$$

$$3xy + 5y^2 = 2. \quad (2)$$

$$(1) \cdot 3, \quad 3xy + 6y^2 = 6. \quad (3)$$

$$-y^2 = -4. \quad (4)$$

$$y = \pm 2. \quad (5)$$

From (1) and (5), $x = \mp 3$.

$$\therefore x = 3, -3.$$

$$y = -2, 2.$$

$$15. \quad x^2 + 2xy + 2y^2 = 10. \quad (1)$$

$$3x^2 - xy - y^2 = 51. \quad (2)$$

$$(2) \cdot 2,$$

$$6x^2 - 2xy - 2y^2 = 102. \quad (3)$$

$$7x^2 = 112. \quad (4)$$

$$x = \pm 4. \quad (5)$$

From (1) and (5),

$$y = -3, -1, 3, 1.$$

$$x = 4, 4, -4, -4.$$

16.

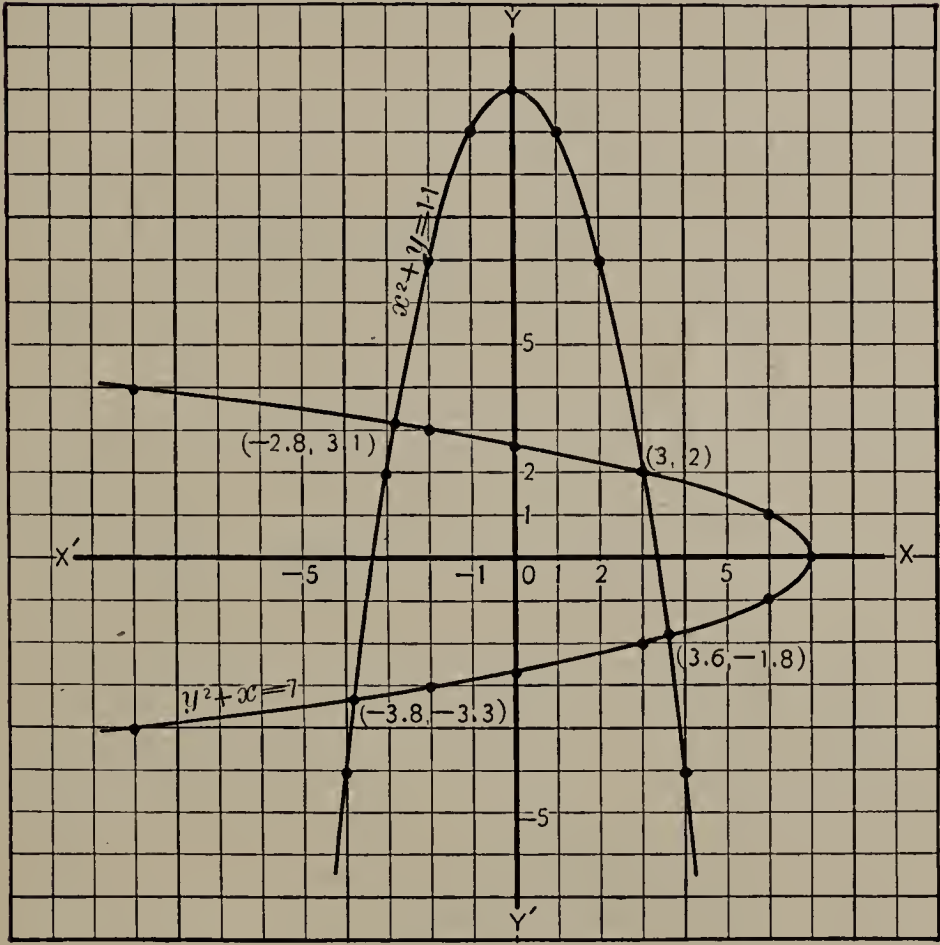
$y^2 + x = 7,$
 $x^2 + y = 11.$

$y^2 + x = 7.$ Then $y = \pm \sqrt{7 - x}.$

x	7	6	3	-2	-9	0
y	0	± 1	± 2	± 3	± 4	± 2.64

$x^2 + y = 11.$ Then $x = \pm \sqrt{11 - y}.$

x	0	± 1	± 2	± 3	± 4	± 5
y	11	10	7	2	-5	-14



The roots are :

x	-2.8	3	3.6	-3.8
y	3.1	2	-1.8	-3.3

$$17. \quad x^2 + xy + y^2 = 7, \quad (1)$$

$$x^2 + y^2 = 10. \quad (2)$$

$$xy = -3. \quad (3)$$

(1) + (3),

$$x^2 + 2xy + y^2 = 4. \quad (4)$$

$$x + y = \pm 2. \quad (5)$$

$$(3) \cdot -3, \quad -3xy = 9. \quad (6)$$

$$(1) - (6), \quad x^2 - 2xy + y^2 = 16. \quad (7)$$

$$x - y = \pm 4. \quad (8)$$

From (5) and (8),

$$\begin{cases} x + y = 2. \\ x - y = \pm 4. \end{cases} \quad \begin{cases} x + y = -2. \\ x - y = \pm 4. \end{cases}$$

$$\therefore x = 3, -3, 1, -1.$$

$$y = -1, 1, -3, 3.$$

$$18. \quad x^2 + xy + x = 0,$$

$$x^2 + xy + 2x = 0.$$

$$x(x + y + 1) = 0.$$

$$x(x + y + 2) = 0.$$

$$\left. \begin{aligned} x + y + 1 &= 0. \\ x + y + 2 &= 0. \end{aligned} \right\} \text{contradictory.}$$

$$\therefore x = 0, y = \text{any number.}$$

$$19. \quad x^2 + xy + y = 0, \quad (1)$$

$$x^2 + xy + x = 0. \quad (2)$$

$$y = x. \quad (3)$$

$$x^2 + x^2 + x = 0. \quad (4)$$

$$x(2x + 1) = 0. \quad (5)$$

Then $x = 0, -\frac{1}{2}.$

And $y = 0, -\frac{1}{2}.$

$$20. \quad \frac{1}{x^2} + \frac{1}{y^2} = 13, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

$$22. \quad x^2 - 2xy + 2y^2 - y = 0, \quad (1)$$

$$2x^2 - 3xy - y^2 + 2y = 0. \quad (2)$$

$$(1) \cdot 2, \quad 2x^2 - 4xy + 4y^2 - 2y = 0. \quad (3)$$

$$(2) + (3), \quad 4x^2 - 7xy + 3y^2 = 0. \quad (4)$$

$$\text{From (4),} \quad x = y, \quad (5)$$

and $x = \frac{3y}{4}. \quad (6)$

Squaring (2),

$$\frac{1}{x^2} - \frac{1}{2xy} + \frac{1}{y^2} = 1. \quad (3)$$

$$(1) - (3), \quad \frac{1}{2xy} = 12. \quad (4)$$

$$(1) + (4), \quad \frac{1}{x^2} + \frac{1}{2xy} + \frac{1}{y^2} = 25. \quad (5)$$

$$\frac{1}{x} + \frac{1}{y} = \pm 5. \quad (6)$$

From (6) and (2),

$$\frac{2}{x} = 6, -4.$$

$$\therefore x = \frac{1}{3}, -\frac{1}{2}.$$

$$y = \frac{1}{2}, -\frac{1}{3}.$$

$$21. \quad \frac{1}{x^3} - \frac{1}{y^3} = 7, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

$$\frac{1}{x^2} + \frac{1}{xy} + \frac{1}{y^2} = 7. \quad (3)$$

Squaring (2),

$$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (4)$$

$$(3) - (4), \quad \frac{3}{xy} = 6. \quad (5)$$

$$\frac{1}{xy} = 2. \quad (6)$$

(3) + (6),

$$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 9. \quad (7)$$

$$\frac{1}{x} + \frac{1}{y} = \pm 3. \quad (8)$$

From (8) and (2), $x = \frac{1}{2}, -1.$

$$y = 1, -\frac{1}{2}.$$

$$\text{From (1) and (5), } y^2 - 2y^2 + 2y^2 - y = 0. \quad (7)$$

$$\text{From (7), } y = 0, 1. \quad (8)$$

$$\text{From (8) and (5), } x = 0, 1. \quad (9)$$

$$\text{From (6) and (1), } \frac{9y^2}{16} - \frac{3y^2}{2} + 2y^2 - y = 0. \quad (10)$$

$$\text{From (10), } y = 0, \frac{1}{17}. \quad (11)$$

$$\begin{aligned} \text{From (11) and (6), } x &= 0, \frac{1}{17}. \\ \therefore x &= 0, 1, \frac{1}{17}. \\ y &= 0, 1, \frac{1}{17}. \end{aligned}$$

$$23. \quad x^2 + z^2 = 34, \quad (1)$$

$$x^2 + y^2 = 25, \quad (2)$$

$$y^2 + z^2 = 41. \quad (3)$$

$$(1) - (2), \quad -y^2 + z^2 = 9. \quad (4)$$

$$(3) + (4), \quad 2z^2 = 50. \quad (5)$$

$$z = \pm 5. \quad (6)$$

$$\text{From (6) and (3), } y^2 + 25 = 41. \quad (7)$$

$$y = \pm 4. \quad (8)$$

$$\text{From (6) and (1), } x^2 + 25 = 34.$$

$$x = \pm 3.$$

Therefore :

$$\begin{array}{cccccccc} x = 3, & 3, & 3, & 3, & -3, & -3, & -3, & -3, \\ y = 4, & -4, & 4, & -4, & 4, & -4, & 4, & -4, \\ z = 5, & 5, & -5, & -5, & 5, & 5, & -5, & -5. \end{array}$$

$$24. \quad 3xy = x^2y^2 - 88, \quad (1)$$

$$x - y = 6. \quad (2)$$

$$\text{From (1), } (xy - 11)(xy + 8) = 0. \quad (3)$$

$$\begin{aligned} \text{From (3) and (2),} \\ (y^2 + 6y - 11)(y^2 + 6y + 8) &= 0. \end{aligned} \quad (4)$$

$$\therefore y = -2, -4, \quad (5)$$

$$\text{and } y = -3 \pm 2\sqrt{5}. \quad (6)$$

$$\text{From (3), (5), and (6), } x = 4, 2, 3 \pm 2\sqrt{5}.$$

$$x = 4, 2, 3 + 2\sqrt{5}, 3 - 2\sqrt{5}.$$

$$y = -2, -4, -3 + 2\sqrt{5}, -3 - 2\sqrt{5}.$$

$$25. \quad x^3 = y^3 + 37, \quad (1)$$

$$x^2y = xy^2 + 12. \quad (2)$$

$$x^3 - y^3 = 37. \quad (3)$$

$$x^2y - xy^2 = 12. \quad (4)$$

$$(3) \div (4), \quad \frac{x^2 + xy + y^2}{xy} = \frac{37}{12}. \quad (5)$$

$$12x^2 + 12xy + 12y^2 = 37xy. \quad (6)$$

From (6), $x = \frac{4y}{3},$ (7)

and $x = \frac{3y}{4}.$ (8)

From (2) and (8), $\frac{9y^3}{16} = \frac{3y^3}{4} + 12.$ (9)

$y^3 + 64 = 0$, or $(y + 4)(y^2 - 4y + 16) = 0.$ (10)

$y = -4, 2 \pm 2\sqrt{-3}.$ (11)

From (11) and (8), $x = -3, \frac{3 \pm 3\sqrt{-3}}{2}.$

From (2) and (7), $\frac{16y^3}{9} = \frac{4y^3}{3} + 12.$ (12)

From (12), $y = 3, \frac{-3 \pm 3\sqrt{-3}}{2}.$ (13)

From (13) and (7), $x = 4, \frac{-2 \pm 2\sqrt{-3}}{2}.$
 $x = 4, -3, \frac{-2 + 2\sqrt{-3}}{2}, \frac{-2 - 2\sqrt{-3}}{2}.$
 $y = 3, -4, \frac{-3 + 3\sqrt{-3}}{2}, \frac{-3 - 3\sqrt{-3}}{2},$

$\frac{3 + 3\sqrt{-3}}{2}, \frac{3 - 3\sqrt{-3}}{2},$
 $2 + 2\sqrt{-3}, 2 - 2\sqrt{-3}.$

26. $\frac{4}{x^2} - \frac{13}{xy} + \frac{9}{y^2} = 9,$ (1) $x = \frac{y}{3}.$ (6)

$\frac{1}{xy} - \frac{1}{y^2} = 3.$ (2) From (6) and (2), $x = y.$ (7)

(2) $\cdot 3,$ $\frac{3}{xy} - \frac{3}{y^2} = 9.$ (3) $\frac{3}{y^2} - \frac{1}{y^2} = 3.$ (8)

(1) - (3), $y^2 = \frac{2}{3}.$ (9)

$\frac{4}{x^2} - \frac{16}{xy} + \frac{12}{y^2} = 0.$ (4) $y = \pm \frac{2}{3}\sqrt{6}.$

From (9) and (6), $x = \pm \frac{1}{9}\sqrt{6}.$

$\left(\frac{1}{x} - \frac{3}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right) = 0.$ (5) No results can be obtained by using (7) and (1), or (7) and (2).

27. $x^2 = 8x + 6y,$ (1)

$y^2 = 6x + 8y.$ (2)

$x^2 - y^2 = 2(x - y).$ (3)

$(x - y)(x + y - 2) = 0.$ (4)

$x = y,$ (5)

$x = 2 - y.$ (6)

From (5) and (1), $y^2 = 14y.$ (7)

$y = 0, 14.$

From (6) and (1), $4 - 4y + y^2 = 16 - 8y + 6y$.

$$y^2 - 2y - 12 = 0.$$

$$y = 1 \pm \sqrt{13}.$$

$$x = 1 \mp \sqrt{13}.$$

$$x = 0, 14, 1 + \sqrt{13}, 1 - \sqrt{13}.$$

$$y = 0, 14, 1 - \sqrt{13}, 1 + \sqrt{13}.$$

$$28 \quad x^3 - y^3 = 2x + y - 4, \quad (1)$$

$$x + 2y = 4. \quad (2)$$

Transposing, $0 = x + 2y - 4. \quad (3)$

Adding $x - y$, $x - y = 2x + y - 4. \quad (4)$

From (1), $(x - y)(x^2 + xy + y^2) = (2x + y - 4)1. \quad (5)$

From (4) and (5), by § 81,

$$\begin{cases} x^2 + xy + y^2 = 1, \\ x + 2y = 4, \end{cases} \quad (6)$$

$$(7)$$

and $\begin{cases} x - y = 0, \\ 2x + y - 4 = 0. \end{cases} \quad (8)$

$$(9)$$

From (6) and (7), $x = \pm 2\sqrt{-1},$

$$y = 2 \pm \sqrt{-1}.$$

From (8) and (9), $x = \frac{4}{3},$

$$y = \frac{4}{3}.$$

$$29. \quad xy = c, \quad (1)$$

$$x + y = a. \quad (2)$$

$$x = a - y. \quad (3)$$

From (1) and (3), $ay - y^2 = c. \quad (4)$

$$y = \frac{a \pm \sqrt{a^2 - 4c}}{2}. \quad (5)$$

From (3) and (5), $x = \frac{a \mp \sqrt{a^2 - 4c}}{2}.$

$$30. \quad x^{-2} - y^{-2} = 6, \quad (1) \quad 31. \quad x - y = 16, \quad (1)$$

$$x^{-1} + y^{-1} = 2. \quad (2) \quad x^{\frac{1}{2}} - y^{\frac{1}{2}} = 2. \quad (2)$$

$$(1) \div (2), \quad x^{-1} - y^{-1} = 3. \quad (3) \quad (1) \div (2), \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 8. \quad (3)$$

$$(3) + (2), \quad 2x^{-1} = 5. \quad (4) \quad (2) + (3), \quad 2x^{\frac{1}{2}} = 10.$$

$$\frac{2}{x} = 5. \quad x = 25.$$

$$(2) - (3), \quad 2y^{-1} = -1. \quad (3) - (2), \quad 2y^{\frac{1}{2}} = 6.$$

$$\frac{2}{y} = -1. \quad y^{\frac{1}{2}} = 3.$$

$$y = -2. \quad y = 9.$$

$$\therefore x = \frac{2}{5}. \quad x = 25.$$

$$y = -2. \quad y = 9.$$

$$32. \quad \frac{1}{x-2} + \frac{1}{y-2} = \frac{3}{4}, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{12}. \quad (2)$$

$$\text{From (2),} \quad 12y - 12x = xy. \quad (3)$$

$$\text{From (1),} \quad 10x + 10y = 3xy + 28. \quad (4)$$

$$(3) \cdot 3, \quad -36x + 36y = 3xy. \quad (5)$$

$$(4) - (5), \quad 46x - 26y = 28. \quad (6)$$

$$x = \frac{13y + 14}{23}. \quad (7)$$

$$\text{From (7) and (3),} \quad 12y - \frac{156y + 168}{23} = \frac{13y^2 + 14y}{23}. \quad (8)$$

$$\text{From (8),} \quad y = 6, \frac{28}{13}. \quad (9)$$

$$\text{From (9) and (7),} \quad x = 4, \frac{42}{13}. \quad (10)$$

$$33. \quad x^2 + 6x^{-2} = 36\frac{1}{6}, \quad (1) \quad (2) - (3), \quad 2xy\sqrt{x} = -256. \quad (4)$$

$$3xy - x^2 = 36. \quad (2) \quad (3) - (4) \cdot 2,$$

$$(1) \cdot x^2, \quad x^4 - 36\frac{1}{6}x^2 + 6 = 0. \quad (3) \quad x^2 + 2xy\sqrt{x} + xy^2 = 64. \quad (5)$$

$$\text{From (3),} \quad x^2 = 36, \frac{1}{6}. \quad (4) \quad x + y\sqrt{x} = \pm 8. \quad (6)$$

$$\text{Whence} \quad x = \pm 6, \pm \frac{1}{6}\sqrt{6}. \quad (5) \quad x - y\sqrt{x} = 24. \quad (7)$$

$$\text{From (2),} \quad y = \frac{36 + x^2}{3x}. \quad (6) \quad (6) + (7), \quad 2x = 32, 16.$$

$$\text{From (5) and (6),} \quad x = 16, 8.$$

$$y = \pm 4, \pm \frac{217}{18}\sqrt{6}. \quad (7) \quad (6) - (7), \quad 2y\sqrt{x} = -16, -32.$$

$$x = \pm 6, \pm \frac{1}{6}\sqrt{6}.$$

$$y = \pm 4, \pm \frac{217}{18}\sqrt{6}. \quad y = \frac{-8}{\sqrt{x}}, \frac{-16}{\sqrt{x}}$$

$$34. \quad x - y\sqrt{x} = 24, \quad (1) \quad = \frac{-8}{4}, \frac{-16}{2\sqrt{2}} = -2, -4\sqrt{2}.$$

$$x^2 + xy^2 = 320. \quad (2)$$

$$\text{Squaring (1),} \quad x = 16, 8.$$

$$x^2 - 2xy\sqrt{x} + xy^2 = 576. \quad (3) \quad y = -2, -4\sqrt{2}.$$

$$35. \quad \frac{x-1}{y-1} = 3, \quad (1)$$

$$\frac{y^2 + y + 1}{x^2 - x + 1} = \frac{13}{43}. \quad (2)$$

$$\text{From (1),} \quad x = 3y - 2. \quad (3)$$

$$\text{From (2),} \quad 43y^2 + 43y + 43 = 13x^2 - 13x + 13. \quad (4)$$

$$\text{From (3) and (4),} \quad 43y^2 + 43y + 30 = 117y^2 - 156y + 52 - 39y + 26. \quad (5)$$

$$\text{From (5),} \quad 37y^2 - 119y + 24 = 0. \quad (6)$$

$$\text{From (6),} \quad y = 3, \frac{8}{37}. \quad (7)$$

$$\text{From (3) and (7),} \quad x = 7, \frac{-50}{37}.$$

$$36. \quad \frac{1}{x^2} + \frac{2}{xy} = 16, \quad (1)$$

$$\frac{3}{x^2} - \frac{4}{xy} + \frac{2}{y^2} = 6. \quad (2)$$

$$(1) \cdot 3, \quad \frac{3}{x^2} + \frac{6}{xy} = 48. \quad (3)$$

$$(2) \cdot 8, \quad \frac{24}{x^2} - \frac{32}{xy} + \frac{16}{y^2} = 48. \quad (4)$$

$$(4) - (3), \quad \frac{21}{x^2} - \frac{38}{xy} + \frac{16}{y^2} = 0. \quad (5)$$

$$\left(\frac{7}{x} - \frac{8}{y}\right)\left(\frac{3}{x} - \frac{2}{y}\right) = 0. \quad (6)$$

$$\text{From (6),} \quad x = \frac{3y}{2}, \quad (7)$$

$$\text{and} \quad x = \frac{7y}{8}. \quad (8)$$

$$\text{From (1) and (8),} \quad \frac{64}{49y^2} + \frac{16}{7y^2} = 16. \quad (9)$$

$$\text{From (9),} \quad y = \pm \frac{1}{7} \sqrt{11}. \quad (10)$$

$$\text{From (8) and (10),} \quad x = \pm \frac{1}{8} \sqrt{11}.$$

$$\text{From (7) and (1),} \quad \frac{4}{9y^2} + \frac{4}{3y^2} = 16. \quad (11)$$

$$\text{From (11),} \quad y = \pm \frac{1}{3}. \quad (12)$$

$$\text{From (7) and (12),} \quad x = \pm \frac{1}{4}.$$

$$x = \pm \frac{1}{2}, \pm \frac{1}{8} \sqrt{11}.$$

$$y = \pm \frac{1}{3}, \pm \frac{1}{7} \sqrt{11}.$$

$$37. \quad \frac{9x}{y} = 18, \quad (1)$$

$$xy = 18. \quad (2)$$

$$\text{From (1),} \quad x = 2y. \quad (3)$$

$$\text{From (2) and (3),} \quad 2y^2 = 18.$$

$$y = \pm 3.$$

$$x = \pm 6.$$

$$38. \quad 4x + \frac{1}{y} = 46, \quad (1)$$

$$\frac{26x}{5} - \frac{1}{y} = 46. \quad (2)$$

$$(1) + (2), \quad \frac{46x}{5} = 92. \quad (3)$$

$$x = 10. \quad (4)$$

From (1) and (4), $40 + \frac{1}{y} = 46.$

$$\frac{1}{y} = 6.$$

$$y = \frac{1}{6}.$$

39. $x^2 + y^2 - (y - x) = 12,$ (1)

$$x^2 - xy = 0. \quad (2)$$

$$x(x - y) = 0. \quad (3)$$

$$x = 0. \quad (4)$$

$$x = y. \quad (5)$$

From (1) and (4), $y^2 - y - 12 = 0. \quad (6)$

From (6), $y = 4, -3.$

From (5) and (1), $y^2 + y^2 = 12.$

$$y^2 = 6.$$

$$y = \pm \sqrt{6}.$$

$$x = 0, 0, \pm \sqrt{6}.$$

$$y = 4, -3, \pm \sqrt{6}.$$

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1. Let x equal one number and y equal the other.

Then $x - y = 4,$

and $x^2 - y^2 = 88.$

Dividing, $x + y = 22.$

$$x - y = 4.$$

$$x = 13, 13 - y = 4, y = 9.$$

2. Let x equal one number and y equal the other.

Then $x + y = 21, \quad (1)$

and $x^2 + y^2 = 281. \quad (2)$

Squaring (1), $x^2 + 2xy + y^2 = 441. \quad (3)$

(3) - (2), $2xy = 160. \quad (4)$

(2) - (4), $x^2 - 2xy + y^2 = 121. \quad (5)$

$$x - y = \pm 11. \quad (6)$$

From (1) and (6), $\begin{cases} x = 16, 5. \\ y = 5, 16. \end{cases}$

3. Let x equal one number and y equal the other.

Then $xy = 192, \quad (1)$

and $\frac{x}{y} = \frac{3}{4}. \quad (2)$

From (1) and (2), $3y^2 = 4 \cdot 192. \quad (3)$

From (3), $y = \pm 16. \quad (4)$

From (2) and (4), $x = \pm 12.$

$$\therefore x = 12, -12. \quad y = 16, -16.$$

4. Let x equal one leg in feet and y equal the other.

$$\frac{xy}{2} = 150. \quad (1)$$

$$x^2 + y^2 = 625. \quad (2)$$

$$(2) - (1) \cdot 4, \quad x^2 - 2xy + y^2 = 25. \quad (3)$$

$$(2) + (1) \cdot 4, \quad x^2 + 2xy + y^2 = 1225. \quad (4)$$

$$\text{From (3),} \quad x - y = \pm 5.$$

$$\text{From (4),} \quad x + y = \pm 35.$$

$$x = 20.$$

$$y = 15.$$

5. Let x and y equal the dimensions in rods.

$$x - y = 8.$$

$$xy = 1280.$$

$$8y + y^2 = 1280.$$

$$y^2 + 8y - 1280 = 0.$$

$$y = 32, -40.$$

$\therefore$ the dimensions are 32 by 40 rods.

6. Let x and y in feet equal the sides of the squares respectively.

$$x^2 - y^2 = 252.$$

$$4x - 4y = 24.$$

$$x - y = 6.$$

$$\text{Dividing,} \quad x + y = 42.$$

$$x = 24.$$

$$y = 18.$$

$\therefore$ the sides are 24 feet and 18 feet.

7. Let x and y equal the dimensions in rods.

$$xy = \frac{18}{5} \cdot 160 = 576. \quad (1)$$

$$x^2 + y^2 = 3600. \quad (2)$$

$$(2) - 2 \cdot (1), \quad x^2 - 2xy + y^2 = 2448.$$

$$(2) + 2 \cdot (1), \quad x^2 + 2xy + y^2 = 4752.$$

$$x - y = \pm 49.47.$$

$$x + y = \pm 68.93.$$

$$x = 59.20.$$

$$y = 9.73.$$

$$2x + 2y = 137.86, \text{ perimeter.}$$

8. Let x and y equal the dimensions in feet.

$$2x + 2y = 112.$$

$$x + y = 56.$$

$$xy = 768.$$

$$56y - y^2 = 768.$$

$$y^2 - 56y + 768 = 0.$$

$$y = 32, 24.$$

$\therefore$ the dimensions are 32 by 24 feet.

9. Let x and y equal the numbers:

$$xy = 56.$$

$$x^2 + y^2 = 113.$$

$$x^2 - 2xy + y^2 = 1.$$

$$x^2 + 2xy + y^2 = 225.$$

$$x - y = \pm 1.$$

$$x + y = \pm 15.$$

$$x = 8, -8.$$

$$y = 7, -7.$$

x = the numerator,

y = the denominator.

$$\frac{x}{y} = \frac{2}{3}.$$

(1)

$$\frac{x^2 - 44}{y^2 - 44} = \frac{5}{14}.$$

(2)

$$x = \frac{2y}{3}.$$

(3)

$$14x^2 - 616 = 5y^2 - 220.$$

(4)

From (3) and (4), $14 \cdot \frac{4y^2}{9} - 396 = 5y^2.$

$$11y^2 = 9 \cdot 396.$$

$$y^2 = 9 \cdot 36.$$

$$y = \pm 18.$$

Then

$$x = \pm 12.$$

$$\therefore \text{the fraction} = \frac{12}{18}.$$

11. Let
and

x = the base,

y = the altitude, each in inches.

$$x = y + 6.$$

$$\frac{xy}{2} = \frac{3}{2} \cdot 144.$$

$$y^2 + 6y = 432.$$

$$y = 18, -24.$$

-24 does not satisfy the conditions.

$$\therefore \text{base} = 24 \text{ inches; altitude} = 18 \text{ inches.}$$

12. Let x and y = the edges respectively, each in inches.

$$\therefore x^3 - y^3 = 1413.$$

(1)

$$x - y = 3.$$

(2)

$$x = y + 3. \quad (3)$$

$$(1) \div (2), \quad x^2 + xy + y^2 = 471. \quad (4)$$

From (3) and (4),

$$y^2 + 6y + 9 + y^2 + 3y + y^2 = 471.$$

$$y^2 + 3y - 154 = 0.$$

$$y = -14, +11. \quad -14 \text{ is not possible.}$$

$\therefore$ 14 inches and 11 inches are the sides.

13. Let x and y = the radii, each in inches.

$$x + y = 25. \quad (1)$$

$$\pi x^2 - \pi y^2 = 125\pi. \quad (2)$$

$$(2) \div (1), \quad x - y = 5.$$

$$\text{Then} \quad x = 15,$$

$$\text{and} \quad y = 10.$$

$\therefore$ 15 inches and 10 inches are the radii.

14. Let x and y = the dimensions.

$$2x + 2y = 5C. \quad (1)$$

$$x = \frac{5C}{2} - y. \quad (2)$$

$$xy = C^2. \quad (3)$$

$$\text{From (2) and (3), } y^2 - \frac{5Cy}{2} + C^2 = 0. \quad (4)$$

$$2y^2 - 5Cy + 2C^2 = 0. \quad (5)$$

$$y = 2C, \frac{C}{2}. \quad (6)$$

$$\text{From (6) and (2),} \quad x = \frac{C}{2}, 2C.$$

$\therefore 2C$ = the length, $\frac{C}{2}$ = the width.

15. Let x and y = the legs.

$$\frac{xy}{2} = 8a^2 - 8b^2. \quad (1)$$

$$x^2 + y^2 = 32a^2 + 32b^2. \quad (2)$$

$$\text{From (1),} \quad 2xy = 32a^2 - 32b^2. \quad (3)$$

$$(2) - (3), \quad (x - y)^2 = 64b^2.$$

$$(2) + (3), \quad (x + y)^2 = 64a^2.$$

$$x - y = \pm 8b.$$

$$x + y = \pm 8a.$$

$$\text{Then} \quad x = 4a + 4b,$$

$$\text{and} \quad y = 4a - 4b.$$

$\therefore$ the legs are $4a + 4b$, and $4a - 4b$.

16. Let $x =$ one leg and $y =$ the other leg, each in feet.

$$y + x + \sqrt{x^2 + y^2} = 56. \quad (1)$$

$$\frac{xy}{2} = 84. \quad (2)$$

$$\text{From (1), } x^2 + 2xy + y^2 - 112x - 112y + 3136 = x^2 + y^2. \quad (3)$$

$$\text{From (2), } 2xy - 336 = 0. \quad (4)$$

$$(3) - (4), \quad -112x - 112y + 3472 = 0. \quad (5)$$

$$x + y = 31. \quad (6)$$

$$\text{From (6) and (2), } y^2 - 31y + 168 = 0.$$

$$y = 7, 24.$$

$$\text{Then } x = 24, 7$$

$\therefore$ the legs are 7 and 24 and the hypotenuse is 25.

17. Let $x =$ the units digit,
and $y =$ the tens digit.

$$(x + y)(10y + x) = 324. \quad (1)$$

$$10y + x + 3y + 3x = 10x + y. \quad (2)$$

$$\text{From (1), } x^2 + 11xy + 10y^2 = 324. \quad (3)$$

$$\text{From (2), } 12y = 6x. \quad (4)$$

From (3) and (4),

$$x^2 + \frac{11x^2}{2} + \frac{5x^2}{2} = 324.$$

$$9x^2 = 324.$$

$$x = \pm 6.$$

$$\text{Then } y = \pm 3.$$

$$\therefore \text{ the number} = 36.$$

18. Let $x =$ the sum in dollars,
and $y =$ the rate per cent.

$$\frac{y}{100}x = \$42. \quad (1)$$

$$\frac{(x + 200)(y - 1)}{100} = \$48. \quad (2)$$

$$xy = 4200. \quad (3)$$

$$xy + 200y - x - 200 = 4800. \quad (4)$$

$$\text{From (3) and (4), } x = 200y - 800. \quad (5)$$

$$\text{From (5) and (3), } 200y^2 - 800y = 4200.$$

$$y^2 - 4y - 21 = 0.$$

$$y = 7, -3.$$

$$x = \$600.$$

$$\therefore \text{ principal} = \$600; \text{ rate} = 7\%.$$

19. Let x = the rate in miles of man going north,
and y = the rate in miles of man going east.

$$\text{Then } \left(\frac{4x}{3}\right)^2 + \left(3 + \frac{4y}{3}\right)^2 = 289. \quad (1)$$

$$\left(\frac{14x}{3}\right)^2 + \left(3 + \frac{14y}{3}\right)^2 = 2809. \quad (2)$$

Solving (1) and (2), $x = 6, y = 9.$

20. Let x = the circumference of fore wheel of carriage in feet,
and y = the circumference of rear wheel of carriage in feet.

$$\frac{5280}{x+1} - \frac{5280}{y+3} = 88. \quad (1) \qquad \frac{60}{x+1} - \frac{60}{y+3} = 1. \quad (3)$$

$$\frac{5280}{x} - \frac{5280}{y} = 40. \quad (2) \qquad \frac{132}{x} - \frac{132}{y} = 1. \quad (4)$$

Solving (3) and (4), $x = 11.$

$$y = 12.$$

$$x + 1 = 12.$$

$$y + 3 = 15.$$

21. $P \qquad \qquad \qquad K \qquad \qquad \qquad Q$

Let K be the point where A and B meet.

First. A goes from P to K , B from Q to K .

(A) $r = x$ (miles per hour).

$t = y$ (hours).

$d = PK = xy.$

$$(B) \quad r = \frac{xy - 40}{y}.$$

$t = y.$

$d = QK = xy - 40.$

Second. A goes from K to Q , B from K to P .

(A) $r = x.$

$$t = \frac{xy - 40}{x} = \left(\frac{d}{r}\right).$$

$d = xy - 40.$

$$(B) \quad r = \frac{xy - 40}{y}.$$

$$t = \frac{xy^2}{xy - 40} = \left(\frac{d}{r}\right).$$

$d = xy.$

Therefore

$$\frac{xy - 40}{x} = 2, \quad (1)$$

and

$$\frac{xy^2}{xy - 40} = 8. \quad (2)$$

(1) $\cdot$ (2),

$$y^2 = 16. \quad (3)$$

$$y = \pm 4. \quad (4)$$

From (4) and (1),

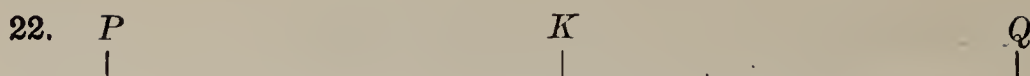
$x = 20$, A's rate.

Then

$$\frac{xy - 40}{y} = 10, \text{ B's rate,}$$

and

$$PQ = 2xy - 40 = 120 \text{ miles.}$$



First. A goes from P to K , and B goes from Q to K .

(A) $r = x$ (in miles per hour).

$t = y$ (in hours).

$d = xy$.

(B) $r = \frac{300 - xy}{y}$.

$t = y$.

$d = 300 - xy$.

Second. A goes from K to Q , and B goes from K to P .

(A) $r = x$.

$t = \frac{300 - xy}{x}$.

$d = 300 - xy$.

(B) $r = \frac{300 - xy}{y}$.

$t = \frac{xy^2}{300 - xy}$.

$d = xy$.

Therefore

$$\frac{300 - xy}{x} = 6\frac{2}{3}. \quad (1)$$

$$\frac{xy^2}{300 - xy} = 15. \quad (2)$$

$$(1) \cdot (2), \quad y^2 = 100. \quad (3)$$

$$y = \pm 10. \quad -10 \text{ is rejected.} \quad (4)$$

From (4) and (1), $x = 18$, or A's rate.

$$\therefore \frac{300 - xy}{y} = 12, \text{ or B's rate.}$$

23. Let $b =$ the breadth.

Then $2b^2 = 1250$.

Whence $b = 25$,

and $2b = 50$, the length.

Now let $x =$ the width of the path.

Then $(50 - 2x)(25 - 2x) = 625$.

$$4x^2 - 150x + 625 = 0.$$

$$x = \frac{150 \pm \sqrt{22,500 - 10,000}}{8} = \frac{75 \pm 25\sqrt{5}}{4}.$$

$$\frac{75 + 25\sqrt{5}}{4} \text{ is rejected.}$$

$$\text{Width of path} = \frac{75 - 25\sqrt{5}}{4} = 4.77 + \text{feet.}$$

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1. $x^2 + y^2 = 36$.

$$(10 - x)^2 + y^2 = 64.$$

$$100 - 20x + x^2 + y^2 = 64.$$

$$-20x + 100 = 28.$$

$$20x = 72.$$

$$x = 3\frac{3}{5}.$$

$$y^2 = \frac{9 \cdot 0 \cdot 0}{2 \cdot 5} - \frac{3 \cdot 2 \cdot 4}{2 \cdot 5} = \frac{5 \cdot 7 \cdot 6}{2 \cdot 5} = 1.$$

$$y = \frac{2 \cdot 4}{5} = 4\frac{4}{5}, \text{ altitude.}$$

2.

$$x^2 + y^2 = 64.$$

$$(17 - x)^2 + y^2 = 225.$$

$$x^2 + y^2 = 64.$$

$$x^2 + y^2 - 34x + 289 = 225.$$

$$34x - 289 = -161.$$

$$34x = 128.$$

$$x = 3\frac{1}{7}.$$

$$y = 7\frac{1}{7}.$$

$$\therefore \text{altitude} = 7\frac{1}{7}; \text{area} = 60.$$

3.

$$x^2 + y^2 = 169.$$

$$(20 - x)^2 + y^2 = 441.$$

Solving,

$$x = 3\frac{1}{5}, y = 12\frac{3}{5}.$$

$$\therefore \text{altitude} = 12\frac{3}{5}; \text{area} = 126.$$

4.

$$x^2 + y^2 = 225.$$

$$(7 - x)^2 + y^2 = 400.$$

Solving,

$$x = 9, y = 12.$$

$$\therefore \text{altitude} = 12; \text{area} = 42.$$

5.

$$x^2 + y^2 = 289.$$

$$(10 - x)^2 + y^2 = 441.$$

Solving,

$$x = -2\frac{3}{5}, y = 16\frac{8}{10}.$$

The minus sign before x indicates that the perpendicular falls outside the triangle.

$$\therefore \text{altitude} = 16\frac{4}{5}; \text{area} = 84.$$

6.

$$x^2 + y^2 = 144.$$

$$(16 - x)^2 + y^2 = 324.$$

Solving,
and

$$x = 2\frac{3}{8},$$

$$y = 11.76, \text{ the altitude.}$$

7.

$$x^2 + y^2 = 100.$$

$$(12 - x)^2 + y^2 = 100.$$

Solving,

$$x = 6, y = 8.$$

$$\therefore \text{altitude} = 8.$$

8.

$$x^2 + y^2 = 144.$$

$$(8 - x)^2 + y^2 = 289.$$

Solving,

$$x = -5\frac{1}{16}, y = 10.88.$$

$$\therefore \text{altitude} = 10.88.$$

9. $x^2 + y^2 = 900.$
 $(5 - x)^2 + y^2 = 841.$
 Solving, $x = -8\frac{2}{5}, y = 28\frac{4}{5},$ altitude.

$$\text{Area } \frac{28\frac{4}{5} \cdot (20 + 15)}{2} = 504.$$

10. $x^2 + y^2 = 144.$
 $(25 - x)^2 + y^2 = 289.$
 Solving, $x = 9\frac{3}{5}, y = 7\frac{1}{5},$ altitude.

$$\text{Area} = \frac{7\frac{1}{5} \cdot (20 + 45)}{2} = 234.$$

11. $x^2 + y^2 = 729.$
 $(19 - x)^2 + y^2 = 900.$
 Solving, $y = 26.53,$ altitude.

$$\text{Area} = \frac{26.53}{2} (21 + 40) = 809.16.$$

12. $x^2 + (100 - 23)^2 = \overline{85}^2.$
 $x^2 = 1296.$
 $x = 36,$ and area $= \frac{1}{2} \cdot 36 \cdot (123) = 2214.$

13. $x^2 + y^2 = \overline{123}^2.$
 $(250 - 42 - x)^2 + y^2 = \overline{325}^2.$
 $x^2 - 416x + y^2 = 105,625 - 43,264.$
 $416x = 47,232.$

$$x = 113\frac{71}{104}.$$

Substituting and solving, $y = 47\frac{4}{13},$ altitude.

Then $\text{area} = 47\frac{4}{13} \left(\frac{250 + 42}{2} \right) = 6906.9 + .$

14. Let x equal the third side, $x + 14$ the second, and y the projection of the third side on the first.

Then $x^2 + y^2 = 256. \quad (1)$

$$(14 + x)^2 = 256 + (18 - y)^2. \quad (2)$$

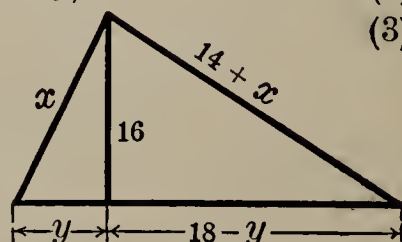
From (1) and (2), $7x + 9y = 32. \quad (3)$

From (3) and (1), $x^2 + 14x - 680 = 0.$

$$\therefore x = -34, 20.$$

$$x + 14 = 34.$$

$\therefore$ the three sides are 18, 34, and 20.



$$9. \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}, \quad \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}, \quad \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}.$$

$$10. 1 + 3\sqrt{2}, \quad 1 + 4\sqrt{2}, \quad 1 + 5\sqrt{2}.$$

Page 408 (First set)

1. Arithmetical progression ; $\frac{8}{3}$.
2. Arithmetical progression ; $6\frac{1}{2}$.
5. Arithmetical progression ; -4 .
6. Arithmetical progression ; -10 .
7. Arithmetical progression ; $-(2a+1)$.
9. Arithmetical progression ; $-\sqrt{3}$.

Page 408 (Second set)

1. $l = 1 + 11 \cdot 4$.
 $l = 45$.
2. $l = -18 + 22 \cdot 3$.
 $l = 48$.
3. $l = 13 + 14 \cdot (-6)$.
 $l = -71$.
4. $l = a + 18 \cdot 2a$.
 $l = 37a$.
5. 7th term $= \frac{7}{3} + 6 \cdot (-\frac{2}{3}) = -\frac{5}{3}$.
12th term $= \frac{7}{3} + 11 \cdot (-\frac{2}{3})$
 $= -\frac{15}{3} = -5$.
6. 5th term $= 1 + 4a$.
20th term $= 1 + 19a$.
7. $l = 7\sqrt{2} + 9 \cdot (-2\sqrt{2})$.
 $l = -11\sqrt{2}$.
8. $l = \frac{\sqrt{3}+5}{2} + 8 \cdot \left(\frac{-\sqrt{3}-1}{2}\right)$.
 $l = \frac{-7\sqrt{3}-3}{2}$.
9. $l = a + (n-2)d$.
10. $l = a + (n-3)d$.
11. $l = \sqrt{5}-1 + (n-4) \cdot (\sqrt{5}-1)$.
 $l = (\sqrt{5}-1)(n-3)$.
12. $l = \frac{1}{n} + (n-1) \left(\frac{n-2}{n}\right)$.
 $l = \frac{n^2-3n+3}{n}$.
13. 3d term $= h + 2(k-h)$
 $= 2k-h$.
nth term $= h + (n-1)(k-h)$.
14. nth term $= h + (n-1) \frac{(k-h)}{2}$.
15. 10th term $= 16 + 9 \cdot 32$
 $= 304$.
nth term $= 16 + (n-1) \cdot 32$
 $= 32n-16$.

Page 410

1. $15 = 3 + 2d$.
 $d = 6$.
 $\therefore$ progression is:
3, 9, 15.
2. $4k = h + 2d$.
 $d = \frac{4k-h}{2}$.
- $\therefore$ progression is:
2, 7, 12, 17.
3. $17 = 2 + 3d$.
 $d = 5$.
 $\therefore$ progression is:
2, 7, 12, 17.

4. $b = a + 3d.$

$$d = \frac{b - a}{3}.$$

$\therefore$ progression is:

$$a, \frac{b + 2a}{3}, \frac{2b + a}{3}, b.$$

5. $16 = -4 + 4d.$

$$d = 5.$$

$\therefore$ progression is:

$$-4, 1, 6, 11, 16.$$

6. $n = m + 4d.$

$$d = \frac{n - m}{4}.$$

$\therefore$ progression is:

$$m, \frac{3m + n}{4}, \frac{m + n}{2}, \frac{m + 3n}{4}, n.$$

7. $45 = 3 + 7d.$

$$d = 6.$$

$\therefore$ progression is:

$$3, 9, 15, 21, 27, 33, 39, 45.$$

8. $\frac{14}{3} = 3 + 10d.$

$$d = \frac{1}{6}.$$

12. $\sqrt{2}(1 - 2\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2} - 1} + 3d.$

$$\sqrt{2} - 4 = \sqrt{4} + \sqrt{2} + 3d.$$

$\therefore$ progression is:

$$\frac{\sqrt{2}}{\sqrt{2} - 1}, \frac{2 - \sqrt{2}}{\sqrt{2} - 1}, \frac{4 - 3\sqrt{2}}{\sqrt{2} - 1}, \sqrt{2}(1 - 2\sqrt{2}).$$

13. One half their sum.

14. Average velocity = $\frac{20 + 30}{2} = 25.$

$$\text{Time} = \frac{1 \text{ hour}}{25} = 2\frac{2}{5} \text{ minutes.}$$

$\therefore$ progression is:

$$3, \frac{19}{6}, \frac{10}{3}, \frac{7}{2}, \frac{11}{3}, \frac{23}{6},$$

$$4, \frac{25}{6}, \frac{13}{3}, \frac{9}{2}, \frac{14}{3}.$$

9. $9\sqrt{2} = -\sqrt{2} + 5d.$

$$d = 2\sqrt{2}.$$

$\therefore$ progression is:

$$-\sqrt{2}, \sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}.$$

10. $13x + 9a = 7x - 3a + 6d.$

$$6d = 6x + 12a.$$

$$d = x + 2a.$$

$\therefore$ progression is:

$$7x - 3a, 8x - a, 9x + a, 10x + 3a, 11x + 5a, 12x + 7a, 13x + 9a.$$

11. $\frac{15\sqrt{5}}{2} = \frac{5}{2\sqrt{5}} + 7d.$

$$d = \sqrt{5}.$$

$\therefore$ progression is:

$$\frac{5}{2\sqrt{5}}, \frac{15}{2\sqrt{5}}, \frac{5\sqrt{5}}{2},$$

$$\frac{7\sqrt{5}}{2}, \frac{9\sqrt{5}}{2}, \frac{11\sqrt{5}}{2},$$

$$\frac{13\sqrt{5}}{2}, \frac{15\sqrt{5}}{2}.$$

15. (a) Average velocity $= \frac{64 + 96}{2} = 80$.
 (b) Distance $= 80$.
16. Average velocity during first second $= \frac{0 + 32}{2} = 16$.
 Distance the first second $= 16$.
 Average velocity the second second $= \frac{32 + 64}{2} = 48$.
 Distance the second second $= 48$.
17. $l = 2 + 24 \cdot 2$.
 $l = 50$.
 Average $= \frac{2 + 50}{2} = \frac{4 + 48}{2}$, etc. $= 26$.
18. $l = 11 + 16 \cdot 2$.
 $l = 43$.
 Mean $= \frac{11 + 43}{2} = \frac{13 + 41}{2}$, etc. $= 27$.
19. 10th term $= 32 + 9 \cdot 32 = 320$.
 Average velocity $= \frac{0 + 320}{2} = 160$.
20. $22 = 1 + 7d$.
 $d = 3$.
 The distances are 1, 4, 7, 10, 13, 16, 19, 22.

Page 412

1. $s = \frac{1}{2} (4 + 9 \cdot 3) = 155$.
 2. $s = \frac{1}{2} [20 + 17(-2)] = -126$.
 3. $s = \frac{1}{2} (6 + 9 \cdot 1\frac{1}{3}) = 90$.
 4. $s = 6 [36 + 11 \cdot (-3\frac{1}{2})] = -15$.
 5. $s = 50 (1 + 100) = 5050$.
 6. $s = 50 (4 + 99 \cdot 2) = 10,100$.
 7. $s = 50 (2 + 99 \cdot 2) = 10,000$.
 8. $s = \frac{1}{2} (188 + 432) = 38,130$.
 9. $s = \frac{n}{2} (2 + [n - 1] 2) = n^2$.
 10. $s = \frac{n}{2} (4 + [n - 1] 2) = n^2 + n$.
 11. $903 = \frac{n}{2} (2 + [n - 1] 1)$.
 $1806 = n^2 + n$.
 $(n + 43)(n - 42) = 0$.
 $n = 42$.
12. $275 = \frac{n}{2} (10 + [n - 1] 4)$.
 $275 = 2n^2 + 3n$.
 $(2n + 25)(n - 11) = 0$.
 $n = 11$.
13. $510 = \frac{n}{2} (160 + [n - 1] 2)$.
 $510 = 79n + n^2$.
 $(n - 6)(n + 85) = 0$.
 $n = 6$.
14. Let $-7 = 1\text{st term}$.
 Then $18 = 6\text{th term}$.
 $18 = -7 + 5d$.
 $d = 5$.
 $-7 = a + 5$.
 $a = -12$.
 11th term $= -12 + 10 \cdot 5$
 $= 38$.

$$\begin{aligned}
 15. \quad s &= \frac{t}{2} \left(\frac{2}{t} + [t-1] \cdot \frac{t-2}{t} \right) \\
 &= 1 + (t-1) \cdot \frac{t-2}{2} \\
 &= \frac{t^2 - 3t + 4}{2}.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad l &= a + (n-1)d. \\
 29 &= 2 + (n-1)3. \\
 9 &= n-1. \\
 n &= 10. \\
 s &= 5(4 + 9 \cdot 3) \\
 &= 155.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad s &= \frac{n}{2} (2a + [n-1]d). \\
 300 &= \frac{n}{2} (6 + [n-1]4) \\
 &= 2n^2 + n. \\
 (n-12)(2n+25) &= 0. \\
 n &= 12. \\
 l &= a + (n-1)d. \\
 l &= 3 + 11 \cdot 4 \\
 &= 47.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad s &= \frac{n}{2} (2a + [n-1]d). \\
 0 &= \frac{13}{2} (2a + 12 \cdot [-11]) \\
 &= 13(a-66). \\
 a &= 66. \\
 l &= 66 + 12 \cdot (-11) \\
 &= -66.
 \end{aligned}$$

$$19. \quad s = \frac{n}{2} (2h + [n-1][k-h]).$$

$$\begin{aligned}
 20. \quad s &= \frac{n}{2} (a + l). \\
 -33 &= \frac{11}{2} (5x + 2 + l). \\
 -6 &= 5x + 2 + l. \\
 l &= -5x - 8. \\
 l &= a + (n-1)d. \\
 -5x - 8 &= 5x + 2 + 10d. \\
 10d &= -10x - 10. \\
 d &= -x - 1.
 \end{aligned}$$

$$21. \quad s = \frac{n}{2} (2a + [n-1]d).$$

$$40\sqrt{2} = \frac{n}{2} (-10\sqrt{2} + [n-1]2\sqrt{2}).$$

$$40\sqrt{2} = \frac{n}{2} (2n\sqrt{2} - 12\sqrt{2}).$$

$$40\sqrt{2} = n^2\sqrt{2} - 6n\sqrt{2}.$$

$$n^2 - 6n - 40 = 0.$$

$$(n-10)(n+4) = 0.$$

$$n = 10.$$

$$l = -5\sqrt{2} + 9 \cdot 2\sqrt{2}.$$

$$l = 13\sqrt{2}.$$

$$22. \quad s = \frac{12}{2} (1 + 12) = 78.$$

$$2 \cdot 78 = 156.$$

$$23. \quad 30 \text{ miles per hour} = 44 \text{ feet per second.}$$

$$l = a + [n-1]d.$$

$$0 = 44 + [n-1][-2].$$

$$46 = 2n.$$

$$n = 23.$$

$$s = \frac{23}{2} (44 + 0).$$

$$s = 506.$$

$$24. \quad l = a + (n-1)d.$$

$$l = 4 + 20 \cdot 8.$$

$$l = 164.$$

$$s = \frac{21}{2} (4 + 164).$$

$$s = 21 \cdot 84.$$

$$s = 1764.$$

$$25. \quad l = a + (n-1)d.$$

$$0 = \frac{39}{2} + (n-1)(-\frac{1}{2}).$$

$$\frac{39}{2} = \frac{1}{2}(n-1).$$

$$n = 40.$$

$$s = 20(2 \cdot 39 + 39[-1]) = 780.$$

$$780 + 20 = 800.$$

$$26. \quad 200 + 10(2 + d) + (2 + 2d)$$

$$= 17\frac{1}{3}(2 + [2 + d] + [2 + 2d]).$$

$$222 + 12d = \frac{86}{5}(6 + 3d).$$

$$1110 + 60d = 516 + 258d.$$

$$594 = 198d.$$

$$d = 3.$$

The number is 258.

27. $l = 75 \cdot 12 + 10 \cdot 50$
 $= 900 + 500 = \$1400$, salary
 for 11th year.
 $s = \frac{900 + 1400}{2} \cdot 11 = \$12,650$.
28. $s = \frac{n}{2}(a + l) = 6(12 + 1) = 78$.
 Interest $= \$0.03 \cdot 50 \cdot 78 = \$117$.
 $12 \cdot 50 = 600 = \text{sum of payments}$.
 $\$600 + \$117 = \$717$.
29. $555 = \frac{n}{2}(2 \cdot 16 + [n - 1] 32)$.
 $555 = 16 n^2$.
 $n^2 = 34.6875$.
 $n = 5.889$ or 5.9 seconds.
 $l = 16 + 4.9 \cdot 32$
 $= 172.8$.
30. $256 = \frac{n}{2}(2 \cdot 16 + [n - 1] 32)$
 $= 16 n^2$.
 $n^2 = 16$.
 $n = 4$ seconds.
 Velocity $= 4 \cdot 32 = 128$ feet
 per second.
31. Time of rising equals time of
 falling.
 $\therefore s = \frac{7}{4}(2 \cdot 16 + [\frac{7}{2} - 1] 32)$.
 $s = \frac{7}{4} \cdot 112$.
 $s = 196$, height.
 Velocity $= \frac{7}{2} \cdot 32 = 112$ feet per
 second.

32. $s_0 = \frac{1}{2}^0(2 \cdot 1 + 9 \cdot 1)$.
 $s_0 = 55$.
 $s_1 = \frac{9}{2}(2 \cdot 1 + 8 \cdot 1)$.
 $s_1 = 45$.
 $s_2 = \frac{8}{2}(2 + 7) = 36$.
 $s_3 = \frac{7}{2}(2 + 6) = 28$.
 $s_4 = \frac{6}{2}(2 + 5) = 21$.
 $s_5 = \frac{5}{2}(2 + 4) = 15$.
 $s_6 = \frac{4}{2}(2 + 3) = 10$.
 $s_7 = \frac{3}{2}(2 + 2) = 6$.
 $s_8 = \frac{2}{2}(2 + 1) = 3$.
 $s_9 = 1$.
 $s = 220$.

33. $s = \frac{n}{2}(2a + [n - 1]d)$.
 $\frac{n}{2}(14 + [n - 1]\frac{1}{2}) = 12n$.
 $14n + \frac{n^2}{2} - \frac{n}{2} = 24n$.
 $n^2 - 21n = 0$.
 $n(n - 21) = 0$.
 $n = 21$.

34. $\frac{n}{2}[2 \cdot 2 + (n - 1)2] = 28(n - 5)$.
 $\frac{n}{2}(2n + 2) = 28n - 140$.
 $n^2 + n = 28n - 140$.
 $n^2 - 27n + 140 = 0$.
 $(n - 7)(n - 20) = 0$.
 $n = 7$ or 20 .

Seven days after A starts, and
 twenty days after A starts.

Page 415 (First set)

1. Geometrical progression ; $r = 3$.
3. Geometrical progression ; $r = -\frac{1}{6}$.
5. Geometrical progression ; $r = -\frac{1}{\sqrt{2}}$.
6. Geometrical progression ; $r = \frac{1}{2}$.
8. Geometrical progression ; $r = 3$.
10. Geometrical progression ; $r = 5a$.
12. Geometrical progression ; $r = 2y$.
13. Geometrical progression ; $r = 4x^{-\frac{1}{2}}y^{\frac{1}{2}}$.
14. When $\frac{a}{b} = \frac{b}{c}$.

Page 415 (Second set)

$$2. \quad a = 3, r = 2, \\ t_{10} = 3 \cdot 2^9 = 1536.$$

$$3. \quad a = 2, r = \frac{3}{2}, \\ t_8 = 2 \cdot \left(\frac{3}{2}\right)^7 = \frac{2187}{64}.$$

$$4. \quad a = 5, r = -2, \\ t_{12} = 5 \cdot (-2)^{11} = -10,240.$$

$$5. \quad a = \$100, r = \frac{53}{50} = 1.06, \\ t_6 = \$100 \cdot (1.06)^5 = \$133.82.$$

$$6. \quad a = 18, r = -\frac{1}{3}, \\ t_9 = 18 \cdot \left(-\frac{1}{3}\right)^8 = \frac{2}{729}.$$

$$7. \quad a = 12a, r = \frac{3}{4}, \\ t_{10} = 12a \cdot \left(\frac{3}{4}\right)^9 = \frac{59049a}{65536}.$$

$$8. \quad a = \frac{-2c}{3}, \\ r = \frac{3}{2c}, \\ t_7 = \frac{-2c}{3} \left(\frac{3}{2c}\right)^6 = -\frac{243}{32c^5}.$$

$$9. \quad a = 4\sqrt{2}, \\ r = \frac{1}{\sqrt{2}}, \\ t_7 = 4\sqrt{2} \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{\sqrt{2}}{2}.$$

$$10. \quad a = \frac{1}{2\sqrt{2}}, \\ r = \frac{\sqrt{2}}{3},$$

$$t_6 = \frac{1}{2\sqrt{2}} \left(\frac{\sqrt{2}}{3}\right)^5 = \frac{2}{243}.$$

$$11. \quad a = \frac{3}{2\sqrt{2}}, \\ r = \frac{2\sqrt{2}}{3},$$

$$t_8 = \frac{3}{2\sqrt{2}} \left(\frac{2\sqrt{2}}{3}\right)^7 = \frac{512}{729}.$$

$$12. \quad \frac{ar^{m-1}}{r} = ar^{m-2}.$$

$$\frac{ar^{m-1}}{r^2} = ar^{m-3}.$$

$$\frac{ar^{m-1}}{r^3} = ar^{m-4}.$$

$$(ar^{m-1})r = ar^m.$$

$$(ar^{m-1})r^2 = ar^{m+1}.$$

$$13. \quad t_1 = h, t_2 = k, r = \frac{k}{h}.$$

$$t_3 = h\left(\frac{k}{h}\right)^2 = \frac{k^2}{h}.$$

$$t_4 = h\left(\frac{k}{h}\right)^3 = \frac{k^3}{h^2}.$$

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$$2. \quad a = 6, n = 4, t_n = t_4 = 48. \\ t_n = ar^{n-1}. \quad 48 = 6r^3. \\ r^3 = 8. \quad r = 2.$$

The progression is:

$$6, 12, 24, 48.$$

$$3. \quad a = 6, n = 5, t_5 = 486. \\ 486 = 6r^4. \\ r^4 = 81. \\ r = \pm 3.$$

The progression is:

$$6, \pm 18, 54, \pm 162, 486.$$

$$4. \quad a = 4, n = 3, t_3 = 9. \\ 9 = 4r^2. \\ r = \pm \frac{3}{2}.$$

The progression is:

$$4, \pm 6, 9.$$

$$5. \quad a = a^{10}, n = 3, t_3 = a^{20}. \\ a^{20} = a^{10}r^2. \\ r = \pm a^5.$$

The progression is:

$$a^{10}, \pm a^{15}, a^{20}.$$

$$\begin{aligned}
 6. \quad a &= -144, n = 5, t_5 = -9. \\
 -9 &= -144r^4. \\
 r^4 &= \frac{1}{16}. \\
 r &= \pm \frac{1}{2}.
 \end{aligned}$$

The progression is :

$$-144, \pm 72, -36, \pm 18, -9.$$

$$7. \quad t_5 = 32, t_9 = 512.$$

$$\text{Let } 32 = t.$$

$$\text{Then } 512 = t_5.$$

$$512 = 32r^4.$$

$$r = 2.$$

$$t_{11} = 2 \cdot 2^{10} = 2048.$$

$$8. \quad t_2 = 3\sqrt{2}, t_5 = \frac{3}{16}.$$

$$t_5 = t_2 r^3.$$

$$\frac{3}{16} = 3\sqrt{2}r^3.$$

$$r^3 = \frac{1}{16\sqrt{2}} = \frac{1}{2^{\frac{9}{2}}}.$$

$$r = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{2\sqrt{2}}.$$

$$a = \frac{3\sqrt{2}}{1} = 12.$$

$$9. \quad a = h, n = 3, t_3 = k.$$

$$k = hr^2.$$

$$r = \pm \sqrt{\frac{k}{h}} = \pm \frac{1}{h} \sqrt{hk}.$$

Geometrical mean is:

$$h\left(\pm \frac{\sqrt{hk}}{h}\right) = \pm \sqrt{hk}.$$

$$10. \quad a = h, n = 4, t_4 = k.$$

$$k = hr^3.$$

$$r = \sqrt[3]{\frac{k}{h}} = \frac{1}{h} \sqrt[3]{h^2 k}.$$

$$2\text{d term} = \sqrt[3]{h^2 k}.$$

$$3\text{d term} = \frac{\sqrt[3]{h^4 k^2}}{h} = \sqrt[3]{hk^2}.$$

$$11. \quad a = a, n = 5, t_5 = c.$$

$$c = ar^4.$$

$$r = \pm \sqrt[4]{\frac{c}{a}}.$$

$$t_2 = \pm a \sqrt[4]{\frac{c}{a}} = \pm \sqrt[4]{a^3 c}.$$

$$t_3 = \sqrt[4]{a^2 c^2} = \sqrt{ac}.$$

$$t_4 = \pm \sqrt[4]{ac^3}.$$

$$a, \pm \sqrt[4]{a^3 c}, \sqrt{ac}, \pm \sqrt{ac^3}, c.$$

$$12. \quad a + ar^2 = 13. \quad (1)$$

$$ar = 6. \quad (2)$$

$$(1) \div (2), \quad \frac{1 + r^2}{r} = \frac{13}{6}. \quad (3)$$

From (3),

$$6r^2 = 13r + 6 = 0.$$

$$\text{Whence } r = \frac{3}{2} \text{ or } \frac{2}{3}.$$

Thus from (2), $a = 4, 9.$

$$1\text{st term} = 4, 9.$$

$$2\text{d term} = 6, 6.$$

$$3\text{d term} = 9, 4.$$

$$13. \quad a = 4, n = 3, t_3 = 9.$$

$$(a) \quad 9 = 4r^2.$$

$$r = \frac{3}{2}.$$

$$t_2 = 6.$$

$$\therefore AD = 6.$$

$$(b) \text{ Let } BD = x,$$

$$\text{and } DC = 26 - x.$$

$$\text{Then } \frac{x}{12} = \frac{12}{26 - x}. \quad (1)$$

$$\text{Solving (1), } x = 8 \text{ or } 18.$$

$$BD = 8, 18.$$

$$CD = 18, 8.$$

$$14. \quad a = 16, n = 3, t_3 = 9.$$

$$(a) \quad 9 = 16r^2.$$

$$r = \frac{3}{4}.$$

$$t_2 = 12.$$

$$AB = 12.$$

$$(b) \text{ Let } AC = x,$$

$$\text{and } AD = 24 + x.$$

$$\text{Then } x(24 + x) = (16)^2. \quad (1)$$

$$\text{Solving (1), } x = 8, -32.$$

The -32 is rejected.

$$AC = 8.$$

$$AD = 32.$$

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$$2. S_n = \frac{a - ar^n}{1 - r}.$$

$$a = 1, r = 5, n = 7.$$

$$S_7 = \frac{1 - 5^7}{1 - 5} = \frac{-78,124}{-4} = 19,531.$$

$$3. a = -2, r = -2, n = 7.$$

$$S = \frac{-2 + 2(-2)^7}{1 + 2} = \frac{-258}{3} = -86.$$

$$4. a = 50, r = \frac{1}{5}, n = 8.$$

$$S = \frac{50 - 50(\frac{1}{5})^8}{1 - \frac{1}{5}} = \frac{50 - \frac{2}{15625}}{\frac{4}{5}} = \frac{125}{2} - \frac{1}{6250}$$

$$= \frac{390624}{6250} = 62.49984.$$

$$5. a = 180, r = -\frac{1}{2}, n = 9.$$

$$S_9 = \frac{180 - 180 \cdot (-\frac{1}{2})^9}{1 + \frac{1}{2}} = \frac{180 + \frac{45}{128}}{\frac{3}{2}} = 120\frac{5}{4}.$$

$$6. a = \frac{2}{5}, r = \frac{5}{2}, n = 5.$$

$$S = \frac{\frac{2}{5} - \frac{2}{5}(\frac{5}{2})^5}{1 - \frac{5}{2}} = \frac{\frac{2}{5} - \frac{625}{16}}{-\frac{3}{2}} = \frac{-\frac{3093}{80}}{-\frac{3}{2}} = \frac{1031}{40} = 25.775.$$

$$7. a = c^3, r = c^2, n = 7.$$

$$S = \frac{c^3 - c^3(c^2)^7}{1 - c^2} = \frac{c^3 - c^{17}}{1 - c^2}.$$

$$8. a = 3\sqrt{2}, r = \frac{2}{\sqrt{2}}, n = 5.$$

$$S = \frac{3\sqrt{2} - 3\sqrt{2}(\sqrt{2})^5}{1 - \sqrt{2}} = 21\sqrt{2} + 18.$$

$$9. a = 81, r = -\frac{1}{\sqrt{3}}, n = 6.$$

$$S = \frac{81 - 81\left(-\frac{1}{\sqrt{3}}\right)^6}{1 + \frac{1}{\sqrt{3}}} = \frac{81 - 3}{\frac{3 + \sqrt{3}}{3}} = \frac{234}{3 + \sqrt{3}} = 39(3 - \sqrt{3}).$$

$$10. a = 3, r = 5.$$

$$S = \frac{3 - 3(5)^n}{1 - 5} = -\frac{3}{4}(1 - 5^n) = \frac{3}{4}(5^n - 1).$$

$$11. a = 2x, r = 2x^3, n = n - 2.$$

$$S = \frac{2x - 2x(2x^3)^{n-2}}{1 - 2x^3} = \frac{2x[1 - (2x^3)^{n-2}]}{1 - 2x^3}.$$

$$12. S = \frac{a - ar^n}{1 - r} = \frac{a - ar^{n-1} \cdot r}{1 - r} = \frac{a - rl}{1 - r}.$$

$$13. l = \$100 (1.04)^3 = \$112.486.$$

$$l = \$100 (1.02)^6 = 112.616.$$

$$14. l = ar^{n-1} = 40 \left(\frac{2}{5}\right)^5 = \frac{2^5 5^6}{6^2 2^5} = .4096.$$

$$S = \frac{32 - 32 \left(\frac{2}{5}\right)^5}{1 - \frac{2}{5}} = \frac{\frac{98976}{3125}}{\frac{3}{5}} = \frac{32992}{625} = 52.7872.$$

$$\text{Total distance} = 40 + 52.7872 = 92.7872 \text{ feet.}$$

$$15. \text{Amount remaining} = l = c \left(\frac{2}{3}\right)^6 = \frac{64}{729} c, \text{ where } c \text{ equals the number of gallons at first.}$$

$$16. l = 400 \left(\frac{9}{10}\right)^{10} = \frac{3486784401}{10^{10}} \cdot 400.$$

Then .3486784401 of original contents remain.

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$$2. a = 1, r = \frac{1}{2}.$$

$$S_{\infty} = \frac{a}{1 - r} = \frac{1}{\frac{1}{2}} = 2.$$

$$3. a = 2, r = -\frac{1}{2}.$$

$$S_{\infty} = \frac{2}{1 + \frac{1}{2}} = \frac{4}{3}.$$

$$4. a = 3, r = -\frac{1}{3}.$$

$$S_{\infty} = \frac{3}{1 + \frac{1}{3}} = \frac{9}{4}.$$

$$5. a = 2, r = \frac{1}{\sqrt{2}}.$$

$$S_{\infty} = \frac{2}{1 - \frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2} - 1}$$

$$= 4 + 2\sqrt{2}.$$

$$6. a = 5a, r = \frac{1}{4}.$$

$$S_{\infty} = \frac{5a}{1 - \frac{1}{4}} = \frac{20a}{3} = 6\frac{2}{3}a.$$

$$7. a = 1, r = x.$$

$$S_{\infty} = \frac{1}{1 - x}.$$

$$8. a = 3, r = \frac{1}{\sqrt{3}}.$$

$$S_{\infty} = \frac{3}{1 - \frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{9 + 3\sqrt{3}}{2}.$$

$$9. a = 1, r = \frac{1}{x}. \quad (x > 1.)$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{x}} = \frac{x}{x - 1}.$$

$$10. a = .51, r = .01.$$

$$S_{\infty} = \frac{.51}{1 - .01} = \frac{51}{99} = \frac{17}{33}.$$

$$11. a = .6, r = .1.$$

$$S_{\infty} = \frac{.6}{1 - .1} = \frac{6}{9} = \frac{2}{3}.$$

$$12. a = .27, r = .01.$$

$$S_{\infty} = \frac{.27}{1 - .01} = \frac{27}{99} = \frac{3}{11}.$$

$$13. a = .39, r = .01.$$

$$S_{\infty} = \frac{.39}{1 - .01} = \frac{39}{99} = \frac{13}{33}.$$

$$14. 25.36\dot{3}\dot{6}.$$

$$S_{\infty} = 25 + \frac{.36}{1 - .01} = 25\frac{4}{11}.$$

15. .72121.

$$S_{\infty} = .7 + \frac{.021}{1 - .01} = \frac{714}{990} = \frac{119}{165}.$$

16. .3091091.

$$S_{\infty} = .3 + \frac{.0091}{1 - .001} = \frac{3088}{9990}$$

$$= \frac{1}{4} \frac{544}{995}.$$

17. $a = 5 \cdot 80.$

$$S_{\infty} = \frac{5 \cdot 80}{1 - .99} = \frac{400}{.01} = 40,000 \text{ ft.}$$

18. $a = \frac{9}{4} \sqrt{3}, r = \frac{1}{4}.$

$$S_{\infty} = \frac{\frac{9}{4} \sqrt{3}}{1 - \frac{1}{4}} = 3 \sqrt{3}.$$

19. (a) $a = 8, r = \frac{1}{\sqrt{2}}.$

$$S_{\infty} = \frac{8}{1 - \frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2} - 1}$$

$$= \frac{8\sqrt{2}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$

$$= 16 + 8\sqrt{2}.$$

(b) $a = 4, r = \frac{1}{2}.$

$$S_{\infty} = \frac{4}{1 - \frac{1}{2}} = 8.$$

21. $p = \frac{Sr(r+1)^n}{(r+1)^n - 1}$

$$p = \frac{\$1000 \times .05(1.05)^3}{(1.05)^3 - 1}$$

$$= \frac{57.88125}{.157625} = \$367.21.$$

22. $p = \frac{Sr(1+r)^n}{(1+r)^n - 1}$

$$= \frac{\$5000 \times .06(1.06)^5}{(1.06)^5 - 1}$$

$$= \$1186.98.$$

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1. $x - y = 1,$

$2y - 2x = -2.$

$$x = \frac{\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix}} = \frac{2 - 2}{2 - 2} = \frac{0}{0}.$$

The system is indeterminate. The graphs of the two equations are identical. Values for any point on either graph will satisfy both equations.

2. $x - y = 0,$

$y - x = 3.$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{1 + 3}{1 - 1} = \frac{4}{0}.$$

The system is inconsistent. The equations are incompatible and the graphs of the two equations are parallel lines.

3. $x + y + z = 0,$

$x - 2y + 3z = 1,$

$2x - y + 4z = 1.$

$$x = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & -1 & 4 \end{vmatrix}} = \frac{-1 + 3 + 2 - 4}{-8 - 1 + 6 + 4 + 3 - 4} = \frac{0}{0}.$$

The system is indeterminate. An infinite number of sets of values will satisfy the system.

$$\begin{aligned} 4. \quad & x + y + z = 1, \\ & x - y - 2z = 2, \\ & 0x + 0y + 0z = 0. \end{aligned}$$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -2 \\ 0 & 0 & 0 \end{vmatrix}} = \frac{0}{0}.$$

The system is indeterminate. An infinite number of sets of values will satisfy the system.

5. The conclusion is that there is an infinite number of values for a system of two linear equations in three variables.

$$\begin{aligned} 6. \quad & x + y + z = 2, \\ & 0x + 0y + 0z = 0, \\ & 0x + 0y + 0z = 0. \end{aligned}$$

$$x = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}} = \frac{0}{0}.$$

The system is indeterminate.

7. Results show that there is an infinite number of sets of roots for one equation in three variables.

$$\begin{array}{lllllll} 8. \ 0. & 10. \ 0. & 12. \ 1. & 14. \ \infty. & 16. \ 1. & 18. \ 0. & 20. \ 0. \\ 9. \ 0. & 11. \ 1. & 13. \ 1. & 15. \ 1. & 17. \ \infty. & 19. \ 0. & 21. \ \infty. \end{array}$$

$$22. \quad \frac{1-x}{1-x^2} = \frac{1}{1+x} \doteq \frac{1}{2} \text{ as } x \doteq 1.$$

$$23. \quad \frac{x^2-5x+6}{x^2-4} = \frac{x-3}{x+2} \doteq -\frac{1}{4} \text{ as } x \doteq 2.$$

$$24. \quad \frac{x-2}{x^3-8} = \frac{1}{x^2+2x+4} = \frac{1}{12} \text{ as } x \doteq 2.$$

Page 434

$$1. \log 2 = .3.$$

$$2. \log 3 = .47.$$

$$3. \log 4 = .6.$$

$$4. \log 5 = .7.$$

$$5. \log 6.2 = .79.$$

$$6. \log 7.4 = .86.$$

$$7. \log 9.6 = .98.$$

$$8. \log 11 = 1.04.$$

$$9. \log 15 = 1.17.$$

$$10. \log .2 = -.7.$$

$$11. \log .5 = -.3.$$

$$12. \log .9 = -.04.$$

$$13. \log 25 = 1.4.$$

$$14. \log 50 = 1.7.$$

$$15. \log 64 = 1.81.$$

$$16. \log 96 = 1.98.$$

- | | |
|------------------------------|-------------------------------|
| 17. $\log 100 = 2.$ | 27. $\log n = - .4, n = .4.$ |
| 18. $\log 10 = 1.$ | 28. $\log n = - .5, n = .3.$ |
| 19. $\log 1 = 0.$ | 29. $\log n = - .25, n = .6.$ |
| 20. $\log 32 = 1.5.$ | 30. $\log n = 1.3, n = 20.$ |
| 21. $\log n = .3, n = 2.$ | 31. $\log n = 1.7, n = 50.$ |
| 22. $\log n = .4, n = 2.5.$ | 32. $\log n = 1.6, n = 40.$ |
| 23. $\log n = .6, n = 4.$ | 33. $\log n = 1.8, n = 63.$ |
| 24. $\log n = .7, n = 5.$ | 34. $\log n = 2, n = 100.$ |
| 25. $\log n = .95, n = 9.$ | 35. $\log n = .84, n = 7.$ |
| 26. $\log n = - .1, n = .8.$ | 36. $\log n = 1.2, n = 16.$ |

Page 435 (First set)

The exercises on pages 435-436 are here solved by means of the table on page 448. The pupil should solve them, however, by means of the curve on page 433.

- | | | | |
|----|---|-----|---|
| 1. | $\log 2 = .3010$
$\log 5 = \underline{.6990}$
1.0000
$\log n = 1, n = 10.$ | 7. | $\log 80 = 1.9031$
$\log .5 = \underline{1.6990}$
$\log n = 1.6021$
$n = 40.$ |
| 2. | $\log 3 = .4771$
$\log n = \underline{\frac{2}{.9542}}$
$n = 9.$ | 8. | $\log 22 = 1.3424$
$\log 4.8 = \underline{.6812}$
$\log n = 2.0236$
$n = 105.6.$ |
| 3. | $\log 4 = .6021$
$\log n = \underline{\frac{2}{1.2042}}$
$n = 16.$ | 9. | $\log 14 = 1.1461$
$\log 6 = \underline{.7782}$
$\log n = 1.9243$
$n = 84.$ |
| 4. | $\log 8 = .9031$
$\log n = \underline{\frac{2}{1.8062}}$
$n = 64.$ | 10. | $\log 80 = 1.9031$
$\log .6 = \underline{1.7782}$
$\log n = 1.6813$
$n = 48.$ |
| 5. | $\log 30 = 1.4771$
$\log 2 = \underline{.3010}$
$\log n = 1.7781$
$n = 60.$ | 11. | $\log 97 = 1.9868$
$\log .7 = \underline{1.8451}$
$\log n = 1.8319$
$n = 67.9.$ |
| 6. | $\log 25 = 1.3979$
$\log 4 = \underline{.6021}$
$\log n = 2.0000$
$n = 100.$ | 12. | $\log 7.2 = .8573$
$\log 1.5 = \underline{.1761}$
$\log n = 1.0334$
$n = 10.8.$ |

Page 435 (Second set)

- | | |
|---|---|
| <p>1. $\log 8 = .9031$
 $\log 2 = \underline{.3010}$
 $\log n = .6021$
 $n = 4.$</p> | <p>6. $\log 48 = 1.6812$
 $\log 6 = \underline{.7782}$
 $\log n = .9030$
 $n = 8.$</p> |
| <p>2. $\log 6 = .7782$
 $\log 3 = \underline{.4771}$
 $\log n = .3011$
 $n = 2.$</p> | <p>7. $\log 22 = 1.3424$
 $\log 11 = \underline{1.0414}$
 $\log n = .3010$
 $n = 2.$</p> |
| <p>3. $\log 40 = 1.6021$
 $\log 5 = \underline{.6990}$
 $\log n = .9031$
 $n = 8.$</p> | <p>8. $\log 56 = 1.7482$
 $\log 8 = \underline{.9031}$
 $\log n = .8451$
 $n = 7.$</p> |
| <p>4. $\log 18 = 1.2553$
 $\log 8 = \underline{.9031}$
 $\log n = .3522$
 $n = 2.25.$</p> | <p>9. $\log 4 = .6021$
 $\log 6 = \underline{.7782}$
 $\log 24 = 1.3803$
 $\log 3 = \underline{.4771}$
 $\log n = .9032$
 $n = 8.$</p> |
| <p>5. $\log 16 = 1.2041$
 $\log 8 = \underline{.9031}$
 $\log n = .3010$
 $n = 2.$</p> | <p>10. $\log 80 = 1.9031$
 $\log 40 = \underline{1.6021}$
 $\log 40 \cdot 80 = \underline{3.5052}$
 $\log 25 = \underline{1.3979}$
 $\log n = 2.1073$
 $n = 128.$</p> |

Page 436 (First set)

- | | |
|---|--|
| <p>1. $\log 3 = .4771$
 $\log n = \underline{.9542}$
 $n = 9.$</p> | <p>3. $\log 3 = .4771$
 $\log n = \underline{1.4313}$
 $n = 27.$</p> |
| <p>2. $\log 5 = .6990$
 $\log n = \underline{1.3980}$
 $n = 25.$</p> | <p>4. $\log 4 = .6021$
 $\log n = \underline{1.8063}$
 $n = 64.$</p> |
| <p>5. $\log 5 = .6990.$
 $\log 10 = 1.$
 $\log 25 = 1.3979.$</p> | <p>$2 \log 5 = 1.3980$
 $3 \log 10 = \underline{3.}$
 4.3980
 $2 \log 25 = \underline{2.7958}$
 $\log n = 1.6022$
 $n = 40.$</p> |

$$\begin{aligned}
 6. \quad 2 \log 2 &= .3010 \times 2 = .6020 \\
 3 \log 3 &= .4771 \times 3 = \frac{1.4313}{2.0333} \\
 \log 6 &= \frac{.7782}{1.2551} \\
 \log n &= \\
 n &= 18.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 2 \log 4 &= 1.2042 \\
 3 \log 3 &= \frac{1.4313}{2.6355} \\
 \log n &= \\
 n &= 432.
 \end{aligned}$$

Page 436 (Second set)

$$1. \quad \frac{\log 3}{2} = \frac{.4771}{2} = .2385 = \log n. \text{ Whence } n = 1.732.$$

$$2. \quad \frac{\log 4}{3} = \frac{.6021}{3} = .2007 = \log n. \text{ Whence } n = 1.587.$$

$$3. \quad \frac{\log 81}{4} = \frac{1.9085}{4} = .4771 = \log n. \text{ Whence } n = 3.$$

$$4. \quad \frac{\log 6}{2} = \frac{.7782}{2} = .3891 = \log n. \text{ Whence } n = 2.45.$$

$$5. \quad \frac{\log 7}{3} = \frac{.8451}{3} = .2817 = \log n. \text{ Whence } n = 1.913.$$

$$\begin{aligned}
 6. \quad 2 \log 5 &= 1.3980 \\
 \frac{\log 3}{2} &= \frac{.2385}{1.6365} \\
 \log n &= \\
 n &= 43.3.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 3 \log 3 &= 1.4313 \\
 \frac{\log 2}{2} &= \frac{.1505}{1.5818} \\
 \log .7 &= \frac{1.8451}{1.7367} \\
 \log n &= \\
 n &= 54.53.
 \end{aligned}$$

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$$1. \log 300 = 2.47.$$

$$2. \log 65 = 1.81.$$

$$3. \log 4 = .60.$$

$$4. \log 1 = 0.$$

$$5. \log .10 = -1.$$

$$6. \log 1730 = 3.238.$$

$$7. \log 173 = 2.238.$$

$$8. \log 1.73 = .238.$$

$$9. \log .173 = \bar{1}.238.$$

$$10. \log .0173 = \bar{2}.238.$$

$$11. 3 = 10^{.47}.$$

$$12. 20 = 10^{1.301}.$$

$$13. 4.9 = 10^{.69}.$$

$$14. 490 = 10^{2.69}.$$

$$15. .0049 = 10^{-3 + .69}.$$

$$16. 381 = 10^{2.58}.$$

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$$1. \log 271 = 2.4330.$$

$$2. \log 344 = 2.5366.$$

$$3. \log 982 = 2.9921.$$

$$4. \log 65 = 1.8129.$$

$$5. \log 650 = 2.8129.$$

$$6. \log 27 = 1.4314.$$

$$7. \log 2.7 = .4314.$$

$$8. \log 2700 = 3.4314.$$

$$9. \log 3 = .4771.$$

$$10. \log 6 = .7782.$$

11. $\log 932 = 2.9694$.
 12. $\log .932 = \bar{1}.9694$.
 13. $\log .643 = \bar{1}.8082$.
 14. $\log .0532 = \bar{2}.7259$.
 15. $\log .00267 = \bar{3}.4265$.
 16. $\log .00579 = \bar{3}.7627$.
 17. $\log .0101 = \bar{2}.0043$.
 18. $\log 825,000 = 5.9165$.

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1. $\log 4630 = 3.6656$
 $\log 4620 = \frac{3.6646}{.0010}$
 $\therefore \log 4625 = 3.6651$.
 2. $\log 365 = 2.5623$
 $\log 364 = \frac{2.5611}{.0012}$
 $\therefore \log 364.7 = 2.5619$.
 3. $\log 42.8 = 1.6314$
 $\log 42.7 = \frac{1.6304}{.0010}$
 $\therefore \log 42.73 = 1.6307$.
 4. $\log 32.8 = 1.5159$
 $\log 32.7 = \frac{1.5145}{.0014}$
 $\therefore \log 32.75 = 1.5152$.
 5. $\log 547 = 2.7380$
 $\log 546 = \frac{2.7372}{.0008}$
 $\therefore \log 546.8 = 2.7378$.
 6. $\log 72.6 = 1.8609$
 $\log 72.5 = \frac{1.8603}{.0006}$
 $\therefore \log 72.543 = 1.8606$.
 7. $\log 10.2 = 1.0086$
 $\log 10.1 = \frac{1.0043}{.0043}$
 $\therefore \log 10.101 = 1.0047$.
 8. $\log 701 = 2.8457$
 $\log 700 = \frac{2.8451}{.0006 \times .35 = .0002 +}$
 $\therefore \log 700.35 = 2.8453$.
 9. $\log 506 = 2.7042$
 $\log 505 = \frac{2.7033}{.0009}$
 $\therefore \log 505.50 = 2.7037$.
 10. $\log 2.01 = .3032$
 $\log 2.00 = \frac{.3010}{.0022 \times .75 = .00165}$
 $\therefore \log 2.0075 = .3026$.
 11. $\log .00386 = \bar{3}.5866$.
 12. $\log .000778 = \bar{4}.8910$
 $\log .000777 = \frac{\bar{4}.8904}{.0006 \times .7 = .00042}$
 $\therefore \log .0007777 = \bar{4}.8908$.
 13. $\log 3.15 = .4983$
 $\log 3.14 = \frac{.4969}{.0014 \times .16 = .000224}$
 $\therefore \log 3.1416 = .4971$.
 14. $\log 2.72 = .4346$
 $\log 2.71 = \frac{.4330}{.0016 \times .828 = .0013248}$
 $\log 2.71828 = .4343$.
 15. $\log .0235 = \bar{2}.3711$
 $\log .0234 = \frac{\bar{2}.3692}{.0019 \times .56 = .001065}$
 $\therefore \log .023456 = \bar{2}.3703$.

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1. antilog 3.9309 = 8530.
2. antilog 1.8162 = 65.5.
3. antilog .6284 = 4.25.
4. antilog $\bar{1}.3541 = .226$.
5. antilog $\bar{2}.5740 = .0375$.
6. antilog $8.5740 - 10 = .0375$.
7. antilog $9.7292 - 10 = .536$.
8. antilog $4.8136 - 10 = .00000651$.
9. antilog .4533 = 2.84.
10. antilog $\bar{4}.6345 = .000431$.
11. antilog 6.9232 = 8,380,000.
12. antilog 8.2148 = 164,000,000.
13. antilog $5.7832 - 6 = .607$.
14. antilog $\bar{5}.9996 = .0000999$.

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1. antilog 1.5717 = 37.3.
antilog 1.5729 = 37.4.
 $\therefore$ antilog 1.5723 = 37.35.
2. antilog 2.3909 = 246.
antilog 2.3927 = 247.
 $\therefore$ antilog 2.3921 = 246.6.
3. antilog .6684 = 4.66.
antilog .6693 = 4.67.
 $\therefore$ antilog .6690 = 4.666.
4. antilog 2.5717 = 373.
antilog 2.5729 = 374.
 $\therefore$ antilog 2.5728 = 373.9.
5. antilog $\bar{1}.2577 = .181$.
antilog $\bar{1}.2601 = .182$.
 $\therefore$ antilog $\bar{1}.2586 = .1813$.
6. antilog $7.3464 - 10 = .00222$.
antilog $7.3483 - 10 = .00223$.
 $\therefore$ antilog $7.3472 - 10 = .002224$.
7. antilog $9.8525 - 10 = .712$.
antilog $9.8531 - 10 = .713$.
 $\therefore$ antilog $9.8527 - 10 = .7123$.
8. antilog $5.9614 - 8 = .00915$.
antilog $5.9619 - 8 = .00916$.
 $\therefore$ antilog $5.9616 - 8 = .009154$.
9. antilog $9.2648 - 10 = .184$.
antilog $9.2672 - 10 = .185$.
 $\therefore$ antilog $9.2654 - 10 = .1842$.
10. antilog .7825 = 6.06.
antilog .7832 = 6.07.
 $\therefore$ antilog .7829 = 6.065.
11. antilog $7.1038 - 10 = .00127$.
antilog $7.1072 - 10 = .00128$.
 $\therefore$ antilog $7.1050 - 10 = .001273$.
12. antilog $6.2304 - 10 = .000170$.
antilog $6.2330 - 10 = .000171$.
 $\therefore$ antilog $6.2308 - 10 = .0001701$.

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2. $\log 37 = 1.5682$
 $\log 23 = \underline{1.3617}$
 $\log 23 \cdot 37 = 2.9299$
antilog 2.9299 = 851.
3. $\log 28 = 1.4472$
 $\log 8 = \underline{.9031}$
 $\log 8 \cdot 28 = 2.3503$
antilog 2.3503 = 224.

4. $\log 9.8 = .9912$
 $\log 5 = \underline{.6990}$
 $\log 5 \cdot 9.8 = \underline{1.6902}$
 $\text{antilog } 1.6902 = 49.0.$
5. $\log 42 = 1.6232$
 $\log 2.2 = \underline{.3424}$
 $\log 42 \cdot 2.2 = \underline{1.9656}$
 $\text{antilog } 1.9656 = 92.38.$
6. $\log 386 = 2.5866$
 $\log 27 = \underline{1.4314}$
 $\log 27 \cdot 386 = \underline{4.0180}$
 $\text{antilog } 4.0180 = 10,420.$
7. $\log 432 = 2.6355$
 $\log 361 = \underline{2.5575}$
 $\log 432 \cdot 361 = \underline{5.1930}$
 $\text{antilog } 5.1930 = 155,900.$
8. $\log 589 = 2.7701$
 $\log 734 = \underline{2.8657}$
 $\log 589 \cdot 734 = \underline{5.6358}$
 $\text{antilog } 5.6358 = 432,300.$
9. $\log 4326 = 3.6361$
 $\log 638 = \underline{2.8048}$
 $\log 638 \cdot 4326 = \underline{6.4409}$
 $\text{antilog } 6.4409 = 2,760,000.$
10. $\log 2870 = 3.4579$
 $\log 3654 = \underline{3.5628}$
 $\log \text{product} = \underline{7.0207}$
 $\text{antilog } 7.0207 = 10,480,000.$
11. $\log 286.7 = 2.4574$
 $\log 2.341 = \underline{.3694}$
 $\log \text{product} = \underline{2.8268}$
 $\text{antilog } 2.8268 = 671.1.$
12. $\log 3.412 = .5331$
 $\log 2.596 = \underline{.4143}$
 $\log \text{product} = \underline{.9474}$
 $\text{antilog } .9474 = 8.860.$
14. $\log 385 = 2.5855$
 $\log .617 = \underline{\bar{1}.7903}$
 $\log \text{product} = \underline{2.3758}$
 $\text{antilog } 2.3758 = 237.6.$
15. $\log 541 = 2.7332$
 $\log .073 = \underline{\bar{2}.8633}$
 $\log \text{product} = \underline{1.5965}$
 $\text{antilog } 1.5965 = 39.49.$
16. $\log 37.6 = 1.5752$
 $\log .00835 = \underline{\bar{3}.9217}$
 $\log \text{product} = \underline{\bar{1}.4969}$
 $\text{antilog } \bar{1}.4969 = .314.$
17. $\log .0876 = \underline{\bar{2}.9425}$
 $\log .673 = \underline{\bar{1}.8280}$
 $\log \text{product} = \underline{\bar{2}.7705}$
 $\text{antilog } \bar{2}.7705 = .05895.$
18. $\log .07325 = \underline{\bar{2}.8648}$
 $\log 6.384 = \underline{.8051}$
 $\log \text{product} = \underline{\bar{1}.6699}$
 $\text{antilog } \bar{1}.6699 = .4676.$
19. $\log .6381 = \underline{\bar{1}.8049}$
 $\log .01897 = \underline{\bar{2}.2782}$
 $\log \text{product} = \underline{\bar{2}.0831}$
 $\text{antilog } \bar{2}.0831 = .0121.$
20. $\log 675 = 2.8293$
 $\log .0236 = \underline{\bar{2}.3729}$
 $\log \text{product} = \underline{1.2022}$
 $\text{antilog } 1.2022 = 15.92.$
21. $\log .437 = \underline{\bar{1}.6405}$
 $\log .0076 = \underline{\bar{3}.8808}$
 $\log \text{product} = \underline{\bar{3}.5213}$
 $\text{antilog } \bar{3}.5213 = .003321.$
23. $\log 96 = 1.9823$
 $\log 12 = \underline{1.0792}$
 $\log \text{quotient} = \underline{.9031}$
 $\text{antilog } .9031 = 8.$

$$\begin{aligned}
 24. \quad & \log 888 = 2.9484 \\
 & \log 37 = \underline{1.5682} \\
 & \log \text{quotient} = 1.3802 \\
 & \text{antilog } 1.3802 = 24.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \log 976 = 2.9894 \\
 & \log 321 = \underline{2.5065} \\
 & \log \text{quotient} = .4829 \\
 & \text{antilog } .4829 = 3.04.
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \log 489 = 2.6893 \\
 & \log 27.1 = \underline{1.4330} \\
 & \log \text{quotient} = 1.2563 \\
 & \text{antilog } 1.2563 = 18.04.
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \log 3460 = 3.5391 \\
 & \log 4.32 = \underline{.6355} \\
 & \log \text{quotient} = 2.9036 \\
 & \text{antilog } 2.9036 = 801.
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \log 4697 = 3.6718 \\
 & \log 281 = \underline{2.4487} \\
 & \log \text{quotient} = 1.2231 \\
 & \text{antilog } 1.2231 = 16.71.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \log 9876 = 3.9946 \\
 & \log 56.78 = \underline{1.7542} \\
 & \log \text{quotient} = 2.2404 \\
 & \text{antilog } 2.2404 = 173.9.
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \log 6432 = 3.8083 \\
 & \log 7.81 = \underline{.8927} \\
 & \log \text{quotient} = 2.9156 \\
 & \text{antilog } 2.9156 = 823.4.
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \log 2.35 = .3711 \\
 & \log .0673 = \underline{2.8280} \\
 & \log \text{quotient} = 1.5431 \\
 & \text{antilog } 1.5431 = 34.92.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \log 4.86 = .6866 \\
 & \log .721 = \underline{1.8579} \\
 & \log \text{quotient} = .8287 \\
 & \text{antilog } .8287 = 6.74.
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & \log .0635 = \bar{2}.8028 \\
 & \log .287 = \underline{1.4579} \\
 & \log \text{quotient} = \bar{1}.3449 \\
 & \text{antilog } \bar{1}.3449 = .2212.
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \log .2674 = \bar{1}.4271 \\
 & \log 3.86 = \underline{.5866} \\
 & \log \text{quotient} = \bar{2}.8405 \\
 & \text{antilog } \bar{2}.8405 = .06926.
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & \log 7635 = 3.8828 \\
 & \log 8692 = \underline{3.9391} \\
 & \log \text{quotient} = \bar{1}.9437 \\
 & \text{antilog } \bar{1}.9437 = .8784.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \log .07382 = \bar{2}.8682 \\
 & \log 68.72 = \underline{1.8371} \\
 & \log \text{quotient} = \bar{3}.0311 \\
 & \text{antilog } \bar{3}.0311 = .001074.
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & \log 256 = 2.4082 \\
 & \log 372 = \underline{2.5705} \\
 & \log \text{product} = 4.9787 \\
 & \log 128 = \underline{2.1072} \\
 & \log \text{quotient} = 2.8715 \\
 & \text{antilog } 2.8715 = 743.8.
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & \log 347 = 2.5403 \\
 & \log 625 = \underline{2.7959} \\
 & \log \text{product} = 5.3362 \\
 & \log 346 = \underline{2.5391} \\
 & \log \text{quotient} = 2.7971 \\
 & \text{antilog } 2.7971 = 626.7. \\
 & \therefore \frac{347 \cdot (-625)}{346} = -626.7.
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & \log 463.2 = 2.6658 \\
 & \log 4.78 = \underline{.6794} \\
 & \log \text{product} = 3.3452 \\
 & \log 68.3 = \underline{1.8344} \\
 & \log \text{quotient} = 1.5108 \\
 & \text{antilog } 1.5108 = 32.42. \\
 & \therefore \frac{463.2 \cdot 4.78}{-68.3} = -32.42.
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & \log 9.63 = .9836 \\
 & \log .0872 = \overline{2}.9405 \\
 & \log \text{product} = \overline{1}.9241 \\
 & \log .00635 = \overline{3}.8028 \\
 & \log \text{quotient} = 2.1213 \\
 & \text{antilog } 2.1213 = 132.2.
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & \log .078 = \overline{2}.8921 \\
 & \log 4.267 = \overline{.}6301 \\
 & \log \text{quotient} = \overline{2}.2620 \\
 & \text{antilog } \overline{2}.2620 = .01827.
 \end{aligned}$$

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$$\begin{aligned}
 2. \quad & \log 6.32 = .8007 \\
 & \log \text{product} = \overline{3}.2028 \\
 & \text{antilog } 3.2028 = 1595.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \log 34.26 = 1.5348 \\
 & \log \text{product} = \overline{3}.0696 \\
 & \text{antilog } 3.0696 = 1173.
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \log 6.715 = .8270 \\
 & \log \text{product} = \overline{2}.4810 \\
 & \text{antilog } 2.4810 = 302.7.
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \log .352 = \overline{1}.5465 \\
 & \log \text{product} = \overline{2}.1860 \\
 & \text{antilog } \overline{2}.1860 = .01534.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \log .0672 = \overline{2}.8274 \\
 & \log \text{product} = \overline{3}.6548 \\
 & \text{antilog } \overline{3}.6548 = .004516.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \log .003567 = \overline{3}.5522 \\
 & \log \text{product} = \overline{13}.7610 \\
 & \text{antilog } \overline{13}.7610 = .0000000000005767.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \log 583 = 2.7657. \\
 & \log \sqrt[3]{583} = .9219. \\
 & \text{antilog } .9219 = 8.354.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \log 1235 = 3.0916. \\
 & \log \sqrt[5]{1235} = .6183. \\
 & \text{antilog } .6183 = 4.153.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \log .0786 = \overline{2}.8954. \\
 & \log \sqrt{.0786} = \overline{1}.4477. \\
 & \text{antilog } \overline{1}.4477 = .2803.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \log .0007324 = \overline{4}.8647. \\
 & \log \sqrt[5]{.0007324} = .3729 - \overline{1}. \\
 & \text{antilog } \overline{1}.3729 = .236.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \log .002679 = \overline{3}.4279. \\
 & \log \sqrt[4]{.002679} = .3569 - 1. \\
 & \text{antilog } \overline{1}.3569 = .2274.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \log 38.4 = 1.5843 \\
 & \log \overline{38.4}^2 = \overline{3}.1686 \\
 & \log \sqrt[3]{38.4}^2 = 1.0562. \\
 & \text{antilog } 1.0562 = 11.38.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \log 4.965 = .6959 \\
 & \log \text{power} = \overline{2}.0877. \\
 & \log \sqrt[4]{4.965}^3 = 1.0439. \\
 & \text{antilog } 1.0439 = 11.06.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \log 6.387 = .8053. \\
 & \log \sqrt[3]{(6.387)^5} = \frac{5}{3} (.8053) \\
 & \quad = 1.3421. \\
 & \text{antilog } 1.3421 = 21.98. \\
 & \therefore (-6.387)^{\frac{5}{3}} = \sqrt[3]{(-6.387)^5} \\
 & \quad = -21.98.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \log 283 = 2.4518 \\
 & \log 4.627 = \underline{.6653} \\
 & \quad \quad 3.1171 \\
 & 3 \log 8.423 = \underline{2.7762} \\
 & \log \text{quotient} = \underline{.3409} \\
 & \log \text{radical} = .1705. \\
 & \text{antilog } .1705 = 1.480.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & 2 \log 23.56 = 2.7444 \\
 & \log 7.384 = \underline{.8683} \\
 & \quad \quad 3.6127 \\
 & 3 \log 4.623 = \underline{1.9947} \\
 & \log \text{quotient} = \underline{1.6180} \\
 & \log \text{radical} = .8090. \\
 & \text{antilog } .8090 = 6.441.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \log 209 = 2.3201. \\
 & \log \sqrt[11]{209} = 2109. \\
 & \text{antilog } .2109 = 1.625.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \log 163 = 2.2122. \\
 & \log \sqrt[4]{163} = \underline{.5430}. \\
 & \text{antilog } .5430 = 3.4915. \\
 & 87 - 3.4915 = 83.5085. \\
 & \log 83.5085 = 1.9217. \\
 & \log \text{radical} = .9609. \\
 & \text{antilog } .9609 = 9.14.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \log 127 = 2.1038 \\
 & \frac{1}{3} \log 872 = \underline{.9802} \\
 & \quad \quad 3.0840 \\
 & \log 67 = \underline{1.8261} \\
 & \log \sqrt[127]{872} = \underline{1.2579} \\
 & \log \sqrt[5]{} = \underline{.2516} \\
 & \log 25 = \underline{1.3979} \\
 & \quad \quad 1.6495 \\
 & \log 361 = \underline{2.5575} \\
 & \log \text{ans.} = \underline{1.0920} \\
 & \text{antilog } 1.0920 = .1236.
 \end{aligned}$$

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$$1. (a) \text{ Circumference} = 2\pi \cdot 42.$$

$$\begin{aligned}
 & \log 2 = .3010 \\
 & \log \pi = .4971 \\
 & \log 42 = \underline{1.6232} \\
 & \log \text{product} = \underline{2.4213} \\
 & \text{antilog } 2.4213 = 263.8.
 \end{aligned}$$

$$2. (a) \text{ Area} = \pi \cdot \overline{3.672}^2.$$

$$\begin{aligned}
 & \log \pi = .4971 \\
 & 2 \log 3.672 = \underline{1.1298} \\
 & \log \text{area} = \underline{1.6269} \\
 & \text{antilog } 1.6269 = 42.36.
 \end{aligned}$$

$$3. (a) \text{ Area} = 4\pi \cdot \overline{3958.79}^2.$$

$$\begin{aligned}
 & \log 4 = .6021 \\
 & \log \pi = .4971 \\
 & 2 \log \text{radius} = \underline{7.1950} \\
 & \log \text{area} = \underline{8.2942} \\
 & \text{antilog } 8.2942 = 196,800,000.
 \end{aligned}$$

$$(b) \text{ Radius} = \frac{6843}{2 \cdot \pi}.$$

$$\begin{aligned}
 & \log 6843 = 3.8353 \\
 & \log 2\pi = \underline{.7981} \\
 & \log \text{quotient} = \underline{3.0372} \\
 & \text{antilog } 3.0372 = 1089.
 \end{aligned}$$

$$(b) R = \sqrt{\frac{\text{area}}{\pi}}.$$

$$\begin{aligned}
 & \log 64.37 = 1.8087 \\
 & \log \pi = \underline{.4971} \\
 & \log \text{quotient} = \underline{1.3116} \\
 & \log \text{radical} = \underline{.6558}. \\
 & \text{antilog } .6558 = 4.527.
 \end{aligned}$$

$$(b) \text{ Equator} = 2\pi R.$$

$$\begin{aligned}
 & \log 2 = .3010 \\
 & \log \pi = .4971 \\
 & \log 3958.79 = \underline{3.5975} \\
 & \log \text{equator} = \underline{4.3956} \\
 & \text{antilog } 4.3956 = 24810 \text{ miles.}
 \end{aligned}$$

$$4. (a) 25 = \frac{4\pi R^2}{3} \therefore R = \sqrt[3]{\frac{75}{4\pi}} \quad (b) \frac{4\pi \cdot D^3}{3 \cdot 8} = 85. \quad D = \sqrt[3]{\frac{8 \cdot 85 \cdot 3}{4\pi}}$$

$$\log 4 = .6021$$

$$\log \pi = .4971$$

$$\log 4\pi = 1.0992$$

$$\log 75 = 1.8750$$

$$\log 4\pi = 1.0992$$

$$\log R^3 = .7758$$

$$\log R = .2586$$

$$\text{antilog} = 1.813 \text{ feet.}$$

$$\log 3 = .4771$$

$$\log 8 = .9031$$

$$\log 85 = 1.9294$$

$$3.3096$$

$$\log 4\pi = 1.0992$$

$$2.2104$$

$$\log \sqrt[3]{} = .7368$$

$$\text{antilog } .7368 = 5.455 \text{ inches.}$$

$$5. (a) \text{ The log of third side} = \frac{1}{2} \log (377 + 288) + \frac{1}{2} \log (377 - 288).$$

$$\frac{\log 665}{2} = \frac{2.8228}{2} = 1.4114.$$

$$\frac{\log 89}{2} = \frac{1.9494}{2} = .9747$$

$$\log \text{ of third side} = 2.3861$$

$$\text{antilog } 2.3861 = 243.2.$$

$$(b) \quad \frac{\log 2025}{2} = \frac{3.3064}{2} = 1.6532.$$

$$\frac{\log 961}{2} = \frac{2.9827}{2} = 1.4913$$

$$\log \text{ side} = 3.1445$$

$$\text{antilog} = 1395.$$

$$6. (a) \text{ Area} = \frac{11.47^2}{4} \sqrt{3}.$$

$$2 \log 11.47 = 2.1190$$

$$\frac{1}{2} \log 3 = .2385$$

$$2.3575$$

$$\log 4 = .6021$$

$$\log 144 = 2.1584$$

$$\log 4 \cdot 144 = 2.7605$$

$$\log \text{ area} = 1.5970$$

$$\text{antilog } 1.5970 = .3953.$$

$$(b) \quad 60 = \frac{s^2}{4} \sqrt{3}.$$

$$s = \sqrt{\frac{4 \cdot 60}{\sqrt{3}}} = \sqrt{80\sqrt{3}}.$$

$$\log 80 = 1.9031$$

$$\log \sqrt{3} = .2385$$

$$\log 80 \sqrt{3} = 2.1416$$

$$\log s = 1.0708$$

$$\text{antilog } 1.0708 = 11.75.$$

$$7. (a) \text{ Area} = \sqrt{23 \cdot 11 \cdot 8 \cdot 4}.$$

$$\log 23 = 1.3617$$

$$\log 11 = 1.0414$$

$$\log 8 = .9031$$

$$\log 4 = .6021$$

$$3.9083$$

$$\log \text{ area} = 1.9542.$$

$$\text{antilog } 1.9542 = 90.0 = \text{area in square inches.}$$

$$(b) \text{ Area} = \sqrt{1203 \cdot 363 \cdot 194 \cdot 646}.$$

$$\log 1203 = 3.0802$$

$$\log 363 = 2.5599$$

$$\log 194 = 2.2878$$

$$\log 646 = 2.8102$$

$$10.7381$$

$$\log \text{ area} = 5.3690.$$

$$\text{antilog } 5.3690 = 233,880.$$

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2. $x \log 3 = \log 25.$

$$x = \frac{\log 25}{\log 3} = \frac{1.3979}{.4771} = 2.929.$$

3. $x \log 64 = \log 4.$

$$x = \frac{\log 4}{\log 64} = \frac{.6021}{1.8062} = .33 +.$$

4. $x \log 16 = \log 1024.$

$$x = \frac{\log 1024}{\log 16} = \frac{3.0103}{1.2041} = 2.50 +.$$

5. $x \log 2 = \log 64.$

$$x = \frac{\log 64}{\log 2} = \frac{1.8062}{.3010} = 6.$$

6. $x \log (1.04) = \log 3.$

$$x = \frac{\log 3}{\log (1.04)} = \frac{.4771}{.0170} = 28.0.$$

7. $x \log 2 = \log 64.$

$$x = \frac{\log 64}{\log 2} = \frac{1.8062}{.3010} = 6.$$

8. $2x \log 4 + \log 4 = \log 84.$

$$2x \log 4 = \log 84 - \log 4 = \log 21.$$

$$x = \frac{\log 21}{2 \log 4} = \frac{1.3222}{1.2041} = 1.09 +.$$

9. $\sqrt[3]{81} \cdot 3^{x+1} = (\sqrt[3]{27})^{2x-1}.$

$$9 \cdot 3^{x+1} = 3^{2x-1}.$$

$$3^2 \cdot 3^{x+1} = 3^{2x-1}.$$

$$\therefore x + 3 = 2x - 1.$$

Whence $x = 4.$

10. $x \log 3 + 7 \log 3 = x \log 5.$

$$x (\log 5 - \log 3) = 7 \log 3,$$

or $x = \frac{7 \log 3}{\log 5 - \log 3}.$

$$x = \frac{7 \log 3}{\log \frac{5}{3}} = \frac{3.3397}{.2219} = 15.05.$$

12. Let $x =$ the number.

$$(1.05)^x = 2.$$

$$x \log (1.05) = \log 2.$$

$$x = \frac{\log 2}{\log 1.05} = \frac{.3010}{.0212} = 14.19 + \text{ years.}$$

13. Let $P =$ amount,
and $x =$ number of years.

$$P(1.04)^x = 3P.$$

$$x \log 1.04 = \log 3.$$

$$x = \frac{\log 3}{\log 1.04} = \frac{.4771}{.0170} = 28.06 + \text{ years.}$$

14. $265(1.035)^x = 2 \cdot 265.$

$$x \log 1.035 = \log 2.$$

$$x = \frac{\log 2}{\log 1.035} = \frac{.3010}{.0149} = 20.20 + \text{ years.}$$

15. $4000 \cdot (1.05)^x = 7360.80.$

$$(1.05)^x = 1.84020.$$

$$x \log 1.05 = \log 1.8402.$$

$$x = \frac{\log 1.8402}{\log 1.05} = \frac{.2648}{.0212} = 12.49 + \text{ years.}$$

16. Let $x =$ amount in dollars.

$$24 \cdot (1.04)^{300} = x.$$

$$300 \log 1.04 + \log 24 = \log x.$$

$$\log 1.04 = .0170$$

$$\frac{300}{5.1000}$$

$$\log 24 = \frac{1.3802}{6.4802}$$

$$\log x = 6.4802$$

$$x = \$3,021,000.$$

17. Let $x =$ the number of years.

$$12(1.02)^{2x} = 24.$$

$$2x \log 1.02 = \log 2.$$

$$2x = \frac{\log 2}{\log 1.02}$$

$$= \frac{.3010}{.0086} = 35.$$

$$x = \frac{35}{2} = 17\frac{1}{2} \text{ years.}$$

$$\begin{array}{rcl}
 19. (a) \ x = 5000 (1.08)^4 & \log 1.08 = .0334 & \\
 = 5000 (1.359) & & \underline{4} \\
 = \$6795, \text{ annually.} & & .1336 \\
 & \text{antilog } .1336 = 1.359. &
 \end{array}$$

$$\begin{array}{rcl}
 (b) \ x = 5000 (1.04)^8 & \log 1.04 = .0170 & \\
 = 5000 (1.367) & & \underline{8} \\
 = \$6835, \text{ semiannually.} & & .1360 \\
 & \text{antilog } .1360 = 1.367. &
 \end{array}$$

$$\begin{array}{rcl}
 (c) \ x = 5000 (1.02)^{16} & \log 1.02 = .0086 & (1) \\
 = 5000 (1.373) & & \underline{16} & (2) \\
 = \$6865, \text{ quarterly.} & (1) \times (2), & .1376 \\
 & \text{antilog } .1376 = 1.373. &
 \end{array}$$

$$20. \text{ From Exercise 20, page 174, } p = \frac{Sr(1+r)^n}{(1+r)^n - 1}. \quad (1)$$

$$\text{Substituting,} \quad \$50 = \frac{\$6000 (.005) (1.005)^n}{(1.005)^n - 1}. \quad (2)$$

$$\text{Solving (2),} \quad n = \frac{\log 2.5}{\log 1.005} = \frac{.3979}{.00215} = 185 + .$$

21. The amount of the first four payments would be :

$$\begin{aligned}
 & \$50 [(1.06)^{15} \times 1.02 + (1.06)^{15} \times 1.015 + (1.06)^{15} \times 1.01 \\
 & \quad + (1.06)^{15} \times 1.005] \\
 & = \$50 (1.06)^{15} \times 4.05 = \$485.30.
 \end{aligned}$$

Similarly, the next twelve payments would amount to :

$$\begin{aligned}
 & \$50 [(1.06)^{14} \times 1.06 + (1.06)^{14} \times 1.055 + \cdots + (1.06)^{14} \times 1.01 \\
 & \quad + (1.06)^{14} \times 1.005] \\
 & = \$50 \times 1.06^{14} \times 12.39.
 \end{aligned}$$

The amount of each succeeding group of twelve payments would be a similar expression. Therefore the last 180 payments would amount to :

$$\begin{aligned}
 & \$50 \times 12.39 [(1.06)^{14} + (1.06)^{13} + \cdots + (1.06)^2 + 1.06] \\
 & = \$50 \times 12.39 \times \frac{1.06 - (1.06)^{15}}{1 - 1.06} = \$13,790.
 \end{aligned}$$

The last payment is \$50. Then the whole amount is approximately \$485.30 + \$13,790 + \$50 = \$14,325.30.

22. From \$6000 the final return is \$14,325.30 - \$6000, or \$8325.30. One year's income will be $\frac{1}{1\frac{2}{3}}$ of \$8325.30, or \$540.01. Dividing 540.01 by 6000 gives 9.00%.

$$23. \quad \log 2^9 = 9(.3010) = 2.7090$$

$$(a) \quad \log 3^8 = 8(.4771) = 3.8168$$

$$\log 5^7 = 7(.6990) = 4.8930$$

11.4188. Twelve digits.

$$(b) \quad \log 3^{52} = 52(.4771) = 24.8092. \text{ Twenty-five digits.}$$

$$(c) \log 2^{340} = 340(.3010) = 102.3400. \text{ One hundred and three digits.}$$

24. No. Most logarithms are incommensurable numbers and therefore neither odd nor even. Hence the signs of the antilogarithms to a negative base could not be determined. Without certainty on this point correct computation is impossible.

$$\begin{aligned} 25. \quad \log_3 9 &= x. \\ 3^x &= 9 = 3^2. \\ \therefore x &= 2. \end{aligned}$$

$$\begin{aligned} 29. \quad 27^x &= 9. \\ 3^{3x} &= 3^2. \\ x &= \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} 26. \quad 2^x &= 8 = 2^3. \\ x &= 3. \end{aligned}$$

$$\begin{aligned} 30. \quad \text{Let } \log_4 8 &= x, \log_8 4 = y. \\ \text{Then } 4^x &= 8, \text{ and } 8^y = 4. \end{aligned}$$

$$\begin{aligned} 27. \quad 8^x &= 2. \\ 2^{3x} &= 2. \\ x &= \frac{1}{3}. \end{aligned}$$

$$\text{Whence } x = \frac{3}{2}, \text{ and } y = \frac{2}{3}.$$

$$\text{Therefore } x + y = \frac{3}{2} + \frac{2}{3} = \frac{13}{6}.$$

$$\begin{aligned} 28. \quad 9^x &= 27. \\ 3^{2x} &= 3^3. \\ x &= \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} 31. \quad \text{Let } \log_{27} 81 &= x, \text{ and } \log_{81} 27 = y. \\ \text{Then } 27^x &= 81, \text{ and } 81^y = 27. \end{aligned}$$

$$\text{Whence } x = \frac{4}{3}, \text{ and } y = \frac{3}{4}.$$

$$\text{Therefore } x - y = \frac{4}{3} - \frac{3}{4} = \frac{7}{12}.$$

$$32. \text{ Let } \log_{25} 125 = x, \log_5 25 = y, \text{ and } \log_{125} 5 = z.$$

$$\text{Then } 25^x = 125, 5^y = 25, \text{ and } 125^z = 5.$$

$$\text{Whence } x = \frac{3}{2}, y = 2, \text{ and } z = \frac{1}{3}.$$

$$\therefore x + y - z = \frac{3}{2} + 2 - \frac{1}{3} = \frac{19}{6}.$$

$$33. \text{ Let } \log_3 \left(\frac{1}{3}\right) = x, \log_9 \left(\frac{1}{27}\right) = y, \text{ and } \log_{27} 9 = z.$$

$$\text{Then } 3^x = \frac{1}{3}, 9^y = \frac{1}{27}, \text{ and } 27^z = 9.$$

$$\text{Whence } x = -1, y = -\frac{3}{2}, \text{ and } z = \frac{2}{3}.$$

$$\therefore x - y + z = -1 + \frac{3}{2} + \frac{2}{3} = \frac{7}{6}.$$

$$34. \log \frac{5}{8} + \log \frac{24}{5} = \log \left(\frac{5}{8} \cdot \frac{24}{5}\right) = \log \frac{3}{2}.$$

$$35. \log \frac{7}{32} - \log \frac{35}{64} = \log \left(\frac{7}{32} \div \frac{35}{64}\right) = \log \frac{2}{5}.$$

$$36. \log \frac{25}{4} + \log \frac{36}{50} - \log \frac{3}{4} = \log \left(\frac{25}{4} \cdot \frac{36}{50} \div \frac{3}{4}\right) = \log 6.$$

$$37. 2 \log 3 + 3 \log 2 = \log (3^2 \cdot 2^3) = \log 72.$$

$$38. 3 \log 4 + 4 \log 3 - 2 \log 6 = \log \frac{4^3 \cdot 3^4}{6^2} = \log 144.$$

39. $a^x = c^{x-1}$.
 $x \log a = (x-1) \log c$.
 $x \log a = x \log c - \log c$.
 $x(\log a - \log c) = -\log c$.
 $x = \frac{-\log c}{\log a - \log c} = \frac{\log c}{\log c - \log a} = \frac{\log c}{\log \frac{c}{a}}$.
40. $a^{x-1} = c^{x-2}$.
 $(x-1) \log a = (x-2) \log c$.
 $x \log a - \log a = x \log c - 2 \log c$.
 $x(\log a - \log c) = \log a - 2 \log c$.
 $x = \frac{\log a - 2 \log c}{\log a - \log c} = \frac{\log \frac{a}{c^2}}{\log \frac{a}{c}}$.
41. $a^{x-1} \cdot b^x = c^{2x}$.
 $x \log a - \log a + x \log b = 2x \log c$.
 $x(\log a + \log b - 2 \log c) = \log a$.
 $x = \frac{\log a}{\log a + \log b - 2 \log c} = \frac{\log a}{\log \frac{ab}{c^2}}$.
42. $3^x \cdot 2^{\frac{1}{x}} = 6$.
 $x \log 3 + \frac{1}{x} \log 2 = \log 6$.
 $x^2 \log 3 - x \log 6 + \log 2 = 0$.
 $x = \frac{\log 6 \pm \sqrt{(\log 6)^2 - 4 \log 2 \log 3}}{2 \log 3}$
 $= \frac{\log 2 + \log 3 \pm \sqrt{(\log 2 + \log 3)^2 - 4 \log 2 \log 3}}{2 \log 3}$
 $= \frac{\log 2 + \log 3 \pm (\log 2 - \log 3)}{2 \log 3}$
 $= \frac{2 \log 2}{2 \log 3} = \frac{\log 2}{\log 3}$, and $= \frac{2 \log 3}{2 \log 3} = 1$.
43. $e^x = e^{-x}$.
 $x \log e = -x \log e$.
 $x = -x$.
 $2x = 0$.
 $x = 0$.
44. $2^{2x} \cdot 3^{\frac{2}{x}} = 36$.
 $2x \log 2 + \frac{2}{x} \log 3 = \log 6^2 = 2 \log 6$.
 $x^2 \log 2 - x \log 6 + \log 3 = 0$.
 $x^2 \log 2 - x \log 2 - x \log 3 + \log 3 = 0$.
 $(x-1)(x \log 2 - \log 3) = 0$.
 $\therefore x = 1, \frac{\log 3}{\log 2}$.

45.

$$a^{x+1} = b^{2x} \div c^{x-1}.$$

$$(x+1) \log a = 2x \log b - (x-1) \log c.$$

$$x \log a + \log a = 2x \log b - x \log c + \log c.$$

$$x(\log a + \log c - 2 \log b) = \log c - \log a.$$

$$x = \frac{\log c - \log a}{\log a - 2 \log b + \log c} = \frac{\log \frac{c}{a}}{\log ac - \log b^2}$$

46.

$$a^{4x} + 8a^{2x} = 6a^{3x}.$$

$$a^{4x} + 8a^{2x} - 6a^{3x} = 0.$$

$$a^{2x}(a^{2x} - 6a^x + 8) = 0.$$

$$\therefore a^{2x} = 0. \text{ Whence } x = -\infty.$$

$$a^{2x} - 6a^x + 8 = 0.$$

$$(a^x - 4)(a^x - 2) = 0.$$

$$\therefore a^x - 4 = 0. \text{ Whence } x = \frac{\log 4}{\log a}.$$

$$a^x - 2 = 0. \text{ Whence } x = \frac{\log 2}{\log a}.$$

47.

$$a^{5x} + a^{4x} = 6a^{4x} - 6a^{3x}.$$

$$a^{5x} + a^{4x} - 6a^{4x} + 6a^{3x} = 0.$$

$$a^{3x}(a^{2x} - 5a^x + 6) = 0.$$

$$\therefore a^{3x} = 0. \text{ Whence } x = -\infty.$$

And

$$a^{2x} - 5a^x + 6 = 0.$$

$$(a^x - 3)(a^x - 2) = 0.$$

$$a^x - 3 = 0. \text{ Whence } x = \frac{\log 3}{\log a}.$$

$$a^x - 2 = 0. \text{ Whence } x = \frac{\log 2}{\log a}.$$

48.

$$2^x = 3^y, \tag{1}$$

$$3^{x-1} = 4^y. \tag{2}$$

$$\text{From (1), } x \log 2 = y \log 3. \tag{3}$$

$$\text{From (2), } x \log 3 - \log 3 = 2y \log 2. \tag{4}$$

$$x \log 2 - y \log 3 = 0. \tag{5}$$

$$x \log 3 - 2y \log 2 = \log 3. \tag{6}$$

$$(5) \cdot \log 3, \quad x \log 2 \log 3 - y (\log 3)^2 = 0. \tag{7}$$

$$(6) \cdot \log 2, \quad x \log 2 \log 3 - 2y (\log 2)^2 = \log 2 \log 3. \tag{8}$$

$$(7) - (8), \quad y [2 (\log 2)^2 - (\log 3)^2] = \log 2 \log 3. \tag{9}$$

$$y = \frac{\log 2 \log 3}{2 (\log 2)^2 - (\log 3)^2}. \tag{10}$$

From (10) and (3),

$$\begin{aligned} x &= \frac{y \log 3}{\log 2} \\ &= \frac{(\log 3)^2}{[2 (\log 2)^2 - (\log 3)^2]}. \end{aligned}$$

$$49. \quad 2x - y = 5, \quad (1)$$

$$3x \cdot 9^3 y = 9^{11}. \quad (2)$$

$$x = \frac{y + 5}{2}. \quad (3)$$

$$3^{\frac{y+5}{2}} \cdot 9^3 y = 9^{11}. \quad (4)$$

$$3^{\frac{y+5}{2}} \cdot 3^6 y = 3^{22}. \quad (5)$$

$$3^{\frac{13y+5}{2}} = 3^{22}. \quad (6)$$

$$\therefore 13y + 5 = 44. \quad (7)$$

$$y = 3. \quad (8)$$

From (3) and (8), $x = 4$.

$$50. \quad 3x + y = 9,$$

$$2x \cdot 8^2 y = 4^{10}.$$

$$y = 9 - 3x.$$

$$2x \cdot 8^{18-6x} = 4^{10}.$$

$$2x \cdot 2^{54-18x} = 2^{20}.$$

$$2^{54-17x} = 2^{20}.$$

$$\therefore 54 - 17x = 20.$$

$$\text{Whence} \quad x = 2.$$

$$\text{Then} \quad y = 3.$$

$$51. \quad 8x \cdot 5y = 50, \quad (1)$$

$$2^{6x} \cdot 3^{2y} = 328. \quad (2)$$

$$x \log 8 + y \log 5 = \log 50. \quad (3)$$

$$6x \log 2 + 2y \log 3 = \log 328.$$

$$\text{Now} \quad 6x \log 2 = 2x \log 2^3 = 2x \log 8. \quad (4)$$

$$(3) \cdot 2, \quad 2x \log 8 + 2y \log 5 = 2 \log 50. \quad (5)$$

$$\text{From (4) and (3),} \quad 2x \log 8 + 2y \log 3 = \log 328. \quad (6)$$

$$(5) - (6), \quad 2y (\log 5 - \log 3) = 2 \log 50 - \log 328. \quad (7)$$

$$y = \frac{2 \log 50 - \log 328}{2 (\log 5 - \log 3)} = \frac{\log \frac{6 \cdot 2 \cdot 5}{8 \cdot 2}}{\log \frac{5}{3}}. \quad (8)$$

$$\text{From (3),} \quad x = \frac{\log 50}{\log 8} - \frac{y \log 5}{\log 8}. \quad (9)$$

$$\text{From (9) and (8),} \quad x = \frac{\log 50}{\log 8} - \frac{\log 5 \cdot \log \frac{6 \cdot 2 \cdot 5}{8 \cdot 2}}{\log 8 \cdot \log \frac{5}{3}}.$$

$$52. \quad 3^x - 6^y = 0, \quad (1)$$

$$3^{x+1} - 6^x = 0. \quad (2)$$

$$\text{From (2),} \quad (x+1) \log 3 = x \log 6. \quad (3)$$

$$x (\log 3 - \log 6) = -\log 3. \quad (4)$$

$$x = \frac{\log 3}{\log 6 - \log 3} = \frac{\log 3}{\log 2}. \quad (5)$$

$$(1) \cdot 3, \quad 3^{x+1} - 3 \cdot 6^y = 0 \quad (6)$$

$$3^{x+1} - 6^x = 0 \quad (7)$$

$$6^x - 3 \cdot 6^y = 0 \quad (8)$$

$$6^x = 3 \cdot 6^y. \quad (9)$$

$$\text{From (9),} \quad x = \frac{\log 3 + y \log 6}{\log 6}. \quad (10)$$

From (5) and (10),

$$\frac{\log 3}{\log 2} = \frac{\log 3 + y \log 6}{\log 6}.$$

$$\frac{\log 3}{\log 2} = \frac{\log 3}{\log 6} + y.$$

$$\begin{aligned} y &= \frac{\log 3 \log 6 - \log 3 \log 2}{\log 2 \log 6} \\ &= \frac{\log 3 (\log 2 + \log 3) - \log 3 \log 2}{\log 2 \log 6} \\ &= \frac{2 \log 3}{\log 6 \cdot \log 2}. \end{aligned}$$

53.

$$xy = y^x, \quad (1)$$

$$y = x^2. \quad (2)$$

Substituting from (2) in (1), $(x)^{x^2} = (x^2)^x$.

$$x^2 \log x = 2x \log x.$$

$$\log x (x^2 - 2x) = 0.$$

$$x^2 - 2x = 0. \text{ Whence } x = 0 \text{ and } 2.$$

$$\log x = 0. \text{ Whence } x = 1.$$

$$\text{If } x = 0, \text{ from (2), } y = 0.$$

$$\text{If } x = 1, \text{ from (2), } y = 1.$$

$$\text{If } x = 2, \text{ from (2), } y = 4.$$

These all check except $x = 0, y = 0$.

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$$1. 42 : 28 = \frac{42}{28} = \frac{3}{2}.$$

$$2. 24a^3 : 56a^2 = \frac{24a^3}{56a^2} = \frac{3a}{7}.$$

$$3. (x^2 - y^2) : (x - y) = \frac{x^2 - y^2}{x - y} = x + y.$$

$$4. (x^3 + 8y^3) : (x + 2y) = \frac{x^3 + 8y^3}{x + 2y} = x^2 - 2xy + 4y^2.$$

$$5. \left(1 - \frac{4}{a^2}\right) : \left(1 - \frac{2}{a}\right) = \frac{1 - \frac{4}{a^2}}{1 - \frac{2}{a}} = 1 + \frac{2}{a} = \frac{a + 2}{a}.$$

$$6. \left(a - \frac{16}{a}\right) : \left(\frac{24}{a^4} + \frac{10}{a^3} + \frac{1}{a^2}\right) = \frac{\frac{a^2 - 16}{a}}{\frac{24 + 10a + a^2}{a^4}} = \frac{a^4 - 4a^3}{a + 6}.$$

$$7. (a) 4 \text{ weeks} : 12 \text{ hours} = 672 \text{ hours} : 12 \text{ hours} = \frac{672}{12} = 56.$$

$$(b) 48000 \text{ inches} : 2 \text{ miles} = 4000 \text{ feet} : 10560 \text{ feet} = \frac{4000}{10560} = \frac{25}{66}.$$

8. 1 mile : 1 kilometer = 63,360 inches : 39,370 inches = $\frac{63360}{39370} = 1.6 +$.

9. Let $4x$, $6x$, and $2x$ be the numbers.

Then $4x + 6x + 2x = 150$.

Whence $x = 12\frac{1}{2}$,

$$4x = 50,$$

$$6x = 75,$$

$$2x = 25.$$

and

$$\begin{aligned} 10. \quad \frac{5+3a}{5+4a} &= \frac{(5+3a)(5+5a)}{(5+4a)(5+5a)} = \frac{15a^2+40a+25}{20a^2+45a+25} \\ \frac{5+4a}{5+5a} &= \frac{(5+4a)(5+4a)}{(5+5a)(5+4a)} = \frac{16a^2+40a+25}{20a^2+45a+25} \\ \therefore \frac{5+3a}{5+4a} &< \frac{5+4a}{5+5a}. \text{ Similarly for (b).} \end{aligned}$$

11. See page 180.

12. See page 180.

13. $a:b=c:d$.

$$\frac{a}{b} = \frac{c}{d}.$$

$$\therefore \frac{a^n}{b^n} = \frac{c^n}{d^n}.$$

14. $a:b=c:d$.

$$\frac{a}{b} = \frac{c}{d}.$$

$$\therefore \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{c}}{\sqrt[n]{d}}.$$

15. See page 181.

16. See page 182.

17. See page 182.

18. See page 182.

19. See page 182.

20. See page 183.

21. $1.44:b=b:.0256$.

$$\begin{aligned} b &= \pm \sqrt{.0256 \times 1.44} \\ &= \pm .192. \end{aligned}$$

22. $15:125=125:a$.

$$15a = 125^2.$$

$$a = 1041\frac{2}{3}.$$

23. $16\frac{1}{4}:8\frac{1}{3}=62\frac{1}{2}:a$.

$$16\frac{1}{4}a = 8\frac{1}{3} \cdot 62\frac{1}{2}.$$

$$a = 32.05.$$

24. $5:15=8:24$.

By addition,

$$20:5=32:8,$$

or $20:15=32:24$.

By subtraction,

$$10:5=16:8,$$

or $10:15=16:24$.

By addition and subtraction,

$$20:10=32:16,$$

or $10:20=16:32$.

By alternation,

$$5:8=15:24.$$

By inversion,

$$15:5=24:8.$$

25. $8:12=(3-x):7$.

$$12(3-x)=56.$$

$$x = -\frac{5}{3}.$$

26. $4:x=x:169$.

$$x = \pm \sqrt{169 \cdot 4} = \pm 26.$$

27. $3:5 = \frac{1}{x} : 2$.

$$\frac{5}{x} = 6.$$

$$x = \frac{5}{6}.$$

28. $8:x=12:(10-x)$.

$$80-8x=12x.$$

$$x=4.$$

29. $3:5=(x-3):(2x+18)$.

$$6x+54=5x-15.$$

$$x=-69.$$

30. $20 : x = x : (10 - x).$

$$x^2 = 200 - 20x.$$

$$x^2 + 20x - 200 = 0.$$

$$x = 7.32 \text{ and } -27.32.$$

31. $S_1 = 4\pi R_1^2. \quad S_2 = 4\pi R_2^2.$

$$\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{R_1^2}{R_2^2}.$$

$$\frac{S_1}{S_2} = \frac{4\pi \left(\frac{D_1}{2}\right)^2}{4\pi \left(\frac{D_2}{2}\right)^2} = \frac{D_1^2}{D_2^2}.$$

$$\frac{S_1}{S_2} = \frac{R_1^2}{R_2^2} = \frac{D_1^2}{D_2^2}.$$

32. $V_1 = \frac{4\pi R_1^3}{3}. \quad V_2 = \frac{4\pi R_2^3}{3}.$

$$\frac{V_1}{V_2} = \frac{\frac{4\pi R_1^3}{3}}{\frac{4\pi R_2^3}{3}} = \frac{R_1^3}{R_2^3}.$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi \left(\frac{D_1}{2}\right)^3}{\frac{4}{3}\pi \left(\frac{D_2}{2}\right)^3} = \frac{D_1^3}{D_2^3}.$$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{D_1^3}{D_2^3}.$$

33. Let

S_1 = surface of the earth,

and

S_2 = surface of the moon.

$$\frac{S_1}{S_2} = \frac{D_1^2}{D_2^2} = \frac{(7920)^2}{(2160)^2} = \left(\frac{11}{3}\right)^2 = \frac{121}{9}.$$

34. Let

V_1 = the volume of the earth,

and

V_2 = the volume of the moon.

$$\frac{V_1}{V_2} = \frac{P_1^3}{P_2^3} = \frac{(7920)^3}{(2160)^3} = \left(\frac{11}{3}\right)^3 = \frac{1331}{27}.$$

35. $\frac{\text{area } ABC}{\text{area } AKR} = \frac{\overline{AB}^2}{\overline{AK}^2}.$

$$\frac{100 \text{ square inches}}{25 \text{ square inches}} = \frac{100}{\overline{AK}^2}.$$

$$\overline{AK} = 5.$$

36. $\frac{\overline{AB}^2}{\overline{AK}^2} = \frac{2 \text{ } AKR}{AKR} = \frac{2}{1}.$

$$\frac{144}{\overline{AK}^2} = \frac{2}{1}.$$

$$\overline{AK} = 6\sqrt{2}.$$

37.

$$KRCB = 8 \text{ } AKR.$$

$$\therefore ABC = 8 \text{ } AKR + AKR = 9 \text{ } AKR.$$

$$\frac{\overline{AC}^2}{\overline{AR}^2} = \frac{9 \text{ } AKR}{AKR}.$$

$$\frac{1600}{\overline{AR}^2} = \frac{9}{1}.$$

$$\overline{AR} = 13\frac{1}{3}.$$

38. Let EF and KR be lines $\parallel$ to BC , cutting AB and AC at points E and K , F and R , respectively, and dividing ABC into three parts of equal area.

$$\frac{\overline{AB}^2}{\overline{AE}^2} = \frac{3 AEF}{AEF}.$$

$$\frac{1024}{\overline{AE}^2} = \frac{3}{1}.$$

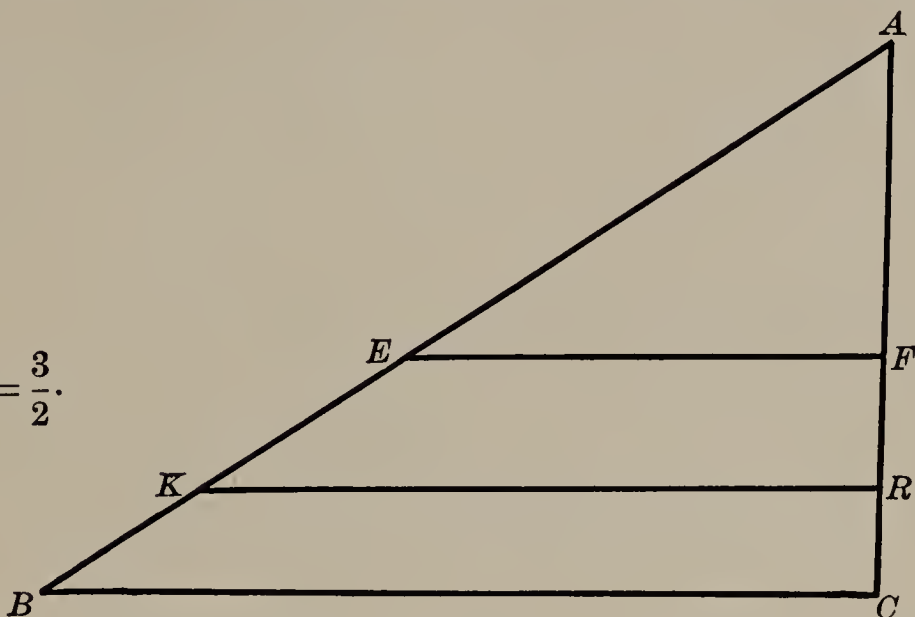
$$AE = 18.474.$$

$$\frac{\overline{AB}^2}{\overline{AK}^2} = \frac{3 AEF}{2 AEF} = \frac{3}{2}.$$

$$AK = 26.127.$$

$$EK = 7.65.$$

$$KB = 5.87.$$



$$\begin{aligned} 39. \quad \frac{D-ABC}{D-KRL} &= \frac{\overline{DH}^3}{\overline{DS}^3}. \\ \frac{32}{4} &= \frac{8}{1} = \frac{(18)^3}{\overline{DS}^3}. \\ \frac{2}{1} &= \frac{18}{\overline{DS}}. \\ DS &= 9. \end{aligned}$$

$$\begin{aligned} 40. \quad \frac{2(D-KRL)}{D-KRL} &= \frac{\overline{DH}^3}{\overline{DS}^3}. \\ \frac{2}{1} &= \frac{12^3}{\overline{DS}^3}. \\ \overline{DS}^3 &= \frac{1728}{2}. \\ DS &= 6\sqrt[3]{4} \\ &= 9.52. \end{aligned}$$

41. Volume of small pyramid = $\frac{8}{27}$ of the volume of large pyramid.

$$\begin{aligned} \frac{D-ABC}{\frac{8}{27}(D-ABC)} &= \frac{36^3}{\overline{DS}^3}. \\ \frac{3}{2} &= \frac{36}{\overline{DS}}. \\ DS &= 24. \end{aligned}$$

42. Let x be the point where DH cuts the first plane below the vertex.

$$\frac{\overline{Dx}^3}{100^3} = \frac{1}{3}.$$

$$\frac{Dx}{100} = \frac{1}{\sqrt[3]{3}}.$$

$$Dx = \frac{100}{\sqrt[3]{3}} = 69.33.$$

Let y be the point where DH cuts the second plane below the vertex.

$$\frac{\overline{Dy}^3}{100^3} = \frac{2}{3}.$$

$$Dy = 100 \sqrt[3]{\frac{2}{3}} = 87.33.$$

$$xy = 18.00.$$

$$\begin{aligned} yH &= 100 - Dy \\ &= 100 - \frac{100}{\sqrt[3]{3}} \sqrt[3]{18} \\ &= 12.66. \end{aligned}$$

Ans. 69.33, 18.00, 12.66.

43. If $a : b = c : d$,

$$\frac{a}{3b} = \frac{c}{3d}.$$

$$\text{Then } \frac{a+3b}{a-3b} = \frac{c+3d}{c-3d}.$$

44. If

$$\frac{a}{b} = \frac{c}{d},$$

then

$$\frac{a^2}{b^2} = \frac{c^2}{d^2}, \text{ and } \frac{a^2}{2b^2} = \frac{c^2}{2d^2}.$$

Therefore

$$\frac{a^2 + 2b^2}{a^2} = \frac{c^2 + 2d^2}{c^2}.$$

45. If

$$\frac{a}{b} = \frac{c}{d},$$

by addition and subtraction,

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

Squaring,

$$\frac{a^2 + 2ab + b^2}{a^2 - 2ab + b^2} = \frac{c^2 + 2cd + d^2}{c^2 - 2cd + d^2}.$$

By addition and subtraction,

$$\frac{2a^2 + 2b^2}{4ab} = \frac{2c^2 + 2d^2}{4cd},$$

or

$$\frac{a^2 + b^2}{2ab} = \frac{c^2 + d^2}{2cd}.$$

46. If

$$\frac{a}{b} = \frac{c}{d},$$

$$\frac{a^3}{b^3} = \frac{c^3}{d^3},$$

and

$$\frac{5a^3}{b^3} = \frac{5c^3}{d^3},$$

then

$$\frac{5a^3 - b^3}{b^3} = \frac{5c^3 - d^3}{d^3}.$$

47. If $\frac{a}{b} = \frac{c}{d}$, by Exercise 45,

$$\frac{a^2 + b^2}{ab} = \frac{c^2 + d^2}{cd}.$$

By subtraction,

$$\frac{a^2 - ab + b^2}{ab} = \frac{c^2 - cd + d^2}{cd}.$$

$$\therefore \frac{a^3 + b^3}{ab(a+b)} = \frac{c^3 + d^3}{cd(c+d)},$$

or

$$\frac{a^3 + b^3}{3a^2b + 3ab^2} = \frac{c^3 + d^3}{3c^2d + 3cd^2}.$$

48. From Exercise 47,

$$\frac{a^3 + b^3}{3a^2b + 3ab^2} = \frac{c^3 + d^3}{3c^2d + 3cd^2},$$

or

$$\frac{a^3 + b^3}{3ab(a+b)} = \frac{c^3 + d^3}{3cd(c+d)}.$$

Then

$$\frac{a^2 - ab + b^2}{3ab} = \frac{c^2 - cd + d^2}{3cd}.$$

40. If $\frac{h}{H} = \frac{a}{A},$
 then $\frac{h}{a} = \frac{H}{A}.$
 Squaring, $\frac{h^2}{a^2} = \frac{H^2}{A^2}.$
 By subtraction, $\frac{h^2 - a^2}{a^2} = \frac{H^2 - A^2}{A^2}.$
 But $h^2 - a^2 = b^2,$
 and $H^2 - A^2 = B^2.$
 $\therefore \frac{b^2}{a^2} = \frac{B^2}{A^2}.$
 Whence $\frac{a}{A} = \frac{B}{b}.$
 Then $\frac{a}{A} = \frac{b}{B} = \frac{c}{C},$ and the triangles are similar.

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2. $\frac{6}{15} = \frac{10}{y}.$
 $y = \frac{150}{6} = 25.$

3. $\frac{h}{m} = \frac{k}{y}.$
 $y = \frac{mk}{h}.$

5. $4 : x = \frac{1}{100} : \frac{1}{10}.$
 $x = 40.$

6. $h : y = \frac{1}{k} : \frac{1}{m}.$
 $y = \frac{hk}{m}.$

8. $\frac{3}{x} = \frac{4 \cdot 5}{20 \cdot 2}.$
 $x = 6.$

9. $10 : x = \frac{4}{9} : \frac{2}{6}.$
 $x = 7\frac{1}{2}.$

10. $\frac{64}{d} = \frac{4}{16}.$
 $d = 256.$

11. $80 : P = \frac{400}{15} : \frac{450}{45}.$
 $P = 30.$

12. Let x equal the weight of the object.

(a) $\frac{100}{x} = \frac{4000}{3000}.$
 $x = 75.$

(b) $\frac{100}{x} = \frac{4000}{2000}.$
 $x = 50.$

(c) $\frac{x}{100} = \frac{0}{4000}.$
 $x = 0.$

13. $4\frac{2}{3} : 5 = \frac{1}{360} : \frac{1}{r}.$
 $r = 336.$

14. $\frac{1}{d} = \frac{4\frac{2}{3}}{2}.$
 $d = \frac{2}{4.4} = \frac{5}{11}$ miles
 $= 2400$ feet.

$$15. \quad \frac{24}{20} = \frac{36}{30}.$$

$$v : v_1 = \frac{1}{P} : \frac{1}{P_1} \text{ (inverse).}$$

$$\frac{24}{v} = \frac{48}{30}.$$

$$v = 15 \text{ cubic inches.}$$

$$16. \quad \frac{16}{d} = \frac{1}{36}.$$

$$d = 576 \text{ feet.}$$

$$17. \quad \frac{x}{1} = \frac{36}{16}.$$

$$x = 2\frac{1}{4}.$$

The page is $2\frac{1}{4}$ times as bright 4 feet from the lamp as it is 6 feet away.

$$18. \quad \frac{1}{4} = \frac{d^2}{81}.$$

$$d^2 = \frac{81}{4}.$$

$$d = \frac{9}{2} = 4.5 \text{ feet away.}$$

$$\frac{1}{2} = \frac{d^2}{81}.$$

$$d = \frac{9}{2} \sqrt{2}$$

$$= 6.36 + \text{ feet away.}$$

19. (a)

$$100 : x = \frac{1}{(4000)^2} : \frac{1}{(5000)^2}.$$

$$\therefore x = 64 \text{ pounds.}$$

$$(b) \quad 100 : x = \frac{1}{(4000)^2} : \frac{1}{(6000)^2}.$$

$$\therefore x = 44\frac{4}{9} \text{ pounds.}$$

$$100 : x = \frac{1}{(4000)^2} : \frac{1}{(8000)^2}.$$

$$\therefore x = 25 \text{ pounds.}$$

$$20. \quad \frac{154}{594} = \frac{49}{r^2}.$$

$$r^2 = \frac{594 \cdot 49}{154} = 189.$$

$$r = 13.74 \text{ inches.}$$

$$21. \quad \frac{6}{x} = \frac{2^3}{6^3}.$$

$$x = 162 \text{ pounds.}$$

$$22. \quad \frac{1}{t} = \frac{\sqrt{100}}{\sqrt{64}}.$$

$$t = .8 \text{ second.}$$

$$23. \quad \frac{1}{2} = \frac{\sqrt{100}}{\sqrt{l}}.$$

$$l = 400 \text{ centimeters.}$$

$$\frac{1}{5} = \frac{\sqrt{100}}{\sqrt{l}}.$$

$$l = 2500 \text{ centimeters.}$$

$$24. \quad P_1 : P_2 = A_1 V_1^2 : A_2 V_2^2.$$

$$.9 : 18 = 1 \cdot (15)^2 : 9 \cdot V_2^2.$$

$$V_2^2 = \frac{18 \cdot (15)^2}{.9 \cdot 9} = 500.$$

$$V_2 = \sqrt{500} = 22.36 \text{ miles per hour.}$$

25.

$$\frac{62.5}{P} = \frac{1 \cdot 1}{96 \cdot 6}.$$

(a)

$$P = 36,000 \text{ pounds.}$$

(b) The pressure at the top of the tank on one end is zero and the pressure at the bottom on one end is equal to the pressure of a column of water 6 feet high. The average pressure per square foot on sides would be equal to the weight of a column of water 3 feet high having a cross section of 1 square foot.

$$\therefore P \text{ on end} = 8 \times 6 \times (3 \cdot 62.5) = 9000 \text{ pounds.}$$

$$P \text{ on side} = 12 \times 6 \times (3 \cdot 62.5) = 13,500 \text{ pounds.}$$

$$26. \quad 1320 : c = \frac{1}{2} : \frac{20}{2\frac{1}{2}}.$$

$$\therefore c = \$21,120.$$

$$30. \quad 1 = k \cdot 4\frac{2}{5}.$$

$$k = .227.$$

k is the distance in miles which sound travels in 1 second.

$$31. \quad 24 \cdot 30 = k.$$

$$k = 720.$$

$$32. \quad a \propto b \cdot h.$$

$$a = k \cdot bh = \frac{bh}{2}.$$

$$\therefore k = \frac{1}{2}.$$

$$28. \quad w \propto r^3.$$

$$w = kr^3.$$

$$6 = k(2)^3.$$

$$k = \frac{3}{4}.$$

$$33. \quad a \propto d^2.$$

$$a = kd^2.$$

$$a = \frac{\pi d^2}{4}.$$

$$\therefore k = .7854.$$

$$34. \quad v \propto d^3.$$

$$v = kd^3.$$

$$v = \frac{\pi d^3}{6}.$$

$$\therefore k = .5236.$$

$$29. \quad \text{In Problem 22,}$$

$$t = k\sqrt{l}.$$

$$1 = k\sqrt{100}.$$

$$k = \frac{1}{10}.$$

35. (Direct.) At any time the heights of two objects are directly proportional to the lengths of the shadows they cast on a horizontal plane.

(Inverse.) The resistance of a wire carrying a current of electricity varies inversely as the square of its diameter.

(Joint.) The amount of light or heat received from the sun by a planet varies inversely as the square of the planet's distance from the sun, and directly as the square of the planet's diameter.

$$36. \quad \text{If} \quad x^2 + y^2 \propto x^2 - y^2, \quad (1)$$

$$x^2 + y^2 = K(x^2 - y^2). \quad (2)$$

$$\text{Then} \quad \frac{x^2 + y^2}{x^2 - y^2} = \frac{K}{1}. \quad (3)$$

By addition and subtraction (3) becomes

$$\frac{x^2}{y^2} = \frac{K+1}{K-1}. \quad (4)$$

$$\frac{x}{y} = \frac{\sqrt{K+1}}{\sqrt{K-1}}. \quad (5)$$

By addition and subtraction (5) becomes

$$\frac{x+y}{x-y} = \frac{\sqrt{K+1} + \sqrt{K-1}}{\sqrt{K+1} - \sqrt{K-1}}. \quad (6)$$

But the right member of (6) is a constant. Call it c .

$$\text{Then} \quad \frac{x+y}{x-y} = c, \text{ or } x+y \propto x-y.$$

$$37. \quad \text{If} \quad x^3 + y^3 \propto x^3 - y^3, \quad (1)$$

$$x^3 + y^3 = K(x^3 - y^3). \quad (2)$$

$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{K}{1}. \quad (3)$$

By addition and subtraction (3) becomes

$$\frac{x^3}{y^3} = \frac{K+1}{K-1}. \quad (4)$$

$$\frac{x}{y} = \frac{\sqrt[3]{K+1}}{\sqrt[3]{K-1}}. \quad (5)$$

By addition and subtraction

$$\frac{x+y}{x-y} = \frac{\sqrt[3]{K+1} + \sqrt[3]{K-1}}{\sqrt[3]{K+1} - \sqrt[3]{K-1}}. \quad (6)$$

But the right member of (6) is a constant.

$$\therefore x+y \propto x-y.$$

38. From the solution of Exercise 37 it follows that

$$x^3 + y^3 = K(x^3 - y^3), \quad (1)$$

and

$$x+y = m(x-y). \quad (2)$$

$$(1) \div (2), \quad x^2 - xy + y^2 = \frac{K}{m}(x^2 + xy + y^2). \quad (3)$$

But $\frac{K}{m}$ is a constant.

Therefore $x^2 - xy + y^2 \propto x^2 + xy + y^2$.

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1. $3\sqrt{-1} + 4\sqrt{-1} - 2\sqrt{-1} = 5\sqrt{-1}.$
2. $\sqrt{-4} + \sqrt{-9} = 2\sqrt{-1} + 3\sqrt{-1} = 5\sqrt{-1}.$
3. $\sqrt{-25} - \sqrt{-16} = 5\sqrt{-1} - 4\sqrt{-1} = \sqrt{-1}.$
4. $5\sqrt{-1} + \sqrt{-9} = 5\sqrt{-1} + 3\sqrt{-1} = 8\sqrt{-1}.$
5. $\sqrt{-4} + \sqrt{-16} = 2\sqrt{-1} + 4\sqrt{-1} = 6\sqrt{-1}.$
6. $5\sqrt{-36x^2} - 2\sqrt{-49x^2} = 30x\sqrt{-1} - 14x\sqrt{-1} = 16x\sqrt{-1}.$
7. $\sqrt{-18} + \sqrt{-8} = 3\sqrt{-2} + 2\sqrt{-2} = 5\sqrt{-2}.$
8. $(-12)^{\frac{1}{2}} + (-27)^{\frac{1}{2}} = 2\sqrt{-3} + 3\sqrt{-3} = 5\sqrt{-3}.$
9. $3 + 2\sqrt{-1} + 5 - 6\sqrt{-1} = 8 - 4\sqrt{-1}.$
10. $5\sqrt{-x^2} - 7a - 3\sqrt{-x^2} = 5x\sqrt{-1} - 7a - 3x\sqrt{-1} = 2x\sqrt{-1} - 7a.$

$$11. 6 - 2\sqrt{-64x^2} - 3\sqrt{-25x^2} + 8 = 14 - 16x\sqrt{-1} - 15x\sqrt{-1} \\ = 14 - 31x\sqrt{-1}.$$

$$12. 5\sqrt{-3} + 3\sqrt{-2} - \sqrt{-27} + 2\sqrt{-8} \\ = 5\sqrt{-3} + 3\sqrt{-2} - 3\sqrt{-3} + 4\sqrt{-2} \\ = 2\sqrt{-3} + 7\sqrt{-2}.$$

$$13. 6\sqrt{-4a^4} - 7a^2\sqrt{-9} + 3\sqrt{-6} - 5\sqrt{-24} \\ = 12a^2\sqrt{-1} - 21a^2\sqrt{-1} + 3\sqrt{-6} - 10\sqrt{-6} \\ = -9a^2\sqrt{-1} - 7\sqrt{-6}.$$

$$14. (12 - 6\sqrt{-9}) - (15 + 2\sqrt{-36}) = 12 - 18\sqrt{-1} - 15 - 12\sqrt{-1} \\ = -3 - 30\sqrt{-1}.$$

$$15. 3a - 2x - (2a\sqrt{-a^2} - 5ra^2\sqrt{-1}) \\ = 3a - 2x - 2a^2\sqrt{-1} + 5ra^2\sqrt{-1} \\ = 3a - 2x - a^2(2 - 5r)\sqrt{-1}.$$

$$16. (x - iy) - (n - iv) = x - iy - n + iv = x - n - (y - v)i.$$

$$17. \sqrt{-10} = \sqrt{10} \cdot \sqrt{-1}.$$

$$21. a\sqrt{-b} = a\sqrt{b} \cdot \sqrt{-1}.$$

$$18. \sqrt{-6} = \sqrt{6} \cdot \sqrt{-1}.$$

$$22. \sqrt{-a-b} = \sqrt{-1(a+b)} \\ = \sqrt{a+b} \cdot \sqrt{-1}.$$

$$19. 2\sqrt{-3} = 2\sqrt{3} \cdot \sqrt{-1}.$$

$$20. \sqrt{-a} = \sqrt{a} \cdot \sqrt{-1}.$$

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$$1. (\sqrt{-1})^5 = (\sqrt{-1})^2 \cdot (\sqrt{-1})^2 \cdot \sqrt{-1} = \sqrt{-1}.$$

$$2. (\sqrt{-1})^6 = (\sqrt{-1})^2 \cdot (\sqrt{-1})^2 \cdot (\sqrt{-1})^2 = -1.$$

$$3. (\sqrt{-1})^7 = -1\sqrt{-1}.$$

$$4. (\sqrt{-1})^8 = 1.$$

$$5. 2\sqrt{-1} \cdot 3\sqrt{-1} = -6.$$

$$6. \sqrt{-9} \cdot \sqrt{-16} = 3\sqrt{-1} \cdot 4\sqrt{-1} = -12.$$

$$7. \sqrt{-5}(-\sqrt{-6}) = \sqrt{5} \cdot \sqrt{-1}(-\sqrt{6} \cdot \sqrt{-1}) = -1(-\sqrt{30}) = \sqrt{30}.$$

$$8. \sqrt{-25} \cdot \sqrt{3} = 5\sqrt{-1} \cdot \sqrt{3} = 5\sqrt{-3}.$$

9. $2\sqrt{-3} \cdot 3\sqrt{-2} = 2\sqrt{3}\sqrt{-1} \cdot 3\sqrt{2}\sqrt{-1} = -6\sqrt{6}.$
10. $\sqrt{-m} \cdot \sqrt{-n} = \sqrt{m}\sqrt{-1} \cdot \sqrt{n}\sqrt{-1} = -\sqrt{mn}.$
11. $4\sqrt{-5}(-3\sqrt{-6}) = 4\sqrt{5}\sqrt{-1}(-3\sqrt{6}\sqrt{-1}) = 12\sqrt{30}.$
12. $\sqrt{a+b} \cdot \sqrt{-a-b} = \sqrt{a+b} \cdot \sqrt{a+b}\sqrt{-1} = (a+b)\sqrt{-1}.$
13. $(2 + \sqrt{-1})(2 - \sqrt{-1}) = 4 - (-1) = 5.$
14. $(3 + \sqrt{-2})(3 - \sqrt{-2}) = (3 + \sqrt{2}\sqrt{-1})(3 - \sqrt{2}\sqrt{-1}) = 11.$
15. $(4 - 2\sqrt{3}i)(4 + 2\sqrt{3}i) = 16 - 4 \cdot 3(-1) = 28.$
16. $(3 + \sqrt{-1})(6 - \sqrt{-2}) = (3 + \sqrt{-1})(6 - \sqrt{2}\sqrt{-1})$
 $= 18 - 3\sqrt{2}\sqrt{-1} + 6\sqrt{-1} - \sqrt{2}(-1)$
 $= 18 - 3\sqrt{-2} + 6\sqrt{-1} + \sqrt{2}.$
17. $(4 - 2i)(3 - 2\sqrt{3}i) = 12 - 6i - 8\sqrt{3}i + 4\sqrt{3}(-1)$
 $= 12 - 6\sqrt{-1} - 8\sqrt{-3} - 4\sqrt{3}.$
18. $(a + ib)(c + id) = ac + iad + ibc + bd(-1)$
 $= ac + ad\sqrt{-1} + bc\sqrt{-1} - bd.$
19. $(a + ib)(a + ib) = a^2 + 2abi + i^2b^2 = a^2 + 2ab\sqrt{-1} - b^2.$
20. $(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2.$
21. $(-\frac{1}{2} + \frac{1}{2}\sqrt{-3})^2 = (-\frac{1}{2} + \frac{1}{2}i\sqrt{3})^2 = \frac{1}{4} - 2 \cdot \frac{1}{4}i\sqrt{3} + \frac{1}{4}(-1)3$
 $= \frac{1}{4} - \frac{1}{2}\sqrt{-3} - \frac{3}{4} = -\frac{1}{2} - \frac{1}{2}\sqrt{-3}.$
22. $(-\frac{1}{2} - \frac{1}{2}\sqrt{-3})^2 = (-\frac{1}{2} - \frac{1}{2}i\sqrt{3})^2 = \frac{1}{4} + 2 \cdot \frac{1}{4}i\sqrt{3} + \frac{1}{4}(-1)3$
 $= \frac{1}{4} + \frac{1}{2}\sqrt{-3} - \frac{3}{4} = -\frac{1}{2} + \frac{1}{2}\sqrt{-3}.$
23. $(x - iy)^2 - (x + iy)^2 = (x^2 - 2ixy + i^2y^2) - (x^2 + 2ixy + i^2y^2)$
 $= -4ixy = -4xy\sqrt{-1}.$
24. $(-\frac{1}{2} + \frac{1}{2}\sqrt{-3})^3 - (-\frac{1}{2} - \frac{1}{2}\sqrt{-3})^3 = (-\frac{1}{2} + \frac{1}{2}i\sqrt{3})^3 - (-\frac{1}{2} - \frac{1}{2}i\sqrt{3})^3$
 $= 1 - 1 = 0.$
25. $(a + i\sqrt{1-x^2})(a - i\sqrt{1-x^2}) = a^2 - (-1)(1-x^2) = a^2 + 1 - x^2.$

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1. $\frac{\sqrt{-8}}{\sqrt{-2}} = \frac{\sqrt{8}i^2\sqrt{2}}{-2} = \frac{\sqrt{16}(-1)}{-2} = 2.$
2. $\frac{\sqrt{-6}}{\sqrt{-3}} = \frac{\sqrt{-6}\sqrt{-3}}{-3} = \frac{-\sqrt{18}}{-3} = \sqrt{2}.$
3. $\frac{2\sqrt{-3}}{3\sqrt{-1}} = \frac{2\sqrt{3}\sqrt{-1}\sqrt{-1}}{3(-1)} = \frac{-2\sqrt{3}}{-3} = \frac{2\sqrt{3}}{3}.$
4. $\frac{\sqrt[4]{-1}}{\sqrt[4]{-4}} = \frac{\sqrt[4]{-1}}{\sqrt[4]{4}\sqrt[4]{-1}} = \frac{1}{\sqrt[4]{4}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$

5. $\frac{\sqrt{6}}{\sqrt{-2}} = \frac{\sqrt{6}\sqrt{2}\sqrt{-1}}{-2} = \frac{\sqrt{-12}}{-2} = -\sqrt{-3}.$
6. $\frac{(-25)^{\frac{1}{4}}}{(-81)^{\frac{1}{4}}} = \frac{(-1)^{\frac{1}{4}}(25)^{\frac{1}{4}}}{(-1)^{\frac{1}{4}}(81)^{\frac{1}{4}}} = \frac{\sqrt{5}}{3}.$
7. $\frac{\sqrt{ax}}{\sqrt{-x}} = \frac{\sqrt{ax}\sqrt{-x}}{-x} = \frac{x\sqrt{-a}}{-x} = -\sqrt{-a}.$
8. $\frac{\sqrt{-a}}{\sqrt{-b}} = \frac{i\sqrt{a}}{i\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}.$
9. $\frac{(-5ax)^{\frac{1}{2}}}{(-2x)^{\frac{1}{2}}} = \frac{(-5ax)^{\frac{1}{2}}(-2x)^{\frac{1}{2}}}{-2x} = \frac{-x(10a)^{\frac{1}{2}}}{-2x} = \frac{\sqrt{10a}}{2}.$
10. $\frac{(-a^6)^{\frac{1}{4}} - (-a^2)^{\frac{1}{4}}}{(-a)^{\frac{1}{4}}} = \frac{(-1)^{\frac{1}{4}}a^{\frac{6}{4}} - (-1)^{\frac{1}{4}}a^{\frac{2}{4}}}{(-1)^{\frac{1}{4}}a^{\frac{1}{4}}} = \frac{a^{\frac{6}{4}} - a^{\frac{2}{4}}}{a^{\frac{1}{4}}} = a^{\frac{5}{4}} - a^{\frac{1}{4}}.$
11. $\frac{2}{1 - \sqrt{-1}} = \frac{2(1 + \sqrt{-1})}{1 - (-1)} = \frac{2 + 2\sqrt{-1}}{2} = 1 + \sqrt{-1}.$
12. $\frac{3}{2 - \sqrt{-2}} = \frac{3(2 + \sqrt{-2})}{4 - (-2)} = \frac{6 + 3\sqrt{-2}}{6} = 1 + \frac{\sqrt{-2}}{2}.$
13. $\frac{2\sqrt{-1}}{\sqrt{-1} + 3} = \frac{2\sqrt{-1}(\sqrt{-1} - 3)}{-1 - 9} = \frac{-2 - 6\sqrt{-1}}{-10} = \frac{1 + 3\sqrt{-1}}{5}.$
14. $\frac{3\sqrt{-2}}{2\sqrt{-3} + 2} = \frac{3\sqrt{-2}(2\sqrt{-3} - 2)}{(2\sqrt{-3} + 2)(2\sqrt{-3} - 2)} = \frac{-6\sqrt{6} - 6\sqrt{-2}}{4(-3) - 4}$
 $= \frac{3\sqrt{6} + 3\sqrt{-2}}{8}.$
15. $\frac{-1 + \sqrt{-3}}{-1 - \sqrt{-3}} = \frac{(1 - \sqrt{-3})(1 - \sqrt{-3})}{(1 + \sqrt{-3})(1 - \sqrt{-3})} = \frac{1 - 2\sqrt{-3} + (-3)}{1 - (-3)}$
 $= \frac{-1 - \sqrt{-3}}{2}.$
16. $\frac{1 + i}{1 - i} = \frac{(1 + i)^2}{1 + 1} = \frac{1 + 2i + (-1)}{2} = i = \sqrt{-1}.$
17. $\frac{a}{a + bi} = \frac{a(a - bi)}{a^2 - b^2(-1)} = \frac{a^2 - abi}{a^2 + b^2}.$
18. $\frac{a + ib}{c + id} = \frac{(a + ib)(c - id)}{c^2 - d^2(-1)} = \frac{ac - iad + ibc + bd}{c^2 + d^2}.$

$$19. \frac{(2+3i)}{(2i-1)(5i-3)} = \frac{2+3i}{-7-11i} = \frac{(2+3i)(-7+11i)}{(-7-11i)(-7+11i)} \\ = \frac{-47+i}{(-7)^2-(11i)^2} = \frac{-47+i}{170}.$$

$$20. (i\sqrt{3}-1)^3 = 8. \text{ Yes.}$$

22. Yes.

$$21. (1-\sqrt{-3})^3 = -8. \text{ Yes.}$$

23. Yes.

24. The sum is 4 and the product 13.

25. Let $a+b\sqrt{-1}$ and $a-b\sqrt{-1}$ be the conjugate complex numbers.

$$a+b\sqrt{-1}+a-b\sqrt{-1}=2a, \text{ a real number.}$$

$$(a+b\sqrt{-1})(a-b\sqrt{-1})=[a^2-(-1)b^2]=a^2+b^2, \text{ a real number.}$$

$$26. \frac{a+b\sqrt{-1}}{a-b\sqrt{-1}} = \frac{(a+b\sqrt{-1})(a+b\sqrt{-1})}{(a-b\sqrt{-1})(a+b\sqrt{-1})} = \frac{a^2+2ab\sqrt{-1}-b^2}{a^2+b^2},$$

a complex number.

27. The error consists in asserting that equation (1) is an identity when it is really an equation of condition true for any real value of x which is less than y . This can be seen by putting 5 for x and 2 for y , and then putting 2 for x and 5 for y .

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$$1. x^2 + 4x + 12 = 0.$$

$$x = \frac{-4 \pm \sqrt{-32}}{2} = -2 \pm 2\sqrt{-2}.$$

$$2. x^2 - 6x + 36 = 0.$$

$$x = \frac{6 \pm \sqrt{-108}}{2} = 3 \pm 3\sqrt{-3}.$$

$$3. x^2 + 5x + 7 = 0.$$

$$x = \frac{-5 \pm \sqrt{25-28}}{2} = \frac{-5 \pm \sqrt{-3}}{2}.$$

$$4. x^2 - 3x + 10 = 0.$$

$$x = \frac{3 \pm \sqrt{9-40}}{2} = \frac{3 \pm \sqrt{-31}}{2}.$$

$$5. 2x^2 + 6x + 5 = 0.$$

$$x = \frac{-6 \pm \sqrt{36-40}}{4} = \frac{-6 \pm 2\sqrt{-1}}{4} = \frac{-3 \pm \sqrt{-1}}{2}.$$

$$6. 3x^2 - 7x + 6 = 0.$$

$$x = \frac{7 \pm \sqrt{49-72}}{6} = \frac{7 \pm \sqrt{-23}}{6}.$$

8. $x^3 = 8.$

$$x^3 - 8 = 0.$$

$$(x-2)(x^2+2x+4) = 0.$$

$$\therefore x - 2 = 0.$$

$$x = 2.$$

$$\therefore x^2 + 2x + 4 = 0.$$

$$x = -1 \pm \sqrt{-3}.$$

9. $x^3 = 27.$

$$x^3 - 27 = 0.$$

$$(x-3)(x^2+3x+9) = 0.$$

$$\therefore x - 3 = 0.$$

$$x = 3.$$

$$\therefore x^2 + 3x + 9 = 0.$$

$$x = \frac{-3 \pm 3\sqrt{-3}}{2}.$$

12. $x^4 = 16.$

$$x^4 - 16 = 0.$$

$$(x-2)(x+2)(x^2+4) = 0.$$

$$\therefore x - 2 = 0.$$

$$x = 2.$$

$$\therefore x + 2 = 0.$$

$$x = -2.$$

$$\therefore x^2 + 4 = 0.$$

$$x = \pm 2\sqrt{-1}.$$

13. $x^6 = 1.$

$$x^6 - 1 = 0.$$

$$(x^3-1)(x^3+1) = 0.$$

$$(x-1)(x^2+x+1)(x+1)(x^2-x+1) = 0.$$

$$\therefore x = 1, \text{ and } x = -1.$$

$$\therefore x^2 + x + 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$

$$\therefore x^2 - x + 1 = 0.$$

$$x = \frac{1 \pm \sqrt{-3}}{2}.$$

14. $x^6 = 64.$

$$(x^3-8)(x^3+8) = 0.$$

$$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4) = 0.$$

$$\therefore x - 2 = 0, \text{ and } x + 2 = 0.$$

$$x = 2, \text{ and } x = -2.$$

$$\therefore x^2 + 2x + 4 = 0.$$

$$x = -1 \pm \sqrt{-3}.$$

$$\therefore x^2 - 2x + 4 = 0.$$

$$x = 1 \pm \sqrt{-3}.$$

10. $x^3 = -8.$

$$x^3 + 8 = 0.$$

$$(x+2)(x^2-2x+4) = 0.$$

$$\therefore x + 2 = 0.$$

$$x = -2.$$

$$\therefore x^2 - 2x + 4 = 0.$$

$$x = 1 \pm \sqrt{-3}.$$

11. $x^4 = 1.$

$$x^4 - 1 = 0.$$

$$(x-1)(x+1)(x^2+1) = 0.$$

$$\therefore x + 1 = 0.$$

$$x = -1.$$

$$\therefore x^2 + 1 = 0.$$

$$x = \pm \sqrt{-1}.$$

$$\therefore x - 1 = 0.$$

$$x = 1.$$

15. Two; three; four; six.

16. Any real number has n n th roots.

17. $8x^3 - 27 = 0$.

$$(2x - 3)(4x^2 + 6x + 9) = 0.$$

$$\therefore 2x - 3 = 0.$$

$$x = \frac{3}{2}.$$

$$\therefore 4x^2 + 6x + 9 = 0.$$

$$x = \frac{-3 \pm 3\sqrt{-3}}{4}.$$

18. $125x^3 + 64 = 0$.

$$(5x + 4)(25x^2 - 20x + 16) = 0.$$

$$\therefore 5x + 4 = 0.$$

$$x = -\frac{4}{5}.$$

$$\therefore 25x^2 - 20x + 16 = 0.$$

$$x = \frac{2 \pm 2\sqrt{-3}}{5}.$$

19. $(x^2 + 5)(x^2 - 7) + 27 = 0$.

$$\therefore x^4 - 2x^2 - 35 + 27 = 0.$$

$$(x^2 + 2)(x^2 - 4) = 0.$$

$$(x^2 + 2)(x + 2)(x - 2) = 0.$$

$$x = \pm \sqrt{-2}, -2, 2.$$

20. $x^3 - x^2 + 2x - 2 = 0$.

$$(x^2 + 2)(x - 1) = 0.$$

$$x = 1, \pm \sqrt{-2}.$$

21. $x^6 + 7x^3 - 8 = 0$.

$$(x^3 - 1)(x^3 + 8) = 0.$$

$$(x - 1)(x^2 + x + 1)(x + 2)(x^2 - 2x + 4) = 0.$$

$$x = 1, \frac{-1 \pm \sqrt{-3}}{2}, -2, 1 \pm \sqrt{-3}.$$

22. $4x^4 + 20x^2 + 21 = 0$.

$$(2x^2 + 7)(2x^2 + 3) = 0.$$

$$2x^2 + 7 = 0.$$

$$x = \pm \frac{\sqrt{-14}}{2}.$$

$$2x^2 + 3 = 0.$$

$$x = \pm \frac{\sqrt{-6}}{2}.$$

23. $64x^4 - 12x^2 - 27 = 0$.

$$x^2 = \frac{12 \pm \sqrt{144 + 6912}}{128} = \frac{12 \pm \sqrt{7056}}{128} = \frac{12 \pm 84}{128}.$$

$$x^2 = \frac{3}{4}, -\frac{9}{16}.$$

$$x = \pm \frac{1}{2}\sqrt{3}, \pm \frac{3}{4}\sqrt{-1}.$$

$$24. 9x^4 + 18x^2 + 8 = 0.$$

$$(3x^2 + 4)(3x^2 + 2) = 0.$$

$$3x^2 + 4 = 0.$$

$$3x^2 + 2 = 0.$$

$$x = \frac{\pm 2\sqrt{-3}}{3}, \frac{\pm \sqrt{-6}}{3}.$$

$$25. 50x^4 + 135x^2 + 36 = 0.$$

$$(5x^2 + 12)(10x^2 + 3) = 0.$$

$$x = \frac{\pm 2\sqrt{-15}}{5}, \frac{\pm \sqrt{-30}}{10}.$$

$$26. (x^2 + 9)(x^2 + 2x + 8) = 0.$$

$$x^2 + 9 = 0.$$

$$x^2 + 2x + 8 = 0.$$

$$x = \pm 3\sqrt{-1}, -1 \pm \sqrt{-7}.$$

$$27. (x^2 + x)^2 + 13(x^2 + x) + 36 = 0.$$

$$(x^2 + x)^2 + 9(x^2 + x) + 4(x^2 + x) + 36 = 0.$$

$$(x^2 + x)(x^2 + x + 9) + 4(x^2 + x + 9) = 0.$$

$$(x^2 + x + 4)(x^2 + x + 9) = 0.$$

$$x = \frac{-1 \pm \sqrt{-15}}{2}, \frac{-1 \pm \sqrt{-35}}{2}.$$

$$28. (x^2 + 5x)^2 + 17(x^2 + 5x) + 66 = 0.$$

$$(x^2 + 5x + 6)(x^2 + 5x + 11) = 0.$$

$$x = \frac{-5 \pm 1}{2} = -2, -3, \text{ and } \frac{-5 \pm \sqrt{-19}}{2}.$$

$$29. \quad x + y = 4, \quad (1)$$

$$x^2 - 3xy - y^2 = -39. \quad (2)$$

From (1) and (2),

$$16 - 8y + y^2 - 3y(4 - y) - y^2 + 39 = 0. \quad (3)$$

$$3y^2 - 20y + 55 = 0. \quad (4)$$

$$y = \frac{10 \pm \sqrt{-65}}{3}. \quad (5)$$

$$\text{From (1) and (6),} \quad x = 4 - \frac{10 \pm \sqrt{-65}}{3} = \frac{2 \mp \sqrt{-65}}{3}.$$

$$30. \quad z^2 + x^2 = 130. \quad (1)$$

$$z + x + 2\sqrt{z + x} = 2. \quad (2)$$

$$z + x + 2\sqrt{z + x} + 1 = 3. \quad (3)$$

$$\sqrt{z + x} + 1 = \pm \sqrt{3}. \quad (4)$$

$$\sqrt{z + x} = -1 \pm \sqrt{3}. \quad (5)$$

$$z + x = 4 \pm 2\sqrt{3}. \quad 4 + 2\sqrt{3} \text{ is impossible.} \quad (6)$$

Squaring in (6),

$$z^2 + 2zx + x^2 = 28 - 16\sqrt{3}. \quad (7)$$

$$(7) - (1), \quad 2zx = -102 - 16\sqrt{3}. \quad (8)$$

$$(1) - (8), \quad z^2 - 2zx + x^2 = 232 + 16\sqrt{3}. \quad (9)$$

$$z - x = \pm \sqrt{232 + 16\sqrt{3}} = \pm 2\sqrt{58 + 4\sqrt{3}}. \quad (10)$$

$$z + x = 4 - 2\sqrt{3}. \quad (11)$$

$$\text{From (10) and (11),} \quad z = 2 - \sqrt{3} \pm \sqrt{58 + 4\sqrt{3}}. \quad (12)$$

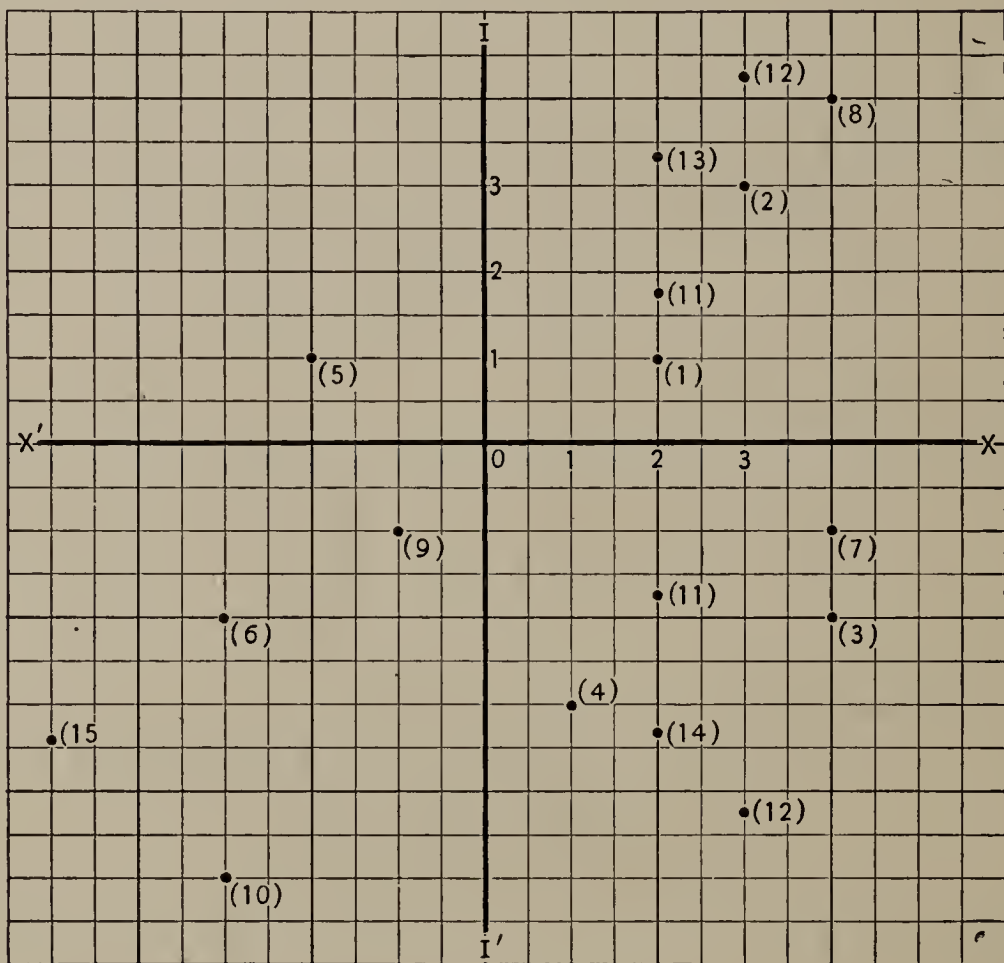
$$\text{From (10) and (11),} \quad x = 2 - \sqrt{3} \mp \sqrt{58 - 4\sqrt{3}}. \quad (13)$$

31. See page 381.

32. See pages 397-398.

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For Exercises 1-15, see the graph below :



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2. $x^2 - 5x + 6 = 0.$

$$b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 6 = 25 - 24 = 1.$$

The roots are real, unequal, and rational.

3. $5x^2 - 11x + 2 = 0.$

$$b^2 - 4ac = (-11)^2 - 4 \cdot 5 \cdot 2 = 121 - 40 = 81.$$

The roots are real, unequal, and rational.

4. $4x^2 - 20x + 25 = 0$.

$$b^2 - 4ac = (-20)^2 - 4 \cdot 4 \cdot 25 = 400 - 400.$$

There is only one root ; it is real and rational.

5. $5x^2 - 3x - 3 = 0$.

$$b^2 - 4ac = (-3)^2 - 4 \cdot 5 \cdot (-3) = 9 + 60 = 69.$$

The roots are real, unequal, and irrational.

6. $7x^2 - 2x + 10 = 0$.

$$b^2 - 4ac = (2)^2 - 4 \cdot 7 \cdot 10 = 4 - 280 = -276.$$

The roots are imaginary.

7. $x^2 - 6x + 6 = 0$.

$$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 6 = 36 - 24 = 12.$$

The roots are real, unequal, and irrational.

8. $4x^2 = 9 - 9x$, or $4x^2 + 9x - 9 = 0$.

$$b^2 - 4ac = 9^2 - 4 \cdot 4 \cdot (-9) = 81 + 144 = 225 = (15)^2.$$

The roots are real, unequal, and rational.

9. $5x = x^2 + 5$, or $-x^2 + 5x - 5 = 0$.

$$b^2 - 4ac = (5)^2 - 4 \cdot (-1) \cdot (-5) = 25 - 20 = 5.$$

The roots are real, unequal, and irrational.

10. $x^2 - 5x + 7 = 0$.

$$b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 7 = 25 - 28 = -3.$$

The roots are imaginary.

11. $12x^2 - 7x + 6 = 0$.

$$b^2 - 4ac = (-7)^2 - 4 \cdot 12 \cdot 6 = 49 - 288 = -239.$$

The roots are imaginary.

12. $x(x - 5) = x - 16$, or $x^2 - 6x + 16 = 0$.

$$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 16 = 36 - 64 = -28.$$

The roots are imaginary.

13. $\frac{3}{x^2} + \frac{11}{x} = 20$, or $20x^2 - 11x - 3 = 0$.

$$b^2 - 4ac = (-11)^2 - 4 \cdot 20 \cdot (-3) = 121 + 240 = 361 = (19)^2.$$

The roots are real, unequal, and rational.

14. $5x - \frac{11}{7} - \frac{6}{7x} = 0$, or $35x^2 - 11x - 6 = 0$.

$$b^2 - 4ac = (-11)^2 - 4 \cdot 35 \cdot (-6) = 121 + 840 = 961 = (31)^2.$$

The roots are real, unequal, and rational.

16. $x^2 - Kx + 36 = 0$.

$$a = 1, b = -K, c = 36.$$

$$b^2 - 4ac = K^2 - 144.$$

In order that there be but one root, $b^2 - 4ac$ must equal 0.

$$\therefore K^2 - 144 = 0.$$

$$K = \pm 12.$$

17. $x^2 - 3Kx + 81 = 0.$

$a = 1, b = -3K, c = 81.$

$b^2 - 4ac = 9K^2 - 324.$

$9K^2 = 324.$

$K = \pm 6.$

18. $2x^2 + 4Kx + 98 = 0.$

$a = 2, b = 4K, c = 98.$

$b^2 - 4ac = 16K^2 - 784.$

$16K^2 = 784.$

$K = \pm 7.$

19. $x^2 - 10x + K = 0.$

$a = 1, b = -10, c = K.$

$b^2 - 4ac = 100 - 4K.$

$K = 25.$

20. $2x^2 + 8x + K = 0.$

$a = 2, b = 8, c = K.$

$b^2 - 4ac = 64 - 8K.$

$K = 8.$

21. $9x^2 + 30x + K + 9 = 0.$

$a = 9, b = 30, c = (K + 9).$

$b^2 - 4ac = 900 - (36K + 324)$

$= 576 - 36K.$

$K = 16.$

22. $4Kx^2 - 60x + 25 = 0.$

$a = 4K, b = -60, c = 25.$

$b^2 - 4ac = 3600 - 400K.$

$K = 9.$

23. $9K^2x^2 - 84x + 49 = 0.$

$a = 9K^2, b = -84, c = 49.$

$b^2 - 4ac = 7056 - 1764K^2.$

$7056 - 1764K^2 = 0.$

$K = \pm 2.$

24. $49x^2 - (K + 3)x + 4 = 0.$

$a = 49, b = -(K + 3), c = 4.$

$b^2 - 4ac = K^2 + 6K + 9 - 784.$

$K^2 + 6K - 775 = 0.$

$K = 25 \text{ or } -31.$

25. $(K^2 + 5)x^2 - 30x + 25 = 0.$

$a = K^2 + 5, b = -30, c = 25.$

$b^2 - 4ac = 900 - (100K^2 + 500).$

$400 - 100K^2 = 0.$

$K = \pm 2.$

26. $(K^2 + 17)x^2 + (5K - 4)x + 4 = 0.$

$a = K^2 + 17, b = 5K - 4, c = 4.$

$b^2 - 4ac = (25K^2 - 40K + 16) - (16K^2 + 272).$

$9K^2 - 40K - 256 = 0.$

$K = 8, \text{ or } -\frac{32}{9}.$

27. $k^2x^2 + 6hx + 9 = 0.$

$a = k^2, b = 6h, c = 9.$

$b^2 - 4ac = 36h^2 - 36k^2.$

$36h^2 - 36k^2 = 0.$

$h^2 = k^2.$

$h = \pm k.$

28. $kx^2 - 2hx + 16 = 0.$

$a = k, b = -2h, c = 16.$

$b^2 - 4ac = 4h^2 - 64k.$

$4h^2 = 64k.$

$h^2 = 16k.$

$h = \pm 4\sqrt{k}.$

29. $x^2 + 4kx + 4h = 0.$

$$a = 1, b = 4k, c = h.$$

$$b^2 - 4ac = 16k^2 - 16h.$$

$$16k^2 - 16h = 0.$$

$$k = \pm \sqrt{h}.$$

30. $kx^2 - 2hx + 6 = 0.$

$$a = k, b = -2h, c = 6.$$

$$b^2 - 4ac = 4h^2 - 24k.$$

$$4h^2 - 24k = 0.$$

$$h = \pm \sqrt{6k}.$$

31. $y^2 = ax. \quad (1)$

$$y = x + 1. \quad (2)$$

Squaring (2), $y^2 = x^2 + 2x + 1. \quad (3)$

(3) - (1), $x^2 + 2x - ax + 1 = 0.$

$$a = 1, b = (2 - a), c = 1.$$

$$b^2 - 4ac = (4 - 4a + a^2) - 4.$$

$$a^2 - 4a = 0.$$

$$a = 0, 4.$$

32. $y^2 = 2x. \quad (1)$

$$y = x + a. \quad (2)$$

Squaring (2), $y^2 = x^2 + 2ax + a^2.$

$$x^2 + 2ax - 2x + a^2 = 0.$$

$$a = 1, b = 2a - 2, c = a^2.$$

$$b^2 - 4ac = 4a^2 - 8a + 4 - 4a^2.$$

$$4 - 8a = 0.$$

$$a = \frac{1}{2}.$$

33. $x^2 + y^2 = a^2. \quad (1)$

$$y = x + 1. \quad (2)$$

Squaring (2), $y^2 = x^2 + 2x + 1. \quad (3)$

(3) - (1), $2x^2 + 2x - a^2 + 1 = 0.$

$$a = 2, b = 2, c = 1 - a^2.$$

$$b^2 - 4ac = 4 - 4 \cdot 2 \cdot (1 - a^2) = 8a^2 - 4.$$

$$8a^2 - 4 = 0.$$

$$a = \pm \frac{\sqrt{2}}{2}.$$

34. $x^2 + y^2 = 2x. \quad (1)$

$$y = x + a. \quad (2)$$

Squaring (2), $y^2 = x^2 + 2ax + a^2. \quad (3)$

(3) - (1), $2x^2 + (2a - 2)x + a^2 = 0.$

$$a = 2, b = 2a - 2, c = a^2.$$

$$b^2 - 4ac = 4a^2 - 8a + 4 - 8a^2.$$

$$a^2 + 2a - 1 = 0.$$

$$a = -1 \pm \sqrt{2}.$$

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2. 2, 7.

 $r_1 + r_2$ with the sign changed is -9 .

$$r_1 \times r_2 = 2 \cdot 7 = 14.$$

$$\therefore x^2 - 9x + 14 = 0.$$

3. $-3, 10$.

$$-(r_1 + r_2) = -(-3 + 10) = -7.$$

$$r_1 \times r_2 = -30.$$

$$\therefore x^2 - 7x - 30 = 0.$$

4. $-4, -5$.

$$-(r_1 + r_2) = -(-4 - 5) = 9.$$

$$r_1 \times r_2 = -4 \cdot (-5) = 20.$$

$$\therefore x^2 + 9x + 20 = 0.$$

5. $-12, -1$.

$$-(r_1 + r_2) = -(-12 - 1) = 13.$$

$$r_1 \times r_2 = -12 \cdot (-1) = 12.$$

$$\therefore x^2 + 13x + 12 = 0.$$

6. $\frac{2}{3}, 5$.

$$-(r_1 + r_2) = -(\frac{2}{3} + 5) = -5\frac{2}{3}, \text{ or } -\frac{17}{3}.$$

$$r_1 \times r_2 = \frac{10}{3}.$$

$$\therefore 3x^2 - 17x + 10 = 0.$$

7. $10, -\frac{1}{5}$.

$$-(r_1 + r_2) = -(10 - \frac{1}{5}) = -9\frac{4}{5}, \text{ or } -\frac{49}{5}.$$

$$r_1 \times r_2 = -\frac{10}{5}.$$

$$\therefore 5x^2 - 49x - 10 = 0.$$

8. $2 + \sqrt{3}, 2 - \sqrt{3}$.

$$-(r_1 + r_2) = -(2 + \sqrt{3} + 2 - \sqrt{3}) = -4.$$

$$r_1 \times r_2 = (2 + \sqrt{3}) \cdot (2 - \sqrt{3}) = 4 - 3 = 1.$$

$$\therefore x^2 - 4x + 1 = 0.$$

9. $-3 \pm \sqrt{5}$.

$$-(r_1 + r_2) = -(-3 + \sqrt{5} - 3 - \sqrt{5}) = 6.$$

$$r_1 \times r_2 = (-3 + \sqrt{5})(-3 - \sqrt{5}) = 4.$$

$$\therefore x^2 + 6x + 4 = 0.$$

10. $\frac{4}{3} \pm \sqrt{7}$.

$$-(r_1 + r_2) = -(\frac{4}{3} + \sqrt{7} + \frac{4}{3} - \sqrt{7}) = -\frac{8}{3}.$$

$$r_1 \times r_2 = (\frac{4}{3} + \sqrt{7})(\frac{4}{3} - \sqrt{7}) = -\frac{47}{9}.$$

$$\therefore 9x^2 - 24x - 47 = 0.$$

11. $\frac{3}{2} \pm \frac{1}{2}\sqrt{6}$.

$$-(r_1 + r_2) = -(\frac{3}{2} + \frac{1}{2}\sqrt{6} + \frac{3}{2} - \frac{1}{2}\sqrt{6}) = -3.$$

$$r_1 \times r_2 = (\frac{3}{2} + \frac{1}{2}\sqrt{6})(\frac{3}{2} - \frac{1}{2}\sqrt{6}) = \frac{3}{4}.$$

$$\therefore 4x^2 - 12x + 3 = 0.$$

$$12. \frac{-6 \pm 2\sqrt{3}}{5}.$$

$$-(r_1 + r_2) = -\left(-\frac{6}{5} + \frac{2\sqrt{3}}{5} - \frac{6}{5} - \frac{2\sqrt{3}}{5}\right) = \frac{12}{5}.$$

$$r_1 \times r_2 = \left(-\frac{6}{5} + \frac{2\sqrt{3}}{5}\right)\left(-\frac{6}{5} - \frac{2\sqrt{3}}{5}\right) = \frac{24}{25}.$$

$$\therefore 25x^2 + 60x + 24 = 0.$$

$$13. \sqrt{5}, -3\sqrt{5}.$$

$$-(r_1 + r_2) = -(\sqrt{5} - 3\sqrt{5}) = 2\sqrt{5}.$$

$$r_1 \times r_2 = \sqrt{5} \cdot (-3\sqrt{5}) = -15.$$

$$\therefore x^2 + 2\sqrt{5}x - 15 = 0.$$

$$14. 3 - \sqrt{2}, 2 + \sqrt{2}.$$

$$-(r_1 + r_2) = -(3 - \sqrt{2} + 2 + \sqrt{2}) = -5.$$

$$r_1 \times r_2 = 4 + \sqrt{2}.$$

$$\therefore x^2 - 5x + (4 + \sqrt{2}) = 0.$$

$$15. a, c.$$

$$-(r_1 + r_2) = -(a + c).$$

$$r_1 \times r_2 = ac.$$

$$\therefore x^2 - ax - cx + ac = 0.$$

$$16. a, \frac{1}{a}.$$

$$-(r_1 + r_2) = -\left(a + \frac{1}{a}\right) = -\frac{a^2 + 1}{a}.$$

$$r_1 \times r_2 = a \cdot \frac{1}{a} = 1.$$

$$\therefore ax^2 - a^2x - x + a = 0.$$

$$17. 3a, \frac{2a}{3}.$$

$$-(r_1 + r_2) = -\left(3a + \frac{2a}{3}\right) = -\frac{11a}{3}.$$

$$r_1 \times r_2 = 3a \cdot \frac{2a}{3} = 2a^2.$$

$$\therefore 3x^2 - 11ax + 6a^2 = 0.$$

$$18. a + 1, \frac{1}{a-1}.$$

$$-(r_1 + r_2) = -\left(a + 1 + \frac{1}{a-1}\right) = -\frac{a^2}{a-1}.$$

$$r_1 \times r_2 = \frac{a+1}{a-1}.$$

$$\therefore ax^2 - x^2 - a^2x + a + 1 = 0.$$

$$19. x^2 - 12x - 13 = 0.$$

$$x = \frac{12 \pm \sqrt{144 + 52}}{2} = \frac{12 \pm 14}{2} = 13 \text{ or } -1.$$

$$-(r_1 + r_2) = -(13 - 1) = -12.$$

$$r_1 \times r_2 = 13 \cdot (-1) = -13.$$

20. $x^2 - 10x + 16 = 0.$

$$x = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm 6}{2} = 8 \text{ or } 2.$$

$$-(r_1 + r_2) = -(8 + 2) = -10.$$

$$r_1 \times r_2 = 16.$$

21. $x^2 + 3x + 3 = 0.$

$$x = \frac{-3 \pm \sqrt{9 - 12}}{2} = \frac{-3 \pm \sqrt{-3}}{2}.$$

$$-(r_1 + r_2) = -\left(\frac{-3 + \sqrt{-3} - 3 - \sqrt{-3}}{2}\right) = 3.$$

$$r_1 \times r_2 = \left(\frac{-3 + \sqrt{-3}}{2}\right)\left(\frac{-3 - \sqrt{-3}}{2}\right) = 3.$$

22. $x^2 - 5x + 20 = 0.$

$$x = \frac{5 \pm \sqrt{25 - 80}}{2} = \frac{5 \pm \sqrt{-55}}{2}.$$

$$-(r_1 + r_2) = -\left(\frac{5 + \sqrt{-55} + 5 - \sqrt{-55}}{2}\right) = -5.$$

$$r_1 \times r_2 = \frac{(5 + \sqrt{-55})}{2} \cdot \frac{(5 - \sqrt{-55})}{2} = 20.$$

23. $x^2 + 2x + 2 = 0.$

$$x = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm \sqrt{-1}.$$

$$-(r_1 + r_2) = -(-1 + \sqrt{-1} - 1 - \sqrt{-1}) = 2.$$

$$r_1 \times r_2 = (-1 + \sqrt{-1})(-1 - \sqrt{-1}) = 2.$$

25. $x^2 + 7x - 18 = 0.$ $r_1 = -9.$

$$-(r_1 + r_2) = -(-9 + r_2) = 7.$$

$$r_2 = 2.$$

26. $x^2 + 2x - c = 0.$ $r_1 = 3.$

$$-(r_1 + r_2) = -(3 + r_2) = 2.$$

$$r_2 = -5.$$

$$r_1 \times r_2 = 2 \cdot (-5) = -10 = c.$$

27. $x^2 - x - c = 0.$ $r_1 = 10.$

$$-(r_1 + r_2) = -(10 + r_2) = -1.$$

$$r_2 = -9.$$

$$r_1 \times r_2 = -90 = c.$$

28. $x^2 + 8x - c = 0.$ $r_1 = -2.$

$$-(r_1 + r_2) = -(-2 + r_2) = 8.$$

$$r_2 = -6.$$

$$r_1 \times r_2 = (-2) \cdot (-6) = 12 = c.$$

$$29. x^2 - cx - 70 = 0. \quad r_1 = 10.$$

$$r_1 \times r_2 = 10 r_2 = -70.$$

$$r_2 = -7.$$

$$c = -(r_1 + r_2) = -3.$$

$$30. x^2 + 2bx + 25 = 0. \quad r_1 = -5.$$

$$r_1 \times r_2 = -5 r_2 = 25.$$

$$r_2 = -5.$$

$$2b = -(-5 - 5) = 10.$$

$$b = 5.$$

$$31. x^2 - 3ax - 52 = 0. \quad r_1 = 4.$$

$$r_1 \times r_2 = 4 r_2 = -52.$$

$$r_2 = -13.$$

$$-3a = -(r_1 + r_2) = -(4 - 13) = 9.$$

$$a = -3.$$

$$32. 2x^2 - 11x + c = 0. \quad r_1 = 5.$$

$$-(r_1 + r_2) = -\frac{11}{2}.$$

$$5 + r_2 = \frac{11}{2}.$$

$$r_2 = \frac{1}{2}.$$

$$r_1 \times r_2 = c.$$

$$c = 5.$$

$$33. ax^2 - 20x + 12 = 0. \quad r_1 = \frac{2}{3}.$$

$$-(r_1 + r_2) = -\frac{20}{a}.$$

$$\frac{2}{3} + r_2 = \frac{20}{a}.$$

$$r_2 = \frac{60 - 2a}{3a}.$$

$$r_1 r_2 = \frac{12}{a}.$$

$$\frac{60 - 2a}{3a} \cdot \frac{2}{3} = \frac{12}{a}.$$

$$a = 3.$$

$$34. ax^2 - 6x - 21. \quad r_1 = -3.$$

$$-(r_1 + r_2) = -\frac{6}{a}.$$

$$-3 + r_2 = \frac{6}{a}.$$

$$r_2 = 3 + \frac{6}{a}.$$

$$r_1 r_2 = -\frac{21}{a}.$$

$$\left(3 + \frac{6}{a}\right)(-3) = -\frac{21}{a}.$$

$$a = \frac{1}{3}.$$

$$35. x^2 - 8x + c = 0. \quad r_1 = 3r_2.$$

$$-(3r_2 + r_2) = -8.$$

$$r_2 = 2.$$

$$c = r_1 \times r_2 = 2 \cdot 6 = 12.$$

$$36. x^2 + 7x + c = 0. \quad r_1 + 1 = r_2.$$

$$-(r_1 + r_1 + 1) = 7.$$

$$r_1 = -4.$$

$$r_2 = -3.$$

$$c = r_1 \times r_2 = 12.$$

$$37. x^2 + 11x + b = 0. \quad r_1 - r_2 = 9.$$

$$-(r_1 + r_1 - 9) = 11.$$

$$r_1 = -1.$$

$$r_2 = -10.$$

$$b = (r_1 \times r_2) = 10.$$

$$38. x^2 - 5x - c = 0. \quad r_1 - r_2 = 7.$$

$$-(r_1 + r_1 - 7) = -5.$$

$$r_1 = 6.$$

$$r_2 = -1.$$

$$c = r_1 \times r_2 = -6.$$

$$39. x^2 - 5x - a = 0. \quad r_1 - r_2 = -13.$$

$$-(r_1 + r_1 + 13) = -5.$$

$$r_1 = -4.$$

$$r_2 = 9.$$

$$a = r_1 \times r_2 = -36.$$

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1. 3, 7.

$$\begin{aligned}(x - 3) &= 0, (x - 7) = 0. \\ \therefore (x - 3)(x - 7) &= 0. \\ x^2 - 10x + 21 &= 0.\end{aligned}$$

2. 4, -5, 6.

$$\begin{aligned}(x - 4) &= 0, (x + 5) = 0, (x - 6) = 0. \\ \therefore (x - 4)(x + 5)(x - 6) &= 0. \\ x^3 - 5x^2 - 26x + 120 &= 0.\end{aligned}$$

3. $1 + \sqrt{3}$, $1 - \sqrt{3}$.

$$\begin{aligned}[x - (1 + \sqrt{3})] &= 0, [x - (1 - \sqrt{3})] = 0. \\ \therefore [x - (1 + \sqrt{3})][x - (1 - \sqrt{3})] &= 0. \\ x^2 - 2x - 2 &= 0.\end{aligned}$$

4. $2 \pm \sqrt{5}$.

$$\begin{aligned}[x - (2 + \sqrt{5})] &= 0, [x - (2 - \sqrt{5})] = 0. \\ \therefore [x - (2 + \sqrt{5})][x - (2 - \sqrt{5})] &= 0. \\ x^2 - 4x - 1 &= 0.\end{aligned}$$

5. $\frac{3 \pm \sqrt{7}}{2}$.

$$\begin{aligned}\left(x - \frac{3 + \sqrt{7}}{2}\right) &= 0, \left(x - \frac{3 - \sqrt{7}}{2}\right) = 0. \\ \therefore \left(x - \frac{3 + \sqrt{7}}{2}\right)\left(x - \frac{3 - \sqrt{7}}{2}\right) &= 0. \\ 2x^2 - 6x + 1 &= 0.\end{aligned}$$

6. 3, -3, 8.

$$\begin{aligned}(x - 3) &= 0, (x + 3) = 0, (x - 8) = 0. \\ \therefore (x - 3)(x + 3)(x - 8) &= 0, \text{ or } x^3 - 8x^2 - 9x + 72 = 0.\end{aligned}$$

7. $1, \frac{3}{2}, -2$.

$$\begin{aligned}(x - 1) &= 0, (x - \frac{3}{2}) = 0, (x + 2) = 0. \\ \therefore (x - 1)(x - \frac{3}{2})(x + 2) &= 0, \text{ or } 2x^3 - x^2 - 7x + 6 = 0.\end{aligned}$$

8. $1 \pm \sqrt{3}$, 3.

$$\begin{aligned}x - (1 + \sqrt{3}) &= 0, x - (1 - \sqrt{3}) = 0, x - 3 = 0. \\ \therefore (x - 1 - \sqrt{3})(x - 1 + \sqrt{3})(x - 3) &= 0. \\ x^3 - 5x^2 + 4x + 6 &= 0.\end{aligned}$$

9. $a + b$, $a - b$.

$$\begin{aligned}x - (a + b) &= 0, x - (a - b) = 0. \\ \therefore (x - a - b)(x - a + b) &= 0. \\ x^2 - 2ax + a^2 - b^2 &= 0.\end{aligned}$$

10. $\frac{1}{a}, 5a.$

$$x - \frac{1}{a} = 0, x - 5a = 0.$$

$$\therefore \left(x - \frac{1}{a}\right)(x - 5a) = 0.$$

$$ax^2 - x - 5a^2x + 5 = 0.$$

11. $3c \pm \sqrt{2a}.$

$$x - (3c + \sqrt{2a}) = 0, x - (3c - \sqrt{2a}) = 0.$$

$$\therefore (x - 3c - \sqrt{2a})(x - 3c + \sqrt{2a}) = 0.$$

$$\therefore x^2 - 6cx + 9c^2 - 2a = 0.$$

12. $\frac{4a \pm \sqrt{3c}}{2}.$

$$x - \frac{4a + \sqrt{3c}}{2} = 0, x - \frac{4a - \sqrt{3c}}{2} = 0.$$

$$\therefore \left(x - \frac{4a + \sqrt{3c}}{2}\right)\left(x - \frac{4a - \sqrt{3c}}{2}\right) = 0.$$

$$4x^2 - 16ax + 16a^2 - 3c = 0.$$

13. $r_1, r_2, r_3.$

$$x - r_1 = 0, x - r_2 = 0, x - r_3 = 0.$$

$$\therefore (x - r_1)(x - r_2)(x - r_3) = 0.$$

$$x^3 - (r_1 + r_2 + r_3)x^2 + (r_1r_2 + r_2r_3 + r_1r_3)x - r_1r_2r_3 = 0.$$

14. $3, 2 \pm \sqrt{a}.$

$$(x - 3) = 0, x - (2 + \sqrt{a}) = 0, x - (2 - \sqrt{a}) = 0.$$

$$\therefore (x - 3)(x - 2 - \sqrt{a})(x - 2 + \sqrt{a}) = 0.$$

$$x^3 - 7x^2 - 8x - ax - 12 + 3a = 0.$$

15. $-5, -7, 6, 8.$

$$x + 5 = 0, x + 7 = 0, x - 6 = 0, x - 8 = 0.$$

$$\therefore (x + 5)(x + 7)(x - 6)(x - 8) = 0.$$

$$x^4 - 2x^3 - 85x^2 + 86x + 1680 = 0.$$

16. $2 \pm \sqrt{3}, 3 \pm \sqrt{2}.$

$$x - (2 + \sqrt{3}) = 0, x - (2 - \sqrt{3}) = 0, x - (3 + \sqrt{2}) = 0, x - (3 - \sqrt{2}) = 0.$$

$$\therefore (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})(x - 3 - \sqrt{2})(x - 3 + \sqrt{2}) = 0.$$

$$x^4 - 10x^3 + 32x^2 - 34x + 7 = 0.$$

17. $1, -2, a \pm \sqrt{a}.$

$$(x - 1) = 0, (x + 2) = 0, x - (a + \sqrt{a}) = 0, x - (a - \sqrt{a}) = 0.$$

$$\therefore (x - 1)(x + 2)(x - a - \sqrt{a})(x - a + \sqrt{a}) = 0.$$

$$x^4 - 2ax^3 + x^3 + a^2x^2 - 3ax^2 - 2x^2 + a^2x + 3ax - 2a^2 + 2a = 0.$$

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1. $x^2 - 3x - 40.$

$$a = 1, \quad b = -3, \quad c = -40.$$

$$b^2 - 4ac = 9 + 160 = 169.$$

The equation has rational factors.

2. $2x^2 + 5x - 7.$

$$a = 2, \quad b = 5, \quad c = -7.$$

$$b^2 - 4ac = 25 + 56 = 81.$$

The equation has rational factors.

3. $7x^2 - 9x + 18.$

$$a = 7, \quad b = -9, \quad c = 18.$$

$$b^2 - 4ac = 81 - 504 = -423.$$

The equation has imaginary factors.

4. $24x^2 - x - 10.$

$$a = 24, \quad b = -1, \quad c = -10.$$

$$b^2 - 4ac = 1 + 960 = 961 = (31)^2.$$

The equation has rational factors.

5. $72x^2 - 17x + 1.$

$$a = 72, \quad b = -17, \quad c = 1.$$

$$b^2 - 4ac = 289 - 288 = 1.$$

The equation has rational factors.

6. $5x^2 + 3x - 20.$

$$a = 5, \quad b = 3, \quad c = -20.$$

$$b^2 - 4ac = 9 + 400 = 409.$$

The equation has irrational factors.

7. $3x^2 - 9x + 28.$

$$a = 3, \quad b = -9, \quad c = 28.$$

$$b^2 - 4ac = 81 - 336 = -255.$$

The equation has imaginary factors.

8. $33h^2 - 233h - 6.$

$$a = 33, \quad b = -233, \quad c = -6.$$

$$b^2 - 4ac = 54,289 + 792 = 55,081.$$

The equation has irrational factors.

9. $x^2 - 2ax + (a^2 - b^2).$

$$a = 1, \quad b = -2a, \quad c = a^2 - b^2.$$

$$\text{The discriminant} = 4a^2 - 4a^2 + 4b^2 = 4b^2.$$

The equation has rational factors.

10. $abx^2 - (b^2 + a^2)x + ab.$

$$a = ab, \quad b = b^2 + a^2, \quad c = ab.$$

$$\text{The discriminant} = b^4 + 2a^2b^2 + b^4 - 4a^2b^2$$

$$= b^4 - 2a^2b^2 + a^4.$$

The equation has rational factors.

12. $x^2 - 7x - 30$.

Let $x^2 - 7x - 30 = 0$.

Solving, $x = \frac{7 \pm \sqrt{49 + 120}}{2} = 10 \text{ or } -3$.

Then $r_1 = 10$, and $r_2 = -3$.

$\therefore x^2 - 7x - 30 = (x - 10)(x + 3)$.

13. $x^2 - 4x - 1$.

Let $x^2 - 4x - 1 = 0$.

$x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}$.

Then $r_1 = 2 + \sqrt{5}$, and $r_2 = 2 - \sqrt{5}$.

$\therefore x^2 - 4x - 1 = (x - 2 - \sqrt{5})(x - 2 + \sqrt{5})$.

14. $x^2 + 2x + 2$.

Let $x^2 + 2x + 2 = 0$.

$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm \sqrt{-1}$.

Then $r_1 = -1 + \sqrt{-1}$, and $r_2 = -1 - \sqrt{-1}$.

$\therefore x^2 + 2x + 2 = (x + 1 - \sqrt{-1})(x + 1 + \sqrt{-1})$.

15. $x^2 + 4x - 9$.

Let $x^2 + 4x - 9 = 0$.

$x = \frac{-4 \pm \sqrt{16 + 36}}{2} = -2 \pm \sqrt{13}$.

Then $r_1 = -2 + \sqrt{13}$, and $r_2 = -2 - \sqrt{13}$.

$\therefore x^2 + 4x - 9 = (x + 2 - \sqrt{13})(x + 2 + \sqrt{13})$.

16. $4x^2 - 12x - 9$.

Let $4x^2 - 12x - 9 = 0$.

$x = \frac{12 \pm \sqrt{144 + 144}}{8} = \frac{3 \pm 3\sqrt{2}}{2}$.

Then $r_1 = \frac{3 + 3\sqrt{2}}{2}$, and $r_2 = \frac{3 - 3\sqrt{2}}{2}$.

$\therefore 4x^2 - 12x - 9 = (2x - 3 - 3\sqrt{2})(2x - 3 + 3\sqrt{2})$.

17. $25x^2 + 20x + 4$.

Let $25x^2 + 20x + 4 = 0$.

$x = \frac{-20 \pm \sqrt{400 - 400}}{50} = -\frac{2}{5}$.

Then $r_1 = -\frac{2}{5}$, and $r_2 = -\frac{2}{5}$.

$\therefore 25x^2 + 20x + 4 = (5x + 2)(5x + 2)$.

18. $6x^2 + 14x - 40$.

Let $6x^2 + 14x - 40 = 0$.

$$x = \frac{-14 \pm \sqrt{196 + 960}}{12} = \frac{-14 \pm 34}{12}$$

$$= \frac{5}{3} \text{ or } -4.$$

Then

$$r_1 = \frac{5}{3}, \text{ and } r_2 = -4.$$

$$\therefore 6x^2 + 14x - 40 = 2(3x - 5)(x + 4).$$

19. $10 - 9x - 9x^2$.

Let $10 - 9x - 9x^2 = 0$.

$$x = \frac{9 \pm \sqrt{81 + 360}}{-18} = -\frac{5}{3} \text{ or } \frac{2}{3}.$$

Then

$$r_1 = -\frac{5}{3}, \text{ and } r_2 = \frac{2}{3}.$$

$$\therefore 10 - 9x - 9x^2 = (-3x - 5)(3x - 2).$$

20. $10x^2 + 12 - 26x$.

Let $10x^2 + 12 - 26x = 0$.

$$x = \frac{26 \pm \sqrt{676 - 480}}{20} = 2 \text{ or } \frac{3}{5}.$$

Then

$$r_1 = 2, \text{ and } r_2 = \frac{3}{5}.$$

$$\therefore 10x^2 + 12 - 26x = 2(x - 2)(5x - 3).$$

21. $x^2 + 7x + 8$.

Let $x^2 + 7x + 8 = 0$.

$$x = \frac{-7 \pm \sqrt{49 - 32}}{2} = \frac{-7 \pm \sqrt{17}}{2}.$$

Then

$$r_1 = \frac{-7 + \sqrt{17}}{2}, \text{ and } r_2 = \frac{-7 - \sqrt{17}}{2}.$$

$$\therefore x^2 + 7x + 8 = \left(\frac{2x + 7 - \sqrt{17}}{2} \right) \left(\frac{2x + 7 + \sqrt{17}}{2} \right).$$

22. $x^2 + x + 1$.

Let $x^2 + x + 1 = 0$.

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

Then

$$r_1 = \frac{-1 + \sqrt{-3}}{2}, \text{ and } r_2 = \frac{-1 - \sqrt{-3}}{2}.$$

$$\therefore x^2 + x + 1 = \left(\frac{2x + 1 - \sqrt{-3}}{2} \right) \left(\frac{2x + 1 + \sqrt{-3}}{2} \right).$$

23. $x^2 + 1$.

Let $x^2 + 1 = 0$.

$$x = \pm \sqrt{-1}.$$

Then

$$r_1 = \sqrt{-1}, \text{ and } r_2 = -\sqrt{-1}.$$

$$\therefore x^2 + 1 = (x - \sqrt{-1})(x + \sqrt{-1}).$$

24. $x^2 + 9$.

Let $x^2 + 9 = 0$.

$$x = \pm 3\sqrt{-1}.$$

Then

$$r_1 = 3\sqrt{-1}, \text{ and } r_2 = -3\sqrt{-1}.$$

$$\therefore x^2 + 9 = (x - 3\sqrt{-1})(x + 3\sqrt{-1}).$$

25. $x^2 - 2ax + a^2 - b$.

Let $x^2 - 2ax + a^2 - b = 0$.

$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b}}{2} = a \pm \sqrt{b}.$$

Then

$$r_1 = a + \sqrt{b}, \text{ and } r_2 = a - \sqrt{b}.$$

$$\therefore x^2 - 2ax + a^2 - b = (x - a - \sqrt{b})(x - a + \sqrt{b}).$$

26. $x^2 + 6ax + 9a^2 - 4b$.

Let $x^2 + 6ax + 9a^2 - 4b = 0$.

$$x = \frac{-6a \pm \sqrt{36a^2 - 36a^2 + 16b}}{2} = -3a \pm 2\sqrt{b}.$$

Then

$$r_1 = -3a + 2\sqrt{b}, \text{ and } r_2 = -3a - 2\sqrt{b}.$$

$$\therefore x^2 + 6ax + 9a^2 - 4b = (x + 3a - 2\sqrt{b})(x + 3a + 2\sqrt{b}).$$

27. $4x^2 + 4ax + a^2 - 4c$.

Let $4x^2 + 4ax + a^2 - 4c = 0$.

$$x = \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 64c}}{8} = \frac{-a \pm 2\sqrt{c}}{2}.$$

Then

$$r_1 = \frac{-a + 2\sqrt{c}}{2}, \text{ and } r_2 = \frac{-a - 2\sqrt{c}}{2}.$$

$$\therefore 4x^2 + 4ax + a^2 - 4c = (2x + a - 2\sqrt{c})(2x + a + 2\sqrt{c}).$$

28. $x^2 - 4ax + 4a^2 + c$.

Let $x^2 - 4ax + 4a^2 + c = 0$.

$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 - 4c}}{2} = 2a \pm \sqrt{-c}.$$

Then

$$r_1 = 2a + \sqrt{-c}, \text{ and } r_2 = 2a - \sqrt{-c}.$$

$$\therefore x^2 - 4ax + 4a^2 + c = (x - 2a - \sqrt{-c})(x - 2a + \sqrt{-c}).$$

29. $ax^2 + bx + c$.

Let $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Then $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, and $r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

$$\begin{aligned} \therefore ax^2 + bx + c &= a \left[x - \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \right] \left[x - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) \right] \\ &= a \left(\frac{2ax + b - \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{2ax + b + \sqrt{b^2 - 4ac}}{2a} \right). \end{aligned}$$

31. $3x^2 - 6xy + 14x - 4y + 8$.

Let $3x^2 - 6xy + 14x - 4y + 8 = 0$.

Then $3x^2 - (6y - 14)x - (4y - 8) = 0$.

$$\begin{aligned} x &= \frac{(6y - 14) \pm \sqrt{(6y - 14)^2 + 4 \cdot 3 \cdot (4y - 8)}}{6} \\ &= \frac{3y - 7 \pm \sqrt{9y^2 - 30y + 25}}{3}. \end{aligned}$$

Then $r_1 = \frac{3y - 7 + 3y - 5}{3} = 2y - 4$.

$$r_2 = \frac{3y - 7 - 3y + 5}{3} = -\frac{2}{3}.$$

$$\begin{aligned} \therefore 3x^2 - 6xy + 14x - 4y + 8 &= 3(x - 2y + 4)(x + \frac{2}{3}) \\ &= (x - 2y + 4)(3x + 2). \end{aligned}$$

32. $x^2 - xy - 2y^2 + 3x - 6y$.

Let $x^2 - xy - 2y^2 + 3x - 6y = 0$.

Then $x^2 - (y - 3)x - 2y^2 - 6y = 0$.

$$x = \frac{(y - 3) \pm \sqrt{(y - 3)^2 + 8y^2 + 24y}}{2} = 2y \text{ or } (-3 - y).$$

Then $r_1 = 2y$, and $r_2 = (-3 - y)$.

$$\therefore x^2 - xy - 2y^2 + 3x - 6y = (x - 2y)(x + 3 + y).$$

33. $x^2 - 4xy - y + 3y^2 - 2 - x$.

Let $x^2 - (4y + 1)x + (3y^2 - y - 2) = 0$.

$$x = \frac{4y + 1 \pm \sqrt{(4y + 1)^2 - 4(3y^2 - y - 2)}}{2}.$$

Then $r_1 = \frac{4y + 1 + \sqrt{4y^2 + 12y + 9}}{2} = \frac{4y + 1 + 2y + 3}{2} = 3y + 2$,

and $r_2 = \frac{4y + 1 - \sqrt{4y^2 + 12y + 9}}{2} = \frac{4y + 1 - 2y - 3}{2} = y - 1$.

$$\therefore x^2 - 4xy - y + 3y^2 - 2 - x = (x - y + 1)(x - 3y - 2).$$

34. $x^2 - 2y^2 - xy + 2x + 5y - 3.$

Let $x^2 - (y - 2)x - (2y^2 - 5y + 3) = 0.$

$$x = \frac{y - 2 \pm \sqrt{(y - 2)^2 + 4(2y^2 - 5y + 3)}}{2} = 2y - 3 \text{ or } -y + 1.$$

Then $r_1 = (2y - 3), \text{ and } r_2 = (-y + 1).$

$$\therefore x^2 - 2y^2 - xy + 2x + 5y - 3 = (x - 2y + 3)(x + y - 1).$$

35. $6x^2 + xy - 12y^2 + x + 10y - 2.$

Let $6x^2 + (y + 1)x - (12y^2 - 10y + 2) = 0.$

$$\begin{aligned} x &= \frac{-(y + 1) \pm \sqrt{(y + 1)^2 + 24(12y^2 - 10y + 2)}}{12} \\ &= \frac{-(y + 1) \pm \sqrt{289y^2 - 238y + 49}}{12}. \end{aligned}$$

Then $r_1 = \frac{-y - 1 + 17y - 7}{12} = \frac{4y - 2}{3},$

and $r_2 = \frac{-y - 1 - 17y + 7}{12} = \frac{-3y + 1}{2}.$

$$\begin{aligned} \therefore 6x^2 + xy - 12y^2 + x + 10y - 2 &= 6\left(x - \frac{4y - 2}{3}\right)\left(x - \frac{-3y + 1}{2}\right) \\ &= (3x - 4y + 2)(2x + 3y - 1). \end{aligned}$$

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$$\begin{aligned} 1. \quad (a + b)^6 &= a^6 + 6a^5b + \frac{30}{\underline{2}}a^4b^2 + \frac{120}{\underline{3}}a^3b^3 + \frac{360}{\underline{4}}a^2b^4 + \frac{720}{\underline{5}}ab^5 \\ &\quad + \frac{720}{\underline{6}}b^6 \\ &= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6. \end{aligned}$$

$$\begin{aligned} 2. \quad (a - 1)^6 &= a^6 - 6a^5 \cdot 1 + \frac{30}{\underline{2}}a^4 \cdot 1^2 - \frac{120}{\underline{3}}a^3 \cdot 1^3 \\ &\quad + \frac{360}{\underline{4}}a^2 \cdot 1^4 - \frac{720}{\underline{5}}a \cdot 1^5 + \frac{720}{\underline{6}} \cdot 1^6 \\ &= a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1. \end{aligned}$$

$$\begin{aligned} 3. \quad (a + 1)^7 &= a^7 + 7a^6 \cdot 1 + \frac{42}{\underline{2}}a^5 \cdot 1^2 + \frac{210}{\underline{3}}a^4 \cdot 1^3 + \frac{840}{\underline{4}}a^3 \cdot 1^4 \\ &\quad + \frac{2520}{\underline{5}}a^2 \cdot 1^5 + \frac{5040}{\underline{6}}a \cdot 1^6 + \frac{5040}{\underline{7}} \cdot 1^7 \\ &= a^7 + 7a^6 + 21a^5 + 35a^4 + 35a^3 + 21a^2 + 7a + 1 \end{aligned}$$

4.
$$\begin{aligned}(a + 2)^6 &= a^6 + 6a^5 \cdot 2 + \frac{30}{\underline{2}}a^4 \cdot 2^2 + \frac{120}{\underline{3}}a^3 \cdot 2^3 + \frac{360}{\underline{4}}a^2 \cdot 2^4 \\ &\quad + \frac{720}{\underline{5}}a \cdot 2^5 + \frac{720}{\underline{6}} \cdot 2^6 \\ &= a^6 + 12a^5 + 60a^4 + 160a^3 + 240a^2 + 192a + 64.\end{aligned}$$
5.
$$\begin{aligned}(a + 3)^7 &= a^7 + 7a^6 \cdot 3 + \frac{42}{\underline{2}}a^5 \cdot 3^2 + \frac{210}{\underline{3}}a^4 \cdot 3^3 + \frac{840}{\underline{4}}a^3 \cdot 3^4 \\ &\quad + \frac{2520}{\underline{5}}a^2 \cdot 3^5 + \frac{5040}{\underline{6}}a \cdot 3^6 + \frac{5040}{\underline{7}} \cdot 3^7 \\ &= a^7 + 21a^6 + 189a^5 + 945a^4 + 2835a^3 + 5103a^2 \\ &\quad + 5103a + 2187.\end{aligned}$$
6.
$$\begin{aligned}(2 - a)^6 &= 2^6 - 6 \cdot 2^5 a + \frac{30}{\underline{2}} \cdot 2^4 a^2 - \frac{120}{\underline{3}} \cdot 2^3 a^3 \\ &\quad + \frac{360}{\underline{4}} \cdot 2^2 a^4 - \frac{720}{\underline{5}} \cdot 2 a^5 + \frac{720}{\underline{6}} a^6 \\ &= 64 - 192a + 240a^2 - 160a^3 + 60a^4 - 12a^5 + a^6.\end{aligned}$$
7.
$$\begin{aligned}(a + b)^{20} &= a^{20} + 20a^{19}b + \frac{380}{\underline{2}}a^{18}b^2 + \frac{6840}{\underline{3}}a^{17}b^3 + \dots \\ &= a^{20} + 20a^{19}b + 190a^{18}b^2 + 1140a^{17}b^3 + \dots.\end{aligned}$$
8.
$$\begin{aligned}(a + b)^{30} &= a^{30} + 30a^{29}b + \frac{870}{\underline{2}}a^{28}b^2 + \frac{24360}{\underline{3}}a^{27}b^3 + \dots \\ &= a^{30} + 30a^{29}b + 435a^{28}b^2 + 4060a^{27}b^3 + \dots.\end{aligned}$$
9.
$$\begin{aligned}(a + 1)^{40} &= a^{40} + 40a^{39} \cdot 1 + \frac{1560}{\underline{2}}a^{38} \cdot 1^2 + \frac{59280}{\underline{3}}a^{37} \cdot 1^3 + \dots \\ &= a^{40} + 40a^{39} + 780a^{38} + 9880a^{37} + \dots.\end{aligned}$$
10.
$$\begin{aligned}(a - 2)^{20} &= a^{20} - 20a^{19} \cdot 2 + \frac{380}{\underline{2}}a^{18} \cdot 2^2 - \frac{6840}{\underline{3}}a^{17} \cdot 2^3 + \dots \\ &= a^{20} - 40a^{19} + 760a^{18} - 9120a^{17} + \dots.\end{aligned}$$
11.
$$\begin{aligned}(a^2 + 2b)^5 &= (a^2)^5 + 5(a^2)^4(2b) + \frac{20}{\underline{2}}(a^2)^3(2b)^2 + \frac{60}{\underline{3}}(a^2)^2(2b)^3 \\ &\quad + \frac{120}{\underline{4}}(a^2)(2b)^4 + \frac{120}{\underline{5}}(2b)^5 \\ &= a^{10} + 10a^8b + 40a^6b^2 + 80a^4b^3 + 80a^2b^4 + 32b^5.\end{aligned}$$

$$\begin{aligned}
 12. \quad (a^2 - 2)^6 &= (a^2)^6 - 6(a^2)^5 \cdot 2 + \frac{30}{\underline{2}}(a^2)^4 \cdot 2^2 - \frac{120}{\underline{3}}(a^2)^3 \cdot 2^3 \\
 &\quad + \frac{360}{\underline{4}}(a^2)^2 \cdot 2^4 - \frac{720}{\underline{5}}(a^2) \cdot 2^5 + \frac{720}{\underline{6}} \cdot 2^6 \\
 &= a^{12} - 12a^{10} + 60a^8 - 160a^6 + 240a^4 - 192a^2 + 64.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad (a^2 + 2b)^7 &= (a^2)^7 + 7(a^2)^6(2b) + \frac{42}{\underline{2}}(a^2)^5(2b)^2 + \frac{210}{\underline{3}}(a^2)^4(2b)^3 \\
 &\quad + \frac{840}{\underline{4}}(a^2)^3(2b)^4 + \frac{2520}{\underline{5}}(a^2)^2(2b)^5 \\
 &\quad + \frac{5040}{\underline{6}}(a^2)(2b)^6 + \frac{5040}{\underline{7}}(2b)^7 \\
 &= a^{14} + 14a^{12}b + 84a^{10}b^2 + 280a^8b^3 + 560a^6b^4 \\
 &\quad + 672a^4b^5 + 448a^2b^6 + 128b^7.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \left(a^2 + \frac{1}{b}\right)^5 &= (a^2)^5 + 5(a^2)^4\left(\frac{1}{b}\right)^1 + \frac{20}{\underline{2}}(a^2)^3\left(\frac{1}{b}\right)^2 + \frac{60}{\underline{3}}(a^2)^2\left(\frac{1}{b}\right)^3 \\
 &\quad + \frac{120}{\underline{4}}(a^2)^1\left(\frac{1}{b}\right)^4 + \frac{120}{\underline{5}}\left(\frac{1}{b}\right)^5 \\
 &= a^{10} + \frac{5a^8}{b} + \frac{10a^6}{b^2} + \frac{10a^4}{b^3} + \frac{5a^2}{b^4} + \frac{1}{b^5}.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \left(a^2 - \frac{1}{a^3}\right)^6 &= (a^2)^6 - 6(a^2)^5\left(\frac{1}{a^3}\right)^1 + \frac{30}{\underline{2}}(a^2)^4\left(\frac{1}{a^3}\right)^2 - \frac{120}{\underline{3}}(a^2)^3\left(\frac{1}{a^3}\right)^3 \\
 &\quad + \frac{360}{\underline{4}}(a^2)^2\left(\frac{1}{a^3}\right)^4 - \frac{720}{\underline{5}}(a^2)^1\left(\frac{1}{a^3}\right)^5 + \frac{720}{\underline{6}}\left(\frac{1}{a^3}\right)^6 \\
 &= a^{12} - 6a^7 + 15a^2 - \frac{20}{a^3} + \frac{15}{a^8} - \frac{6}{a^{13}} + \frac{1}{a^{18}}.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad (a^2 + 2b)^{20} &= (a^2)^{20} + 20(a^2)^{19}(2b)^1 + \frac{380}{\underline{2}}(a^2)^{18}(2b)^2 \\
 &\quad + \frac{6840}{\underline{3}}(a^2)^{17}(2b)^3 + \dots \\
 &= a^{40} + 40a^{38}b + 760a^{36}b^2 + 9120a^{34}b^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \left(a^2 - \frac{2}{a}\right)^{30} &= (a^2)^{30} - 30(a^2)^{29}\left(\frac{2}{a}\right)^1 \\
 &\quad + \frac{870}{\underline{2}}(a^2)^{28}\left(\frac{2}{a}\right)^2 - \frac{24360}{\underline{3}}(a^2)^{27}\left(\frac{2}{a}\right)^3 + \dots \\
 &= a^{60} - 60a^{57} + 1740a^{54} - 32,480a^{51} + \dots
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \left(\frac{a}{b} + \frac{3b}{a}\right)^{20} &= \left(\frac{a}{b}\right)^{20} + 20\left(\frac{a}{b}\right)^{19}\left(\frac{3b}{a}\right)^1 + \frac{380}{2}\left(\frac{a}{b}\right)^{18}\left(\frac{3b}{a}\right)^2 \\
 &\quad + \frac{6840}{3}\left(\frac{a}{b}\right)^{17}\left(\frac{3b}{a}\right)^3 + \dots \\
 &= \frac{a^{20}}{b^{20}} + 60\frac{a^{18}}{b^{18}} + 1710\frac{a^{16}}{b^{16}} + 30,780\frac{a^{14}}{b^{14}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \left(\frac{2x}{y^3} - \frac{y^4}{6x^5}\right)^7 &= \left(\frac{2x}{y^3}\right)^7 - 7\left(\frac{2x}{y^3}\right)^6\left(\frac{y^4}{6x^5}\right)^1 \\
 &\quad + \frac{42}{2}\left(\frac{2x}{y^3}\right)^5\left(\frac{y^4}{6x^5}\right)^2 - \frac{210}{3}\left(\frac{2x}{y^3}\right)^4\left(\frac{y^4}{6x^5}\right)^3 + \dots \\
 &= 128\frac{x^7}{y^{21}} - \frac{224x}{3y^{14}} + \frac{56}{3x^5y^7} - \frac{70}{27x^{11}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 20. \quad (a^2 - 3b^2)^{10} &= (a^2)^{10} - 10(a^2)^9(3b^2)^1 \\
 &\quad + \frac{90}{2}(a^2)^8(3b^2)^2 - \frac{720}{3}(a^2)^7(3b^2)^3 + \dots \\
 &= a^{20} - 30a^{18}b^2 + 405a^{16}b^4 - 3240a^{14}b^6 + \dots
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \left(\frac{3x^5}{y^3} - \frac{2y^{15}}{9x^{12}}\right)^6 &= \left(\frac{3x^5}{y^3}\right)^6 - 6\left(\frac{3x^5}{y^3}\right)^5\left(\frac{2y^{15}}{9x^{12}}\right)^1 \\
 &\quad + \frac{30}{2}\left(\frac{3x^5}{y^3}\right)^4\left(\frac{2y^{15}}{9x^{12}}\right)^2 - \frac{120}{3}\left(\frac{3x^5}{y^3}\right)^3\left(\frac{2y^{15}}{9x^{12}}\right)^3 \dots \\
 &= \frac{729x^{30}}{y^{18}} - 324x^{13} + \frac{60y^{18}}{x^4} - \frac{160y^{36}}{27x^{21}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \left(\frac{a^2}{b^3} - \frac{2b^2}{a^4}\right)^{12} &= \left(\frac{a^2}{b^3}\right)^{12} - 12\left(\frac{a^2}{b^3}\right)^{11}\left(\frac{2b^2}{a^4}\right)^1 \\
 &\quad + \frac{132}{2}\left(\frac{a^2}{b^3}\right)^{10}\left(\frac{2b^2}{a^4}\right)^2 - \frac{1320}{3}\left(\frac{a^2}{b^3}\right)^9\left(\frac{2b^2}{a^4}\right)^3 + \dots \\
 &= \frac{a^{24}}{b^{36}} - \frac{12a^{22} \cdot 2b^2}{b^{33}a^4} + \frac{132a^{20} \cdot 4b^4}{2b^{30}a^8} - \frac{1320a^{18} \cdot 8b^6}{6b^{27}a^{12}} + \dots \\
 &= \frac{a^{24}}{b^{36}} - 24\frac{a^{18}}{b^{31}} + 264\frac{a^{12}}{b^{26}} - 1760\frac{a^6}{b^{21}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \left(\frac{\sqrt{x}}{y} + \frac{\sqrt{y}}{x}\right)^{12} &= \left(\frac{\sqrt{x}}{y}\right)^{12} + 12\left(\frac{\sqrt{x}}{y}\right)^{11}\left(\frac{\sqrt{y}}{x}\right)^1 + \frac{132}{2}\left(\frac{\sqrt{x}}{y}\right)^{10}\left(\frac{\sqrt{y}}{x}\right)^2 \\
 &\quad + \frac{1320}{3}\left(\frac{\sqrt{x}}{y}\right)^9\left(\frac{\sqrt{y}}{x}\right)^3 + \dots \\
 &= \frac{x^6}{y^{12}} + 12\frac{x^{\frac{9}{2}}}{y^{\frac{21}{2}}} + 66\frac{x^3}{y^9} + 220\frac{x^{\frac{3}{2}}}{y^{\frac{15}{2}}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 24. \quad (a+b)^n &= a^n + na^{n-1}b + \frac{n(n-1)}{\underline{2}} a^{n-2}b^2 \\
 &+ \frac{n(n-1)(n-2)}{\underline{3}} a^{n-3}b^3 \\
 &+ \frac{n(n-1)(n-2)(n-3)}{\underline{4}} a^{n-4}b^4 \\
 &+ \frac{n(n-1)(n-2)(n-3)(n-4)}{\underline{5}} a^{n-5}b^5 + \dots \\
 (a+b)^3 &= a^3 + 3a^{3-1}b + \frac{3(3-1)}{\underline{2}} a^{3-2}b^2 \\
 &+ \frac{3(3-1)(3-2)}{\underline{3}} a^{3-3}b^3 + 0 = a^3 + 3a^2b + 3ab^2 + b^3.
 \end{aligned}$$

For $n = 1, 2, \text{ or } 4$, $(a+b)^n = a+b$, $a^2 + 2ab + b^2$,
and $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$, respectively.

The number of terms in each expansion is $n+1$.

The value of each coefficient after the $(n+1)$ st is zero.

The expansion does not extend to more than six terms when $n = 5$, because $(5-5)$ or 0 appears as a factor in each coefficient after the sixth.

$$\begin{aligned}
 25. \quad \left(1 + \frac{1}{n}\right)^n &= 1^n + n \cdot 1^{n-1} \left(\frac{1}{n}\right)^1 + \frac{n(n-1)}{\underline{2}} \cdot 1^{n-2} \left(\frac{1}{n}\right)^2 \\
 &+ \frac{n(n-1)(n-2)}{\underline{3}} \cdot 1^{n-3} \left(\frac{1}{n}\right)^3 + \dots \\
 &= 1 + 1 + \frac{n(n-1)}{n^2 \underline{2}} + \frac{n(n-1)(n-2)}{n^3 \underline{3}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 26. \quad (1 + .1)^{10} &= 1^{10} + 10 \cdot 1^9 \cdot .1 + \frac{90}{\underline{2}} \cdot 1^8 \cdot .1^2 + \frac{720}{\underline{3}} \cdot 1^7 \cdot .1^3 \\
 &+ \frac{5040}{\underline{4}} \cdot 1^6 \cdot .1^4 + \dots \\
 &= 1 + 1 + .450 + .120 + .021 + \dots = 2.59+.
 \end{aligned}$$

$$\begin{aligned}
 27. \quad (1 - .02)^{11} &= 1^{11} - 11 \cdot 1^{10} \cdot .02 + \frac{110}{\underline{2}} \cdot 1^9 \cdot (.02)^2 - \frac{990}{\underline{3}} \cdot 1^8 \cdot (.02)^3 + \dots \\
 &= 1 - .220 + .022 - .00132 + \dots = .80+.
 \end{aligned}$$

$$\begin{aligned}
 28. \quad (3 - .1)^8 &= 3^8 - 8 \cdot 3^7 \cdot .1 + \frac{56}{\underline{2}} 3^6 \cdot .1^2 - \frac{336}{\underline{3}} 3^5 \cdot .1^3 \\
 &+ \frac{1680}{\underline{4}} 3^4 \cdot .1^4 - \frac{6720}{\underline{5}} 3^3 \cdot .1^5 + \dots \\
 &= 6561 - 1749.6 + 204.12 - 13.608 + .567 - .01512 + \dots \\
 &= 5002.46+.
 \end{aligned}$$

$$29. (1 + .06)^6 = 1^6 + 6 \cdot 1^5 \cdot .06 + \frac{30}{2} 1^4 (.06)^2 + \dots$$

$$= 1 + .360 + .054 + \dots = 1.41+.$$

$$30. \underline{6} = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720.$$

$$31. \underline{2} \cdot \underline{4} = 2 \cdot 4 \cdot 3 \cdot 2 = 48.$$

$$32. \underline{6} \div \underline{3} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2} = 120.$$

$$33. \underline{4} - \underline{3} \cdot \underline{2} \cdot \underline{2} = 24 - 24 = 0.$$

$$34. 2 + \frac{1}{\underline{2}} + \frac{1}{\underline{3}} + \frac{1}{\underline{4}} + \frac{1}{\underline{5}} = 2 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = \frac{326}{120} = 2\frac{43}{60} = 2.71+.$$

$$35. \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{\underline{7}} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 6435.$$

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$$1. (26)^{\frac{1}{2}} = (25 + 1)^{\frac{1}{2}} = 25^{\frac{1}{2}} + \frac{1}{2} \cdot 25^{-\frac{1}{2}} \cdot 1 - \frac{1}{8} \cdot 25^{-\frac{3}{2}} \cdot 1^2 + \dots$$

$$= 5 + \frac{1}{10} - \frac{1}{1000} + \dots = 5.09+.$$

$$2. (38)^{\frac{1}{2}} = (36 + 2)^{\frac{1}{2}} = 36^{\frac{1}{2}} + \frac{1}{2} \cdot 36^{-\frac{1}{2}} \cdot 2 - \dots$$

$$= 6 + \frac{1}{6} - \dots = 6.16+.$$

$$3. (79)^{\frac{1}{2}} = (81 - 2)^{\frac{1}{2}} = 81^{\frac{1}{2}} - \frac{1}{2} \cdot 81^{-\frac{1}{2}} \cdot 2 + \dots$$

$$= 9 - \frac{1}{9} + \dots$$

$$= 8.88+.$$

$$4. (120)^{\frac{1}{2}} = (121 - 1)^{\frac{1}{2}} = 121^{\frac{1}{2}} - \frac{1}{2} \cdot 121^{-\frac{1}{2}} \cdot 1 + \dots$$

$$= 11 - \frac{1}{22} + \dots$$

$$= 10.95+.$$

$$5. (28)^{\frac{1}{3}} = (27 + 1)^{\frac{1}{3}} = 27^{\frac{1}{3}} + \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 1 - \dots$$

$$= 3 + \frac{1}{27} - \dots$$

$$= 3.03+.$$

$$6. (66)^{\frac{1}{3}} = (64 + 2)^{\frac{1}{3}} = 64^{\frac{1}{3}} + \frac{1}{3} \cdot 64^{-\frac{2}{3}} \cdot 2 - \dots$$

$$= 4 + \frac{1}{24} - \dots$$

$$= 4.04+.$$

$$7. (25)^{\frac{1}{3}} = (27 - 2)^{\frac{1}{3}} = 27^{\frac{1}{3}} - \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 2 + \dots$$

$$= 3 - \frac{2}{27} + \dots$$

$$= 2.92+.$$

$$8. (720)^{\frac{1}{3}} = (729 - 9)^{\frac{1}{3}} = 729^{\frac{1}{3}} - \frac{1}{3} (729)^{-\frac{2}{3}} \cdot 9 + \dots$$

$$= 9 - .037 + \dots$$

$$= 8.96+.$$

$$9. (1 + x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2} \cdot 1^{-\frac{1}{2}} \cdot x - \frac{1}{8} \cdot 1^{-\frac{3}{2}} \cdot x^2 + \frac{1}{16} \cdot 1^{-\frac{5}{2}} \cdot x^3 - \dots$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots.$$

$$10. \quad (2+x)^{\frac{1}{2}} = 2^{\frac{1}{2}} + \frac{1}{2} \cdot 2^{-\frac{1}{2}} \cdot x - \frac{1}{8} \cdot 2^{-\frac{3}{2}} \cdot x^2 + \frac{1}{16} \cdot 2^{-\frac{5}{2}} \cdot x^3 - \dots$$

$$= \sqrt{2} + \frac{x}{2\sqrt{2}} - \frac{x^2}{16\sqrt{2}} + \frac{x^3}{64\sqrt{2}} - \dots$$

$$11. \quad (3-x)^{\frac{1}{2}} = 3^{\frac{1}{2}} - \frac{1}{2} \cdot 3^{-\frac{1}{2}} \cdot x + \frac{1}{8} \cdot 3^{-\frac{3}{2}} \cdot x^2 - \frac{1}{16} \cdot 3^{-\frac{5}{2}} \cdot x^3 + \dots$$

$$= \sqrt{3} - \frac{x}{2\sqrt{3}} + \frac{x^2}{24\sqrt{3}} - \frac{x^3}{144\sqrt{3}} + \dots$$

$$12. \quad (1+x)^{\frac{1}{3}} = 1^{\frac{1}{3}} + \frac{1}{3} \cdot 1^{-\frac{2}{3}} \cdot x - \frac{1}{9} \cdot 1^{-\frac{5}{3}} \cdot x^2 + \frac{5}{81} \cdot 1^{-\frac{8}{3}} \cdot x^3 - \dots$$

$$= 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \dots$$

$$13. \quad (2+x)^{\frac{1}{3}} = 2^{\frac{1}{3}} + \frac{1}{3} \cdot 2^{-\frac{2}{3}} \cdot x - \frac{1}{9} \cdot 2^{-\frac{5}{3}} \cdot x^2 + \frac{5}{81} \cdot 2^{-\frac{8}{3}} \cdot x^3 - \dots$$

$$= \sqrt[3]{2} + \frac{x}{3\sqrt[3]{4}} - \frac{x^2}{18\sqrt[3]{4}} + \frac{5x^3}{324\sqrt[3]{4}} - \dots$$

$$14. \quad (3-x)^{\frac{1}{3}} = 3^{\frac{1}{3}} - \frac{1}{3} \cdot 3^{-\frac{2}{3}} \cdot x + \frac{1}{9} \cdot 3^{-\frac{5}{3}} \cdot x^2 - \frac{5}{81} \cdot 3^{-\frac{8}{3}} \cdot x^3 + \dots$$

$$= \sqrt[3]{3} - \frac{x}{3\sqrt[3]{9}} + \frac{x^2}{27\sqrt[3]{9}} - \frac{5x^3}{729\sqrt[3]{9}} + \dots$$

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$$2. \quad 6\text{th term of } (a+b)^9 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 4 \cdot 3 \cdot 2} a^4 b^5 = 126 a^4 b^5.$$

$$3. \quad 4\text{th term of } (a+b)^{20} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2} a^{17} b^3 = 1140 a^{17} b^3.$$

$$4. \quad 7\text{th term of } (a-b)^{10} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^4 b^6 = 210 a^4 b^6.$$

$$5. \quad 8\text{th term of } (a-b)^{15} = -\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^8 b^7 = -6435 a^8 b^7.$$

$$6. \quad 4\text{th term of } \left(a + \frac{1}{a}\right)^{30} = \frac{30 \cdot 29 \cdot 28}{3 \cdot 2} a^{27} \left(\frac{1}{a}\right)^3 = 4060 a^{24}.$$

$$7. \quad 5\text{th term of } (a^2 - b)^{20} = \frac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2} (a^2)^{16} b^4 = 4845 a^{32} b^4.$$

$$8. \quad 6\text{th term of } \left(\frac{a}{b} - \frac{b^2}{a}\right)^{18} = -\frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14}{5 \cdot 4 \cdot 3 \cdot 2} \left(\frac{a}{b}\right)^{13} \left(\frac{b^2}{a}\right)^5 = -8658 \frac{a^8}{b^3}.$$

$$9. \quad 7\text{th term of } \left(\frac{a^2}{b} - \frac{2b^2}{a}\right)^{14} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \left(\frac{a^2}{b}\right)^8 \left(\frac{2b^2}{a}\right)^6$$

$$= 192,192 a^{10} b^4.$$

$$10. \quad 9\text{th term of } (x^2 - x)^{16} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} (x^2)^8 x^8$$

$$= 12,870 x^{24}.$$

11. 5th term of $\left(\sqrt{x} - \sqrt{\frac{y}{x}}\right)^{15} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2} (\sqrt{x})^{11} \left(\sqrt{\frac{y}{x}}\right)^4$
 $= 1365 x^{\frac{7}{2}} y^2.$

12. If $x^{r-1} = x^5$, $r = 6$. The coefficient of $x^5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2} = 252.$

13. If $(x^2)^{r-1} = x^8$, $r = 5$. The coefficient of $x^8 = \frac{16 \cdot 15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2} = 1820.$

14. If $(x^3)^{15-r+1} = x^{15}$, $r = 11$. The coefficient of x^{15}
 $= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 3003.$

15. If $(x^2)^{14-r+1} (x^{-1})^{r-1} = x^{10}$, $r = 7$. The coefficient of x^{10}
 $= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 3003.$

16. $(3+1)^{-1} = 3^{-1} - 1 \cdot 3^{-2} \cdot 1 + \frac{2}{2} \cdot 3^{-3} \cdot 1^2 - \frac{6}{3} \cdot 3^{-4} \cdot 1^3 + \dots$
 $= \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$
 $= (\text{by } B, \text{ page 418}) \frac{\frac{1}{3}}{\frac{4}{3}} - \frac{\frac{1}{3}(-\frac{1}{3})^\infty}{\frac{4}{3}} = \frac{1}{4} - 0 = \frac{1}{4}.$

$(1+3)^{-1} = 1^{-1} - 1 \cdot 1^{-2} \cdot 3 + \frac{2}{2} \cdot 1^{-3} \cdot 3^2 - \frac{6}{3} \cdot 1^{-4} \cdot 3^3 + \dots$
 $= 1 - 3 + 9 - 27 + \dots$
 $= \text{no definite number.}$

17. $(2+1)^{-1} = 2^{-1} - 1 \cdot 2^{-2} + 1 \cdot 2^{-3} - 1 \cdot 2^{-4} + 1 \cdot 2^{-5} - 1 \cdot 2^{-6} + \dots$
 $= \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64} + \dots$
 $= (\text{by } B, \text{ page 418}) \frac{\frac{1}{2} - 0}{1 - (-\frac{1}{2})} = \frac{1}{3}.$

$(1+2)^{-1} = 1^{-1} - 1 \cdot 1^{-2} \cdot 2 + 1 \cdot 1^{-3} \cdot 2^2 - 1 \cdot 1^{-4} \cdot 2^3 + 1 \cdot 1^{-5} \cdot 2^4 - \dots$
 $= 1 - 2 + 4 - 8 + 16 - \dots$
 $= \text{no definite number.}$

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1.

$$\begin{array}{r}
 x^3 + 3x^2 + 3x + 1 \overline{) x + 1} \\
 \underline{x^3} \\
 3 \cdot x^2 = 3x^2 \\
 \underline{3x^2 \div 3x^2 = 1} \\
 3 \cdot x \cdot 1 = 3x \\
 \underline{1^2 = 1} \\
 3x^2 + 3x + 1 \overline{) 3x^2 + 3x + 1}
 \end{array}$$

$$\begin{array}{r}
 2. \qquad \qquad \qquad 8x^3 - 12x^2 + 6x - 1 \overline{) 2x - 1} \\
 \qquad \qquad \qquad \underline{8x^3} \\
 3(2x)^2 = \qquad 12x^2 \quad -12x^2 + 6x - 1 \\
 -12x^2 \div 12x^2 = -1 \\
 3 \cdot 2x \cdot (-1) = \qquad -6x \\
 (-1)^2 = \qquad \qquad 1 \\
 \underline{12x^2 - 6x + 1} \quad -12x^2 + 6x - 1
 \end{array}$$

$$\begin{array}{r}
 3. \qquad \qquad \qquad 27x^3 + 27x^2y + 9xy^2 + y^3 \overline{) 3x + y} \\
 \qquad \qquad \qquad \underline{27x^3} \\
 3(3x)^2 = \qquad 27x^2 \quad 27x^2y + 9xy^2 + y^3 \\
 27x^2y \div 27x^2 = y \\
 3 \cdot 3x \cdot y = \qquad 9xy \\
 \underline{y^2} = \qquad \qquad y^2 \\
 27x^2 + 9xy + y^2 \quad 27x^2y + 9xy^2 + y^3
 \end{array}$$

$$\begin{array}{r}
 4. \qquad \qquad \qquad 64a^3 - 144a^2c + 108ac^2 - 27c^3 \overline{) 4a - 3c} \\
 \qquad \qquad \qquad \underline{64a^3} \\
 3(4a)^2 = \qquad 48a^2 \quad -144a^2c + 108ac^2 - 27c^3 \\
 -144a^2c \div 48a^2 = -3c \\
 3 \cdot 4a(-3c) = \qquad -36ac \\
 (-3c)^2 = \qquad \qquad 9c^2 \\
 \underline{48a^2 - 36ac + 9c^2} \quad -144a^2c + 108ac^2 - 27c^3
 \end{array}$$

$$\begin{array}{r}
 5. \qquad \qquad \qquad x^{12} - 15x^{10} + 75x^8 - 125x^6 \overline{) x^4 - 5x^2} \\
 \qquad \qquad \qquad \underline{x^{12}} \\
 3(x^4)^2 = \qquad 3x^8 \quad -15x^{10} + 75x^8 - 125x^6 \\
 -15x^{10} \div 3x^8 = -5x^2 \\
 3 \cdot x^4(-5x^2) = \qquad -15x^6 \\
 (-5x^2)^2 = \qquad \qquad 25x^4 \\
 \underline{3x^8 - 15x^6 + 25x^4} \quad -15x^{10} + 75x^8 - 125x^6
 \end{array}$$

$$\begin{array}{r}
 6. \qquad \qquad \qquad x^6 - \frac{3x^4}{2} + \frac{3x^2}{4} - \frac{1}{8} \overline{) x^2 - \frac{1}{2}} \\
 \qquad \qquad \qquad \underline{x^6} \\
 3(x^2)^2 = \qquad 3x^4 \quad -\frac{3x^4}{2} + \frac{3x^2}{4} - \frac{1}{8} \\
 -\frac{3x^4}{2} \div 3x^4 = -\frac{1}{2} \\
 3 \cdot x^2 \left(-\frac{1}{2}\right) = \qquad -\frac{3x^2}{2} \\
 \left(-\frac{1}{2}\right)^2 = \qquad \qquad \frac{1}{4} \\
 \underline{3x^4 - \frac{3x^2}{2} + \frac{1}{4}} \quad -\frac{3x^4}{2} + \frac{3x^2}{4} - \frac{1}{8}
 \end{array}$$

7.

$$\begin{array}{r|l}
 \frac{x^3}{27} - \frac{2}{3} + \frac{4}{x^3} - \frac{8}{x^6} & \left| \frac{x}{3} - \frac{2}{x^2} \right. \\
 \hline
 \frac{x^2}{27} & \\
 \hline
 3 \left(\frac{x}{3} \right)^2 = & \frac{x^2}{3} \\
 - \frac{2}{3} \div \frac{x^2}{3} = & - \frac{2}{x^2} \\
 3 \cdot \frac{x}{3} \left(- \frac{2}{x^2} \right) = & - \frac{2}{x} \\
 \left(- \frac{2}{x^2} \right)^2 = & \frac{4}{x^4} \\
 \hline
 \frac{x^2}{3} - \frac{2}{x} + \frac{4}{x^4} & - \frac{2}{3} + \frac{4}{x^3} - \frac{8}{x^6}
 \end{array}$$

8.

$$\begin{array}{r|l}
 \frac{a^3}{c^6} - \frac{3}{c^3} + \frac{3}{a^3} - \frac{c^3}{a^6} & \left| \frac{a}{c^2} - \frac{c}{a^2} \right. \\
 \hline
 \frac{a^3}{c^6} & \\
 \hline
 3 \left(\frac{a}{c^2} \right)^2 = & \frac{3a^2}{c^4} \\
 - \frac{3}{c^3} \div \frac{3a^2}{c^4} = & - \frac{c}{a^2} \\
 3 \cdot \frac{a}{c^2} \left(- \frac{c}{a^2} \right) = & - \frac{3}{ac} \\
 \left(- \frac{c}{a^2} \right)^2 = & \frac{c^2}{a^4} \\
 \hline
 \frac{3a^2}{c^4} - \frac{3}{ac} + \frac{c^2}{a^4} & - \frac{3}{c^3} + \frac{3}{a^3} - \frac{c^3}{a^6}
 \end{array}$$

9.

$$\begin{array}{r|l}
 x^{\frac{3}{2}} - \frac{6x}{a} + \frac{12x^{\frac{1}{2}}}{a^2} - \frac{8}{a^3} & \left| x^{\frac{1}{2}} - \frac{2}{a} \right. \\
 \hline
 x^{\frac{1}{2}} & \\
 \hline
 3 (x^{\frac{1}{2}})^2 = & 3x \\
 - \frac{6x}{a} \div 3x = & - \frac{2}{a} \\
 3 \cdot x^{\frac{1}{2}} \left(- \frac{2}{a} \right) = & - \frac{6x^{\frac{1}{2}}}{a} \\
 \left(- \frac{2}{a} \right)^2 = & \frac{4}{a^2} \\
 \hline
 3x - \frac{6x^{\frac{1}{2}}}{a} + \frac{4}{a^2} & - \frac{6x}{a} + \frac{12x^{\frac{1}{2}}}{a^2} - \frac{8}{a^3}
 \end{array}$$

10.
$$\frac{a^3 + 3a^2b + 3ab^2 + b^3 - 3a^2 - 6ab - 3b^2 + 3a + 3b - 1}{a^3} \mid a + b - 1$$

$$\begin{array}{r|l} 3a^2 = 3a^2 & 3a^2b + 3ab^2 + b^3 - 3a^2 - 6ab - 3b^2 + 3a + 3b - 1 \\ 3a^2b \div 3a^2 = b & \\ 3 \cdot a \cdot b = 3ab & \\ b^2 = b^2 & \\ \hline 3a^2 + 3ab + b^2 & 3a^2b + 3ab^2 + b^3 \\ 3(a+b)^2 = 3a^2 + 6ab + 3b^2 & -3a^2 - 6ab - 3b^2 + 3a + 3b - 1 \\ -3a^2 \div 3a^2 = -1 & \\ 3(a+b)(-1) = & -3a - 3b \\ (-1)^2 = & 1 \\ \hline 3a^2 + 6ab + 3b^2 - 3a - 3b + 1 & -3a^2 - 6ab - 3b^2 + 3a + 3b - 1 \end{array}$$

11.
$$\frac{x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8}{x^6} \mid x^2 - 3x + 2$$

$$\begin{array}{r|l} 3(x^2)^2 = 3x^4 & -9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8 \\ -9x^5 \div 3x^4 = -3x & \\ 3 \cdot x^2(-3x) = & -9x^3 \\ (-3x)^2 = & 9x^2 \\ \hline 3x^4 - 9x^3 + 9x^2 & -9x^5 + 27x^4 - 27x^3 \\ 3(x^2 - 3x)^2 = 3x^4 - 18x^3 + 27x^2 & 6x^4 - 36x^3 + 66x^2 - 36x + 8 \\ 6x^4 \div 3x^4 = 2 & \\ 3(x^2 - 3x) \cdot 2 = & 6x^2 - 18x \\ 2^2 = & 4 \\ \hline 3x^4 - 18x^3 + 33x^2 - 18x + 4 & 6x^4 - 36x^3 + 66x^2 - 36x + 8 \end{array}$$

12. To find the sixth root take the square root of the cube root.

$$\begin{array}{r|l} & x^2 - 4x + 4 \\ x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64 & \\ x^6 & \\ 3(x^2)^2 = 3x^4 & -12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64 \\ -12x^5 \div 3x^4 = -4x & \\ 3 \cdot x^2(-4x) = & -12x^3 \\ (-4x)^2 = & 16x^2 \\ \hline 3x^4 - 12x^3 + 16x^2 & -12x^5 + 48x^4 - 64x^3 \\ 3(x^2 - 4x)^2 = 3x^4 - 24x^3 + 48x^2 & 12x^4 - 96x^3 + 240x^2 - 192x + 64 \\ 12x^4 \div 3x^4 = 4 & \\ 3(x^2 - 4x) \cdot 4 = & 12x^2 - 48x \\ 4^2 = & 16 \\ \hline 3x^4 - 24x^3 + 60x^2 - 48x + 16 & 12x^4 - 96x^3 + 240x^2 - 192x + 64 \end{array}$$

The square roots of $x^2 - 4x + 4 = \pm(x - 2)$.

13.

$$\begin{array}{r}
 1 + 3x \overline{) 1 + x - x^2} \\
 \underline{1} \\
 3 \overline{) 3x} \\
 \underline{3x} \\
 3x \overline{) 3x} \\
 \underline{3x} \\
 x^2 \overline{) x^2} \\
 \underline{x^2} \\
 3 + 3x + x^2 \overline{) 3x + 3x^2 + x^3} \\
 \underline{3 + 6x + 3x^2} \\
 -3x^2 - 3x^3 \overline{) -3x^2 - x^3} \\
 \underline{-3x^2 - 3x^3} \\
 (-x^2)^2 = x^4 \overline{) x^4} \\
 \underline{x^4} \\
 3 + 6x - 3x^3 + x^4 \overline{) -3x^2 - 18x^3 + 3x^5 - 3x^6}
 \end{array}$$

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1.

$$\begin{array}{r}
 15'625 \overline{) 20 + 5 = 25} \\
 \underline{8\ 000} \\
 3(20)^2 = 1200 \overline{) 7\ 625} \\
 \underline{7\ 625} \\
 7625 \div 1200 = 5 \\
 3 \cdot 20 \cdot 5 = 300 \\
 5^2 = 25 \\
 \underline{1525} \overline{) 7\ 625}
 \end{array}$$

2.

$$\begin{array}{r}
 12'167 \overline{) 20 + 3 = 23} \\
 \underline{8\ 000} \\
 3(20)^2 = 1200 \overline{) 4\ 167} \\
 \underline{4\ 167} \\
 4167 \div 1200 = 3 \\
 3 \cdot 20 \cdot 3 = 180 \\
 3^2 = 9 \\
 \underline{1389} \overline{) 4\ 167}
 \end{array}$$

3.

$$\begin{array}{r}
 1'404'928 \overline{) 100 + 10 + 2 = 112} \\
 \underline{1\ 000\ 000} \\
 3(100)^2 = 30000 \overline{) 404\ 928} \\
 \underline{404\ 928} \\
 404928 \div 30000 = 10 \\
 3 \cdot 100 \cdot 10 = 3000 \\
 (10)^2 = 100 \\
 \underline{33100} \overline{) 331\ 000} \\
 \underline{331\ 000} \\
 3(110)^2 = 36300 \overline{) 73\ 928} \\
 \underline{73\ 928} \\
 73928 \div 36300 = 2 \\
 3 \cdot 110 \cdot 2 = 660 \\
 2^2 = 4 \\
 \underline{36964} \overline{) 73\ 928}
 \end{array}$$

4. $13'481'272 \mid 200 + 30 + 8 = 238$

$(200)^3 =$	8 000 000	
$3(200)^2 =$	120000	5 481 272
$5481272 \div 120000 = 30$		
$3 \cdot 200 \cdot 30 =$	18000	
$(30)^2 =$	900	
	138900	4 167 000
$3(230)^2 =$	158700	1 314 272
$1314272 \div 158700 = 8$		
$3 \cdot 230 \cdot 8 =$	5520	
$8^2 =$	64	
	164284	1 314 272

5. $41'063.625 \mid 30 + 4 + .5 = 34.5$

$(30)^3 =$	27 000.000	
$3(30)^2 =$	2700	14 063.625
$14063 \div 2700 = 4$		
$3 \cdot 30 \cdot 4 =$	360	
$4^2 =$	16	
	3076	12 304.000
$3(34)^2 =$	3468	1 759.625
$1759.625 \div 3468 = .5$		
$3 \cdot 34 \cdot .5 =$	51	
$.5^2 =$	.25	
	3519.25	1 759.625

6. $1.052'8 \mid 1 + .01 + .007 = 1.017 +$

$1^3 =$	1	
$3 \cdot 1^2 =$	3	.052 800
$.0528 \div 3 = .01$		
$3 \cdot 1 \cdot .01 =$	.03	
$(.01)^2 =$	.0001	
	3.0301	.030 301
$3(1.01)^2 =$	3.0603	.022 499
$.022499 \div 3.0603 = .007$		
$3 \cdot 1 \cdot .01 \cdot .007 =$	.02121	
$(.007)^2 =$	.000049	
	3.081559	.021 570 913

7.

$.2^3 =$	$.017'3 \mid .2 + .05 + .008 = .258 +$
$3 \cdot .2^2 = .12$	$.008$
$.0093 \div .12 = .05$	$.009\ 300$
$3 \cdot .2 \cdot .05 = .03$	
$(.05)^2 =$	
$\underline{.0025}$	
$.1525$	$.007\ 625$
$3 (.25)^2 = .1875$	$.001\ 675\ 000$
$.001675 \div .1875 = .008$	
$3 \cdot .25 \cdot .008 = .006$	
$(.008)^2 =$	
$\underline{.000064}$	
$.188164$	$.001\ 507\ 312$

8.

$.1^3 =$	$.004'913 \mid .1 + .07 = .17$
$3 \cdot .1^2 = .03$	$.001$
$.003913 \div .03 = .07$	$.003\ 913$
$3 \cdot .1 \cdot .07 = .021$	
$(.07)^2 =$	
$\underline{.0049}$	
$.0559$	$.003\ 913$

9.

$(.03)^3 =$	$.000'062 \mid .03 + .009 = .039 +$
$3 (.03)^2 = .0027$	$.000\ 027$
$.000035 \div .0027 = .009$	$.000\ 035\ 000$
$3 \cdot .03 \cdot .009 = .00081$	
$(.009)^2 =$	
$\underline{.000081}$	
$.003591$	$.000\ 032\ 319$

10.

		<u>35</u> <u>$3 + .2 + .07 + .001 = 3.271 +$</u>
$3^3 =$		27
$3 \cdot 3^2 =$	27.	8.000
$8 \div 27 = .2$		
$3 \cdot 3 \cdot .2 =$	1.8	
$.2^2 =$	<u>.04</u>	
	28.84	5.768
$3(3 \cdot 2)^2 =$	30.72	<u>2.232 000</u>
$2 \cdot 232 \div 30.72 = .07$		
$3 \cdot 3 \cdot 2 \cdot .07 =$	.672	
$(.07)^2 =$	<u>.0049</u>	
	31.3969	2.197 783
$3(3.27)^2 =$	32.0787	<u>.034 217 000</u>
$.034217 \div 32.0787 = .001$		
$3 \cdot 3.27 \cdot .001 =$	.00981	
$(.001)^2 =$	<u>.000001</u>	
	32.088511	<u>.032 088 511</u>

11. $\sqrt[3]{\frac{2}{3}} = \sqrt[3]{.66666 +}$

		<u>.666'666'666</u> <u>$.8 + .07 + .003 = .873 +$</u>
$.8^3 =$		.512
$3 \cdot .8^2 =$	1.92	.154 666 666
$.1546 \div 1.92 = .07$		
$3 \cdot .8 \cdot .07 =$	.168	
$(.07)^2 =$	<u>.0049</u>	
	2.0929	.146 503
$3(.87)^2 =$	2.2707	<u>.008 163 666</u>
$.0081636 \div 2.2707 = .003$		
$3 \cdot .87 \cdot .003 =$	.00783	
$(.003)^2 =$	<u>.000009</u>	
	2.278539	<u>.006 835 587</u>

12. The edge is $\sqrt[3]{5832}$ inches long.

		<u>5'832</u> <u>$10 + 8 = 18$</u>
$(10)^3 =$		1 000
$3(10)^2 =$	300	4 832
$4832 \div 300 = 8$		
$3 \cdot 10 \cdot 8 =$	240	
$8^2 =$	<u>64</u>	
	604	<u>4 832</u>

13. The diagonal is the hypotenuse of a right triangle whose one leg is the edge of the cube and whose other leg is the diagonal of a face of the cube.

$$\begin{array}{r}
 (30)^3 \\
 3(30)^2 = 2700 \\
 19656 \div 2700 = 6 \\
 3 \cdot 30 \cdot 6 = 540 \\
 6^2 = 36 \\
 \hline
 3276
 \end{array}
 \begin{array}{r}
 46'656 \mid 30 + 6 = 36 \\
 27\ 000 \\
 \hline
 19\ 656 \\
 \hline
 19\ 656
 \end{array}$$

$$\text{The diagonal} = \sqrt{(\sqrt{(36)^2 + (36)^2})^2 + (36)^2} = \sqrt{2(36)^2 + (36)^2} = 36\sqrt{3}.$$

$$\begin{array}{r}
 3.00'00'00'00'00 \mid 1.7320 + \\
 1 \\
 2.7 \mid 2.00 \\
 \hline
 1.89 \\
 343 \mid .1100 \\
 \hline
 .1029 \\
 3.462 \mid .007100 \\
 \hline
 .006924 \\
 3.4640 \mid .0001760000
 \end{array}$$

The diagonal equals $36 \cdot 1.7320 + = 62.35 +$ meters.

14. The sixth root equals the square root of the cube root.

$$\begin{array}{r}
 (300)^3 = \\
 3(300)^2 = 270000 \\
 19656000 \div 270000 = 60 \\
 3 \cdot 300 \cdot 60 = 54000 \\
 (60)^2 = 3600 \\
 \hline
 327600
 \end{array}
 \begin{array}{r}
 46'656'000 \mid 300 + 60 = 360 \\
 27\ 000\ 000 \\
 \hline
 19\ 656\ 000 \\
 \hline
 19\ 456\ 000
 \end{array}$$

$$\begin{array}{r}
 3'60.000000 \mid + 18.973 + \\
 1 \\
 28 \mid 260 \\
 \hline
 224 \\
 36.9 \mid 36.00 \\
 \hline
 33.21 \\
 37.87 \mid 2.7900 \\
 \hline
 2.6509 \\
 37.943 \mid .139100 \\
 \hline
 .113829 \\
 .026271
 \end{array}$$

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1. $x^2 + 4x - 7 = 0.$ (1)

Let the roots of (1) be r_1 and r_2 .

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2. \quad (2)$$

But $r_1 + r_2 = -4$, and $r_1r_2 = -7$.

Substituting in (2), $(-4)^2 = r_1^2 + r_2^2 + 2(-7).$

Hence $r_1^2 + r_2^2 = 30.$

2. $x^3 - 3x - 2 = 0.$ (1)

Let the roots of (1) be r_1 and r_2 .

$$(r_1 + r_2)^3 = r_1^3 + r_2^3 + 3r_1r_2(r_1 + r_2). \quad (2)$$

But $r_1 + r_2 = 3$, and $r_1r_2 = -2$.

Substituting in (2), $3^3 = r_1^3 + r_2^3 + 3(-2)3.$

Whence $r_1^3 + r_2^3 = 45.$

3. $x^2 + 7x + 1 = 0.$ (1)

Let the roots of (1) be r_1 and r_2 .

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1r_2}. \quad (2)$$

But $r_1 + r_2 = -7$, and $r_1r_2 = 1$.

Substituting in (2), $\frac{1}{r_1} + \frac{1}{r_2} = -7.$

4. $x^2 - x - 5 = 0.$ (1)

Let the roots of (1) be r_1 and r_2 .

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2. \quad (2)$$

But $r_1 + r_2 = 1$, and $r_1r_2 = -5$.

Substituting in (2), $1^2 = r_1^2 + r_2^2 + 2(-5). \quad (3)$

Whence $r_1^2 + r_2^2 = 11. \quad (4)$

Let the required equation be

$$x^2 + bx + c = 0. \quad (5)$$

Then $b = -(r_1^2 + r_2^2) = -11,$

and $c = (r_1r_2)^2 = 25.$

Hence the required equation is

$$x^2 - 11x + 25 = 0.$$

5. Let r_1 and r_2 be the roots of the equation

$$x^2 + x - 3 = 0.$$

Then $r_1 + r_2 = -1$, and $r_1r_2 = -3.$

Let the required equation be

$$x^2 + bx + c.$$

Then

$$b = -\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = -\frac{r_1 + r_2}{r_1 r_2} = \frac{1}{3},$$

and

$$c = \frac{1}{r_1 r_2} = -\frac{1}{3}.$$

Hence the required equation is

$$x^2 + \frac{1}{3}x - \frac{1}{3} = 0.$$

6. If r_1 and r_2 are the roots of the equation

$$x^2 - 3x + 5 = 0,$$

$$r_1 + r_2 = 3, \text{ and } r_1 r_2 = 5.$$

$$r_1^2 r_2 + r_1 r_2^2 = r_1 r_2 (r_1 + r_2). \quad (1)$$

Substituting in (1), $r_1^2 r_2 + r_1 r_2^2 = 5 \cdot 3 = 15.$

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